Math Fundamentals for Statistics (Math 95)

Homework Unit 2: Probability

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"The 'How' and 'Whys' Guys"

2.1: Playing a New Game

Vocabulary and symbols – write out what the following mean:

- Full House
- Straight
- 3 of a kind

Concept questions:

1. What must you score on the top section to get the bonus?
2. With this bonus, what does that equate to for each category on the top section?
3. If you had to put a 0 down in the ones place (top section), could you still score the bonus? How?
4. If you had to put a 0 down in the sixes place (top section), could you still score the bonus? Explain.

Exercises:

5. If you rolled all 3 times and ended with 1-1-1-3-5, with a fully open board, which of these locations would be the best place to write your score (explain your answer):
   a. Ones
   b. Threes
   c. Fives
   d. 3-of-a-kind
   e. Chance
   f. Yahtzee

6. Some people play where they leave the Yahtzee open as long as possible, while others fight to get the top bonus and will fill in the Yahtzee with a 0 if needed, even early in the game. Explain why both can be winning strategies.

7. If you ended with 5-5-5-6-6, would you score it under 3 of a kind or full house? Explain.

8. If you rolled 1-1-1-3-5 on your first roll, with a totally open score-card, which die or dice would you re-roll? Explain.

Wrap-up and look back:

9. Martin only has open the 3 of a kind and Fives. He rolls 1-1-5-3-2. Which dice should he roll again? Explain.

10. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.2: Introduction to Probability

Vocabulary and symbols – write out what the following mean:

- Probability
- Procedure
- Outcome
- Event
- Simple Event
- Sample Space
- Impossible Event
- Certain Event
- Equally Likely Outcomes
- Quantify

Concept questions:

1. When you run an experiment or procedure, do you end up with events or outcomes? Explain.
2. Give an example of a procedure that doesn't produce equally likely outcomes.
3. A bag has 5 red marbles, 2 green marbles and 3 blue marbles. Marylou said that with 3 outcomes, the probability of drawing red is $P(R)=\frac{1}{3}$ because red is one of the 3 outcomes, so you put 1 on top and 3 on bottom. Is Marylou correct? Is her reasoning correct? Why?
4. A bag has 5 red marbles, 2 green marbles and 8 blue marbles. Marylou said that with 3 outcomes, the probability of drawing red is $P(R)=\frac{1}{3}$ because red is one of the 3 outcomes, so you put 1 on top and 3 on bottom. Is Marylou correct? Is her reasoning correct? Why?
5. If you were designing a game where one outcome was very challenging, would you make that worth more points or less points? Why?
6. If you were playing Yahtzee and rolled 5-5-5-2-1 on your first roll, would you roll again or score it as 3-of-a-kind right away? Why?
7. If you were playing Yahtzee and rolled 5-5-5-5-5 on your first roll, would you roll again or score it as Yahtzee right away? Why?
8. If you were playing Yahtzee and rolled 5-5-5-5-6 on your first roll, would you roll again or score it right away as 4-of-a-kind, Chance, or 'fives'? Why?
9. If someone wrote $P(6)$, is this enough to know what is meant?

Exercises:

10. Yahtzee questions
    a. How many different dice outcomes are there when rolling the five dice?
    b. Are the outcomes equally likely?
    c. Is rolling two-2's and three-3's on the five dice considered an outcome or an event? Explain.
    d. Julio said that the outcomes are not equally likely because rolling a Yahtzee is less likely than rolling 3 of a kind. How do you respond?
    e. What are your chances for rolling a Yahtzee with sixes on the first roll?
    f. What are your chances for rolling any Yahtzee on the first roll?

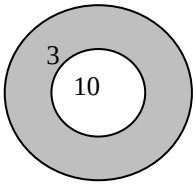
11. Write out all the outcomes in the events from the procedure listed.
    a. Procedure: Rolling a 12-sided die once.

	Event	Outcomes	Simple Event?
i.	Roll an odd number		Yes No
ii.	Roll a number greater than 9		Yes No
iii.	Roll a number greater than or equal to 9		Yes No
iv.	Roll a 5		Yes No
v.	Roll a 13		Yes No

b. Procedure: Rolling a 4-sided die once.

	Event	Outcomes	Simple Event?
i.	Roll an odd number		Yes No
ii.	Roll a number greater than 4		Yes No
iii.	Roll a number greater than or equal to 4		Yes No
iv.	Roll a 2		Yes No
v.	Roll a prime		Yes No

12. Which of the following produce equally likely outcomes?
    a. Flipping a thumb-tack and seeing how it ends (point in the air or point touching the table).
    b. Rolling a standard 20-sided die.
    c. Rolling a loaded (weighted) die.
    d. Spinning a spinner with 8 slices all having the same central angle.
    e. Spinning a spinner where one slice takes up 50% of the wheel, and the others are split equally.
    f. Randomly drawing an m-&-m out of a package and seeing what color it is.
    g. Randomly drawing a card from a standard deck of cards.
    h. Randomly selecting a course from the MCC catalog and writing down the subject code only (Math, Eng, Hist, etc.)
    i. Randomly throwing a dart at the dartboard shown here and recording a 10.



13. For the following events done with a jug that has 10 red chips, 27 white chips, 15 blue chips, and 9 green chips, find the probability of each event.
    a. Draw a green chip.
    b. Draw a red chip.
    c. Draw a chip that is red or white.
    d. Draw a chip that is not blue.
    e. Draw a chip that is blue and white.
    f. Draw a chip that is blue or white.
    g. Draw a chip that is red or white or green.
    h. Draw a chip that is not blue and not green.
    i. Do any of the events have the same probability? Which one(s) and why? [It could be blind luck or the events could be connected.]

14. Find the sample space, Event space, and probability related to the following:

	Event	Sample Space	Event	Probability
a.	Roll a 4-sided die; try to get 2			
b.	Flip a single coin; try to get T			
c.	Roll an 8-sided die; try to get less than 3			
d.	Spin a spinner with 1, 1, 3, 5, 8; try to get a 1 (all regions equally likely)			
e.	Draw a marble from a jug with 7 red and 12 blue; try to get blue.			

15. Identify how likely the event is to occur with the probability. Use the terms: Impossible, Highly Unlikely, Unlikely, Likely, Highly Likely, or Certain.

a. $P(R) = 0.2$
b. $P(A) = 0.72$
c. $P(H) = 1$
d. $P(B) = 0.02$
e. $P(E) = 0.0000000005$
f. $P(M) = \frac{78}{85}$
g. $P(T) = \frac{78}{850}$
h. $P(G) = 0$
i. $P(L) = \frac{78}{85{,}000{,}000}$
j. $P(W) = 0.98$
k. $P(V) = 0.49$
l. $P(X) = 0.56$

16. Compare the probabilities given to see which is more likely, as well as how likely it is. Circle the event that is more likely, then describe in words. The first is done for you.

	Event 1	Event 2	Explanation
a.	$P(E) = 0.0000000005$	$P(B) = 0.02$	Neither is likely. *B* is more likely to occur, but is still highly unlikely.
b.	$P(M) = \frac{7}{8}$	$P(T) = \frac{6}{8}$	
c.	$P(A) = 0.72$	$P(R) = 0.9$	
d.	$P(S) = \frac{78}{85}$	$P(D) = \frac{9}{10}$	
e.	$P(V) = 0.49$	$P(X) = 0.56$	

Wrap-up and look back:

17. Samuel computes $P(R) = 1.3$. Does this make sense or not? Explain.

18. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.3: More Advanced Probability

Vocabulary and symbols – write out what the following mean:

- None

Concept questions:

1. If you flip a coin 10 times in a row, is it more likely that you'll get all heads or that you'll get exactly 2 heads?

2. If you flip a coin 10 times in a row, what number of heads is most likely?

3. Sammi said that since flipping a coin has a 50-50 chance of getting H, then when you flip a coin 10 times in a row, you should get 5H and 5T all the time. Is she correct?

Exercises:

4. Flipping a coin 10 times in a row.
   a. If you flip a coin 10 times in a row, how many different outcomes are possible?
   b. How many of these outcomes are all H or all T?
   c. What is the probability of flipping a coin 10 times in a row and getting all H or all T?
   d. If you had 5 heads and 5 tails, you could think of them as letters: HTHTHTHHTT. How many different ways could you rearrange the outcomes?
   e. What is the probability that you'll flip a coin 10 times and get exactly 5 H and 5 T?
   f. Would you consider this event (5H and 5T) likely or not?
   g. Margie said that since flipping a coin has a 50-50 chance of getting H, then when you flip a coin 10 times in a row, you should get 5H and 5T half the time. Is she correct?

5. Pascal's triangle is shown here – to get to the next row, you add the two numbers above. So the first 3 that you see came from 1 + 2. And the 6 comes from 3 + 3. Compute the next 3 rows of Pascal's triangle.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

6. Compute the following:
   a. ${}_2C_0$, ${}_2C_1$, ${}_2C_2$
   b. ${}_3C_0$, ${}_3C_1$, ${}_3C_2$, ${}_3C_3$
   c. ${}_4C_0$, ${}_4C_1$, ${}_4C_2$, ${}_4C_3$, ${}_4C_4$
   d. ${}_5C_0$, ${}_5C_1$, ${}_5C_2$, ${}_5C_3$, ${}_5C_4$, ${}_5C_5$,
   e. How does Pascal's triangle compare with combinations from Unit 1?
   f. To check the first few rows of Pascal's triangle, what combination would you use?
   g. Does it check out with the rest of the pattern?

7. Sums in Pascal's triangle.
   a. What do you notice about the sums?
   b. The first row in Pascal's triangle is often called the 0^{th} row, not the 1^{st} row. So the 1^{st} row is 1 – 1. What is the sum of the 1^{st} row?
   c. For the 2^{nd} row, the one with 1 – 2 – 1, what is the sum?
   d. For the 4^{th} row, the one with 1 – 4 – 6 – 4 – 1, what is the sum?
   e. Relate this back to a type of sequence from Math 52 – what type is it?
   f. What is the sum of the 9^{th} row? (You don't have to write it all out)
   g. What is the sum of the n^{th} row?

8. How does Pascal's triangle compare to the sample space/outcomes for flipping coins? Explain where the pattern comes from.

9. Try to not use Pascal's triangle here. If you flipped a coin 10 times in a row…
   a. Is each coin flip set of outcomes equally likely?
   b. How many different outcomes are possible?
   c. How many different ways could you get exactly 3 heads?
   d. What is the probability of getting exactly 3 heads?
   e. What is the probability of getting exactly 5 heads?
   f. Does this match with what we did in question 6?
   g. How does rearranging 5 each of the letters T and H relate back to ${}_{10}C_5$?

10. When rolling two 6-sided dice, determine the following probabilities:
   a. Roll a sum of 5.
   b. Roll a sum of 11.
   c. Roll a sum of 13.
   d. Roll a sum of 10.
   e. Roll a sum of 3.
   f. Roll a sum of 2.
   g. Roll a sum that is even.
   h. Roll a sum that is greater than 6.
   i. Roll a sum that is even or a sum that is 5.
   j. Roll a sum that is less than or equal to 5.
   k. Roll a sum that is greater than 6 or a sum that is odd.

11. When rolling two 4-sided dice, create all possible outcomes (sample space). Then answer the questions:
   a. What is the sample space?
   b. What is the probability of rolling a sum of 6?
   c. What is the probability of rolling a sum that is odd?
   d. What is the probability of rolling a sum that is greater than 2?
   e. What is the probability of rolling a sum that is less than or equal to 7?
   f. What is the probability or rolling a sum of 8?

12. If we flip a coin twice and then roll a single 4-sided die, determine…
   a. The sample space.
   b. The probability of getting H-T-3.
   c. The probability of getting T first.
   d. The probability of rolling a 2.
   e. The probability that you end with H first and an even number last.
   f. The probability of getting both heads in your outcome.

13. If you use the numbers 1, 2, 3, 4, 5, and 6, determine…
   a. How many combinations of two numbers can be made?
   b. How many permutations of two numbers can be made?
   c. What is the probability of randomly selecting a combination of two numbers and getting (2,5)?
   d. What is the probability of randomly selecting a permutation of two numbers and getting (2,5)?
   e. How many combinations of three numbers can be made?
   f. How many permutations of three numbers can be made?
   g. What is the probability of randomly selecting a combination of three numbers and getting (2,5,1)?
   h. What is the probability of randomly selecting a permutation of three numbers and getting (2,5,1)?
   i. In all of these, can the numbers be repeated or not? Explain.
   j. If the numbers can be repeated, how many permutations of two numbers can be made?
   k. If the numbers can be repeated, what is the probability of randomly selecting a permutation of two numbers and getting (2,5)?

Wrap-up and look back:

14. Pascal's triangle is cool. Why don't we use it after the first few rows? Why don't we use it for flipping 20 coins?

15. Can you use Pascal's Triangle for problems like babies being born (Boys and Girls)? What about for Winning and losing games as a team (W and L)? When does it seem like Pascal's triangle works?

16. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.4: Probability and Counting

Vocabulary and symbols – write out what the following mean:

- Fundamental Counting Principle
- Factorial
- Permutations
- Combinations
- Permutations of Like Objects

Concept questions:

1. When do we use the Fundamental Counting Principle?

2. Which of these do we use when the objects cannot be repeated?

Exercises:

3. If a family has 3 children (use B for boy and G for girl)…
   a. What is the event that the family has exactly one son?
   b. What is the probability that the family has exactly one son?
   c. What is the event that the family has at least one daughter?
   d. What is the probability that the family at least one daughter?

4. Toss a coin 3 times (use H for Heads and T for Tails)…
   a. What is the event that the outcomes are all the same?
   b. What is the probability that the outcomes are all the same?
   c. What is the probability that the outcomes are not all the same?
   d. Which is more likely: that the coins all match, or that they don't all match? Explain.

5. A conference has 29 short sessions, 18 long sessions, and 5 main sessions. The fee to attend includes one of each type.
   a. How many different ways could you select your 3 sessions?
   b. You pick your sessions out of a hat and end up taking each session with Leia. What is the probability that you randomly select the sessions Leia already chose? (4 decimal places)
   c. Does it seem more likely that this randomly happened or that Leia is following you because you are strong with the FORCE? (Answering this does not guarantee you are a Jedi Knight. Your secret is safe with me.)

6. 8 people walk in to be served at a restaurant. They arrived at exactly the same time.
   a. How many ways can they be served in order?
   b. What is the probability that they are served in alphabetical order (A to Z)?
   c. If the restaurant was closing and could only serve 3 people, how many ways could this be done?
   d. How many ways could the 3 people being selected be served?

7. You have a group of 20 people, but only 3 applicants are selected to be interviewed. All are equally qualified, so the names are put in a hat and drawn out.
   a. How many ways can the interviewees be selected?
   b. What is the probability that the 3 applicants drawn are the 3 youngest?
8. My children snuck into the office one day and accidentally knocked over a stack of books. There are 9 books in the stack, and the kids didn't see the order of the books.
   a. What is the probability that the kids can stack the books back (randomly) and have the stack match the precise order that they were in? (this is another way to say that they didn't get caught)
   b. What is the probability that the kids don't stack them correct and are caught for sneaking into the office?
   c. How likely is it that they are caught? Explain.

9. There are 10 art exhibits to hang up in a gallery that only can hold 4 of them.
   a. How many ways are there to select the exhibits if we <u>don't care</u> where they are displayed?
   b. Assume we don't care about how they are displayed. If someone put a number on each exhibit and then chose randomly from a hat, what is the probability that the four selected are the largest four in order of weight?
   c. How many ways are there to select the exhibits if we <u>do care</u> where they are displayed?
   d. Assume we do care about how they are displayed. If someone put a number on each exhibit and then chose randomly from a hat, what is the probability that the four selected are the largest four in order of weight?

10. In a dog race, there are 5 Wiener dogs (W), 2 Rat dogs (R), 3 Ugly dogs (U), and 1 Pee-in-the-house dog (P).
    a. How many ways can the race finish if we only consider the type of dog?
    b. If the order of finish is random, what is the probability that the rat dogs finish last?
    c. If the order of finish is random, what is the probability that the ugly dogs finish first?
    d. If the order of finish is random, what is the probability of: W-R-U-W-W-U-P-R-W-U-W?
    e. If you bet $30 on the previous outcome, what are your chances of winning? Explain.

11. You have a group of 20 people, but only 3 applicants are to be interviewed. The people are grouped into 3 categories: 5 people in the youngest (under 30 – Y), 7 people in the middle (between 30 and 45 – M), and 8 people in the final group (over 45 – F). Again, names are put into a hat and drawn at random…
    a. What is the probability that the group interviewed is two Y and one F?
    b. What is the probability that the group interviewed is one of each group?
    c. What is the probability that the group interviewed is all F?
    d. What is the probability that the group interviewed has none of the F group?
    e. How likely is the last event (none of the F group)? Explain.
    f. Typically, an event must occur less than 5% of the time (probability of less than 0.05) in order to have a case for age discrimination. Is there a case for age discrimination here if none of the F group were chosen?

12. There are 13 teams playing in a beach volleyball league. At the end of the season…
    a. How many ways can the teams finish 1st, 2nd, and 3rd?
    b. If each team must play every other team, how many games must be played total?
    c. Is it possible that 8 teams each lost 10 games? Explain.
    d. If the team names were put in a hat, then 3 names drawn to be in a tournament, what is the probability that the 3 names drawn are the worst 3 teams in the league?

13. There is a bag with 16 batteries that work perfectly. Rick and his son Carl have flashlights that hold 2 batteries each, but the batteries are dead. Carl accidentally drop their dead batteries into the bag and then, with the zombies closing in, drops his flashlight and it breaks. Rick quickly grabs 4 batteries from the bag and the two run away.
    a. How many different ways could Rick select the 4 batteries?
    b. A flashlight only works if both batteries are working. How many different ways could Rick test the four batteries he grabbed in groups of two at a time?
    c. What is the probability that Rick grabbed 2 dead and 2 working batteries?
    d. What is the probability that Rick grabbed 1 dead and 3 working batteries?
    e. What is the probability that Rick grabbed 0 dead and 4 working batteries?
    f. What is the probability that some arrangement of the batteries will make the flashlight work?
    g. How likely is the last event (some arrangement will make the flashlight work)? Explain.

14. A lock has a spinning dial with numbers from 0 to 39. Three numbers must be selected in order to open the lock.
    a. Sometimes people call these combination locks. Is this a good word to use or not? Explain.
    b. How many 'combinations' are possible for this lock?
    c. If it takes you 30 seconds to try one 'combination' and are determined to get into the lock, figure out how long it could take (worst case scenario).
    d. When someone says that they "won't stop until they get into this lock" – do you believe that statement is true?
    e. What is the probability that you are able to open the lock on your first try?

15. In the 2008 version of Powerball, there were 55 White and 42 Red options. Players would select 5 white numbers and 1 red (power) ball.
    a. How many ticket options are possible?
    b. What is the probability of winning the first prize? (matching all 5 white and 1 red)
    c. Tell me in your words about how likely it is that you would win this version of Powerball. [Note: the chances of being struck by lightning during your lifetime is approximately 1 in 9 million.]
    d. What is the probability that you get everything wrong on your ticket?
    e. What is the probability that you get exactly one white correct (no red)?
    f. What is the probability that you get exactly two whites correct (no red)?
    g. The last 3 options show the probability of winning nothing. What is that probability?
    h. How likely is it that you'll win absolutely nothing?
    i. Create another experiment with a smaller number of objects that would mirror your chances of winning (just winning overall, not the grand prize).
    j. There are about 2,500 miles from NewYork City to Los Angeles. How many feet is this (use your Math 52 skills)? How many inches?
    k. What if you put a small piece of paper (for each inch from NYC to LA) into a bag, with only one piece having an X on it and the rest all blank. What are your chances of randomly drawing out the one with the X on it?

(#15 continued)

l. How did this NYC to LA probability relate back to the probability that you'll win the grand prize in this version of Powerball?

m. In 2015, the Powerball modified the game again so now it has nearly double the options as the version in 2008. Describe a way put in words (similar to NYC to LA) the chances of winning the newest version?

16. There are 15 pills that are put in a bag – they all look exactly the same. However, 9 of them are actual pills and 6 are placebos (fake pills made of sugar). If you randomly choose 4 pills...
    a. What is the probability that you have exactly 2 real pills?
    b. What is the probability that you have exactly 0 real pills?
    c. What is the probability that you have at least 1 real pill?
    d. What is the probability that you have at least 1 placebo?
    e. Which is more likely – that you end with at least 1 real pill or that you end with at least 1 placebo?

Wrap-up and look back:

17. Describe your chances of losing the Powerball (2008 version) to the chances of winning (grand prize)?

18. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.5: Classical vs Empirical Probability (Relative Frequency)

Vocabulary and symbols – write out what the following mean:

- Relative Frequency
- Empirical Probability

Concept questions:

1. If you can't figure out the classical probability, what could you do to determine the chances from the procedure?

2. Were the probability pigs an opportunity to see equally likely outcomes? Explain.

3. How can you tell the difference between classical and empirical probability procedures?

4. When teaching a statistics course, a good way to help students understand randomness is to have them go home and pretend to flip a coin 200 times, writing down H and T when they feel it would come up. Students do this and then turn it in the next day for 'extra credit.' The following day in class, students are told to actually go home and flip a coin 200 times – which takes quite a bit longer. Even though they are told to actually flip the coin, many students don't because… really… how in the world can a teacher ever know? Right? I mean it's 200 coin flips – who's going to count those. Scott will often bring up the two pieces of paper and mix them together, then try to figure out which one was real… or if someone cheated.
    a. What were the chances of getting 7 Heads in a row at some point in the 200 flips (from class)?
    b. How do you think Scott was able to use this to find the folks who cheated? [He was accurate more than 80% of the time.]

Exercises:

5. If the probability is very hard to determine, creating a computer program that runs options based on certain conditions is known as a **Monte Carlo simulation**. We did some of these in class. Your turn to try some at home:
   a. Create a Monte Carlo simulation that will help you find the chances of getting one of each toy in a cereal box assuming there are 6 different toys and you purchase 12 boxes. [Run your simulation at least 20 times.]
   b. Create a Monte Carlo simulation that will help you find the chances of rolling at least 3 sixes on a standard 6-sided die if you roll 10 times. [Run your simulation at least 20 times.]
   c. Create a Monte Carlo simulation that will help you find the chances of flipping a coin 15 times and getting at least 9 heads. [Run your simulation at least 20 times.]

6. Are these classical or empirical? Explain your answer.
   a. Trying to see the weather for the next day, so you check the weather from the past 10 days.
   b. Trying to determine the probability that a student is taking a math class by asking 300 students on campus.
   c. Trying to determine the probability that a student is taking a math class by determining the total number of students in any math classes and then finding the total number of students enrolled.
   d. Trying to find the probability of rolling a sum of 9 using a 12-sided and 4-sided dice by rolling the two dice 400 times and writing the results.
   e. Trying to find the probability of rolling a sum of 9 using a 12-sided and 4-sided dice by creating the sample space and finding all outcomes in the "sum of 9" event.
   f. Trying to find the probability of winning Powerball by running a Monte Carlo simulation.
   g. Trying to find the probability of winning Powerball by determining all the outcomes for winning choices and then creates the probability.
   h. Trying to find the probability of winning Powerball by playing $100 in Powerball tickets every week for a year.

7. Use the table to determine the answers after a poll is conducted and the results to "Would you vote for Measure A in November?"

	Yes	No	Not Sure	Totals
Republicans	513	79	16	
Democrats	174	811	103	
Undecided	42	61	18	
Totals				

   a. Find the probability that you randomly select a Democrat?
   b. Find the probability that you randomly select a person voting Yes?
   c. Find the probability that you randomly select a person who is Democrat or votes No?
   d. Find the probability that you randomly select a person who is Undecided and Not Sure?
   e. Looking only at the Republicans, find the probability of randomly selecting a No voter?
   f. Someone claims that because of this poll, it is clear that there are more Republicans voting No than Undecided people voting No, so Republicans are more opposed to Measure A than Undecided voters. Do you agree or disagree?
   g. Are your answers based on classical or empirical probability?

Wrap-up and look back:

8. Why are computers used so much for probability?
9. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.6: Some Rules in Probability

Vocabulary and symbols – write out what the following mean:

- Complementary Events

Concept questions:

1. Can a probability ever end up being greater than 1? Explain.

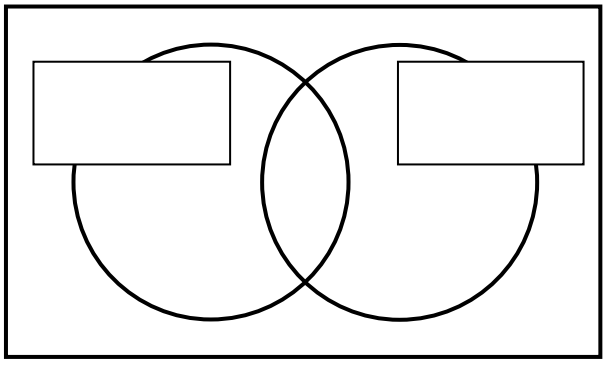
2. Can a probability ever end up being less than 0? Explain.

3. When Sharlene plugs in everything he knows into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, he gets this equation: $0.80 = P(A) + 0.75 - 0.2$. What is the missing piece? Does this make sense? Explain.

4. When Shane plugs in everything he knows into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, he gets this equation: $0.67 = P(A) + 0.95 - 0.2$. What is the missing piece? Does this make sense? Explain.

Exercises:

5. Use the given information to find the probabilities requested. $P(A) = 0.38$, $P(\overline{B}) = \frac{1}{4}$, $P(A \cup B) = 0.8$.
   a. $P(\overline{A})$
   b. $P(B)$
   c. $P(\overline{A \cup B})$
   d. $P(A \cap B)$
   e. $P(\overline{A \cap B})$
   f. Use all of your knowledge to fill out the probability Venn diagram… instead of how many are in each region, write the probability for that region.

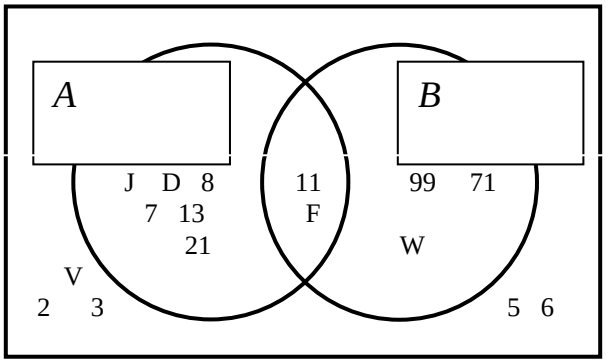


6. Find the requested probabilities.
   a. Event *J*: drawing a joker out of a deck of 54 cards – 52 standard and 2 jokers. Find $P(J)$ and $P(\overline{J})$.
   b. Event *D*: drawing a diamond out of a standard deck of 52 cards. Find $P(D)$ and $P(\overline{D})$.
   c. Event *F*: rolling two 6-sided dice and getting a sum of 5. Find $P(F)$ and $P(\overline{F})$.
   d. Event *T*: rolling one 4-sided die and getting a 2. Find $P(T)$ and $P(\overline{T})$.
   e. Event *W*: rolling two 6-sided dice and getting a one. Find $P(W)$ and $P(\overline{W})$.

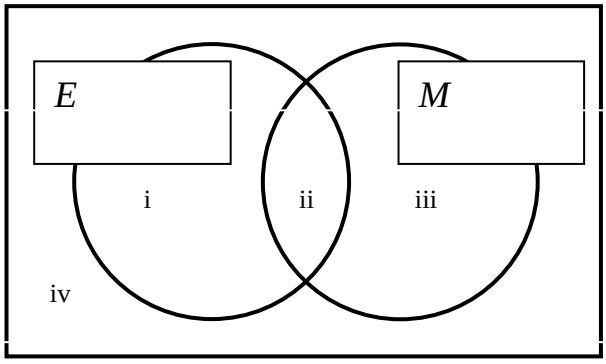
7. Consider a standard deck of cards (52 total). Find the probability that you will randomly ***draw one card*** from the deck and get…
   a. An ace.
   b. A heart
   c. A spade or a heart or a club.
   d. A spade or an ace.
   e. A card that is not a diamond.
   f. A card that is a 10 or an ace.
   g. A card that is a 7.
   h. A 5 or 7 or 10.
   i. A red card or a queen.
   j. A red card and a queen.

8. Consider a Pinochle deck of cards (only cards from 9 up to A, same suits, but two of each card). Find the probability that you will randomly ***draw one card*** from the deck and get…
   a. An ace.
   b. A heart.
   c. A spade or a heart or a club.
   d. A spade or an ace.
   e. A card that is not a diamond.
   f. A card that is a 10 or an ace.
   g. A card that is a 7.
   h. A 9 or jack or queen.
   i. A red card or a queen.
   j. A red card and a queen.

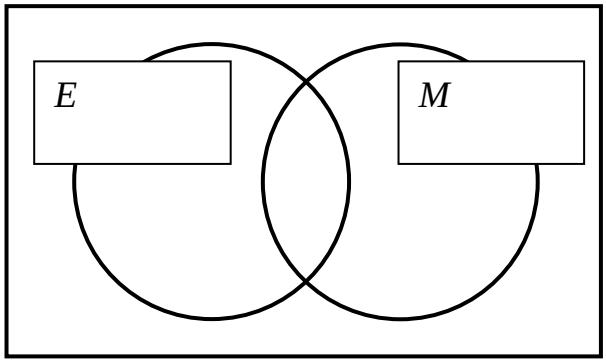
9. In the Venn diagram, the symbols represent individual elements.
   a. $P(A)$
   b. $P(\overline{A})$
   c. $P(B)$
   d. $P(\overline{B})$
   e. $P(A \cup B)$
   f. $P(A \cap B)$
   g. $P(\overline{A \cup B})$
   h. $P(\overline{A \cap B})$



10. In this Venn diagram, we represent two classes (at MiraCosta): English and math. If someone is in the circle, then that person is passing the class. Describe each region in the Venn diagram.
    i.
    ii.
    iii.
    iv.



11. In the Venn diagram, the numbers represent probabilities, but it still relates to English and math. If someone is in the circle, then that person is passing the class. Fill out the Venn diagram based on these clues: $P(E) = 0.61$, $P(M) = 0.75$, and $P(E \cap M) = 0.42$.



a. What is the probability that a student passes both classes?

b. What is the probability that a student doesn't path math but does pass English?

c. What is the probability that a student doesn't pass math?

d. What is the probability that a student doesn't pass English and doesn't pass math?

e. What is the probability that a student passes exactly one of the two classes?

f. Are the events "Passing English" and "Passing math" disjoint? Explain.

12. A company is discussing using a new medical test that is much cheaper than a current test. In order to be put in use, it must be at least 90% accurate. The table is included to summarize their results.

	Have the disease	Don't have the disease	Totals
Test Positive (+)	113	93	
Test Negative (–)	9	1,285	
Totals			

a. How many total people were in this set of data?
b. How many people were accurately placed? (had the disease and tested positive or didn't have the disease and tested negative)
c. What is the probability of accuracy based on this information?
d. What is the probability of testing positive?
e. What is the probability that someone tests positive or has the disease?
f. What is the probability that someone tests negative and doesn't have the disease?
g. Are the events "Have the disease" and "Test Positive" disjoint? Explain.

13. Which of these events are disjoint and which are not? Explain your answers for each.

	Events
a.	Randomly choosing a person who is a CEO of a major company. Randomly choosing a person who is a man.
b.	Randomly choosing a person who was a supreme court justice. Randomly choosing a person who is a woman.
c.	Randomly choosing a nurse. Randomly choosing someone who works in a hospital.
d.	Randomly choosing a toy from Disneyland. Randomly choosing a Toyota truck.

14. Lie detectors are sometimes used to help law enforcement as it is claimed that the rate of accuracy is 90% or better. However, critics have research showing that the rate can be as low as 70%. Here are some results from lie detector tests early on in a case. Later, information can be used to determine if the individual was lying or telling the truth. Here a positive test (+) indicates that the machine believes the person is lying. A false positive is when the machine indicates a lie but the person is telling the truth. A false negative is when the machine indicates truth but the person is actually lying.

	Actually Truthful	Actually Lying	Totals
Test Positive (+)	16	47	
Test Negative (–)	133	4	
Totals			

a. How many total people were in this set of data?
b. How many people were accurately placed? (liars who tested positive or honest people who tested negative)
c. What is the probability of accuracy based on this information?
d. What is the probability of testing positive?
e. What is the probability that someone tests positive or is actually lying?
f. What is the probability that someone tests negative and is actually truthful?
g. Are the events "Actually Truthful" and "Test Positive" disjoint? Explain.

Wrap-up and look back:

15. Can you use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ if the events are disjoint? Explain.

16. Can you use $P(A \cup B) = P(A) + P(B)$ if the events are not disjoint? Explain.

17. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.7: Conditional Probability, and Independent Events

Vocabulary and symbols – write out what the following mean:

- Independent Events
- Dependent Events
- Conditional Probability
- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{n(A \cap B)}{n(B)}$
- $P(A \cap B) = P(B) \cdot P(A \mid B)$

Concept questions:

1. Waylan says that two events are dependent if they are disjoint. Is he correct?

2. Shelli says that two events are independent when they can't happen at the same time. Is she correct?

3. Noel says that with conditional probability, you don't need to do anything different – just find the events and divide them. Is she correct?

4. Is conditional probability commutative – is this true: $P(A \mid B) = P(B \mid A)$? Explain why or give a counter-example to show why not.
5. When can we use $P(A \cap B) = P(B) \cdot P(A)$ compared to $P(A \cap B) = P(B) \cdot P(A \mid B)$? Explain.
6. If two events are independent, does that mean they must be disjoint? Explain.

Exercises:

7. Determine whether these events are independent or dependent.

	Events
a.	Randomly choosing a person who is a CEO of a major company. Randomly choosing a person who is a man.
b.	Randomly choosing a nurse. Randomly choosing someone who works in a hospital.
c.	Randomly choosing an ace from a standard deck and putting it back. Randomly choosing an ace as your second card.
d.	Randomly choosing an ace from a standard deck and keeping it. Randomly choosing an ace as your second card.
e.	Randomly choosing a person who was a supreme court justice. Randomly choosing a person who is a woman.
f.	Flipping a coin and getting H. Flipping the same coin again and getting T.
g.	Rolling a six on a 6-sided die. Flipping a coin and getting H.
h.	Winning the Powerball this week. Winning the MegaMillions this week.
i.	It is cloudy today. It is raining today.
j.	Your microwave clock is blank. Your TV doesn't work.

8. In the medical testing case from the last section, we could also consider conditional probability. Use the table to answer the questions that follow.

	Have the disease	Don't have the disease	Totals
Test Positive (+)	113	93	206
Test Negative (–)	9	1,285	1,294
Totals	122	1,378	1,500

a. $P(+)$
b. $P(+ \mid \text{don't have disease})$
c. $P(\text{don't have disease})$
d. $P(\text{don't have disease} \mid +)$
e. $P(-)$
f. $P(- \mid \text{have disease})$
g. $P(\text{have disease})$
h. $P(\text{have disease} \mid -)$

i. What would you tell someone who tested positive? (How likely is it that they don't have the disease?)
j. What would you tell someone who tested negative? (How likely is it that they have the disease?)
k. Which group would a doctor recommend to have a second test – people who test negative or people who test positive? Why?

9. In the lie detector test from the last section, we could also consider conditional probability. Use the table to answer the questions that follow.

	Actually Truthful	Actually Lying	Totals
Test Positive (+)	16	47	
Test Negative (–)	133	4	
Totals			

a. $P(+)$
b. $P(+ \mid \text{actually truthful})$
c. $P(\text{actually truthful})$
d. $P(\text{actually truthful} \mid +)$
e. $P(-)$
f. $P(- \mid \text{actually lying})$
g. $P(\text{actually lying})$
h. $P(\text{actually lying} \mid -)$
i. If you were in a jury case and were told that the person who was testifying had tested positive for lying. Would you be likely to believe the testimony or not? Is it likely that the person was actually truthful?
j. If you were in a jury case and were told that the person who was testifying had tested negative for lying. Would you be likely to believe the testimony or not? Is it likely that the person was really lying?
k. Why are lie detector tests not used much in court cases?

10. If $P(A) = 0.278$ and $P(A \mid B) = 0.278$, what can we conclude about events *A* and *B*? Are they independent or dependent? Are they disjoint or not. Explain.

11. If $P(A) = 0.278$ and $P(A \mid B) = 0$, what can we conclude about events *A* and *B*? Are they independent or dependent? Are they disjoint or not. Explain.

12. If $P(A) = 0$ and $P(A \mid B) = 0$, what can we conclude about events *A* and *B*? Are they independent or dependent? Are they disjoint or not. Explain.

13. If $P(A) = 0.25$ and $P(A \mid B) = 0.45$, what can we conclude about events *A* and *B*? Are they independent or dependent? Are they disjoint or not. Explain.

14. If you know that *A* and *B* are independent, what must be true about $P(B)$ and $P(B \mid A)$? Explain.

15. If you know that *A* and *B* are dependent, what must be true about $P(B)$ and $P(B \mid A)$? Explain.

16. If you know that *A* and *B* are disjoint, what must be true about $P(B \mid A)$? Explain.

<u>Wrap-up and look back:</u>

17. Can $P(B \mid A)$ ever be the same as $P(A \mid B)$? Find an example or explain why not.

18. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.8: Multiplication Rule, Complementary Events and "At Least 1"

Vocabulary and symbols – write out what the following mean:

- $P(A \cap B) = P(A) \cdot P(B)$
- $P(A \cap B) = P(A) \cdot P(B \mid A)$

Concept questions:

1. Are both of these true: $P(A \cap B) = P(A) \cdot P(B \mid A)$ and $P(A \cap B) = P(B) \cdot P(A \mid B)$? Explain using the formulas.
2. In the work/text, it was seen that the probability that at least 2 people in a group of 35 share a birthday was about 81.44%. Explain why this probability is so high… there are 365 days in the year and only 35 people in the group? What's up with that?

Exercises:

3. There is a multiple choice test with 5 questions – each has 4 options.
   a. If you guess on all questions, what is the probability that you get all of them correct?
   b. How many ways can you rearrange CWCCW? This shows how many different ways we could answer exactly 3 questions correctly.
   c. How does your answer to part (b) compare to ${}_5C_3$?
   d. What is the probability of randomly guessing and getting exactly 3 correct?
   e. What is the probability of randomly guessing and getting none correct?
   f. What is the probability of randomly guessing and getting at least one correct?
   g. If you can eliminate 2 options from each question, what is the new probability that you get them all correct?
   h. Why is it better to eliminate options when guessing?

4. A true-false test had 10 questions (each with 2 options).
   a. If you guess on all questions, what is the probability that you get all of them correct?
   b. How many ways can you get exactly 2 correct? (remember that this is just rearranging 8 W and 2 C letters.)
   c. What is the probability of guessing and getting exactly 80% on the test?
   d. How many ways can you get exactly 3 correct?
   e. How many total ways are there where you can get at least a 70% on the test?
   f. What is the probability of guessing and getting at least 70% on the test?
   g. Why is it better to eliminate options when guessing?

5. Find the probability that with a standard deck of cards…
   a. That you draw 2 red cards in a row with replacement.
   b. That you draw 2 red cards in a row without replacement.
   c. That you draw A(spades) and then K(spades) with replacement.
   d. That you draw A(spades) and then K(spades) without replacement.
   e. That you draw 4 hearts in a row with replacement.
   f. That you draw 4 hearts in a row without replacement.

6. Mark bought sees that bags of Skittles say that there is a 20% chance that each bag will win a prize.
   a. What is the probability that he selects 2 bags and both win?
   b. What is the probability that he selects 2 bags and wins exactly once?
   c. What is the probability that he selects 2 bags and wins nothing?
   d. What is the probability that he selects 10 bags and wins nothing?
   e. What is the probability that he selects 10 bags and wins at least once?
   f. What is the probability that you get 8 losers in a row and then 2 winners?
   g. How many ways can you arrange 8 L and 2 W as letters?
   h. What is the probability that you get 8 losers and 2 winners (in any order)?

7. Find the probability that…
   a. You roll at least one 5 in the next four rolls on a 6-sided die?
   b. You roll at least one 5 in the next four rolls on a 12-sided die?
   c. You roll at least one 5 in the next ten rolls on a 6-sided die?
   d. You roll at least one 5 in the next ten rolls on a 12-sided die?

8. In a game show, one contestant is selected who picks their actual favorite song. In the audience, people are selected to see if they can determine it. There are 100 songs put on a board and only 1 is correct.
   a. What is the probability that a group of 20 contestants miss every time?
   b. What is the probability that a group of 20 wins (someone in the group picks the winning song)?
   c. What is the probability that a group of 50 contestants miss every time?
   d. What is the probability that a group of 50 wins (someone in the group picks the winning song)?

9. Consider a standard deck of cards. Find the following…
   a. The probability of drawing one card and it is a Queen of hearts.
   b. The probability of drawing one card and it is a Queen.
   c. The probability of drawing one card and it is a heart.
   d. The probability of drawing one card and it is a Queen or a heart.
   e. The probability of drawing two cards, a both hearts, with replacement.
   f. The probability of drawing two cards, a both hearts, without replacement.
   g. The probability of drawing 6 cards in a row (with replacement) and drawing at least one heart.

10. Consider a group of 20 numbers in a hat (from 1 to 20). Find the following…
    a. The probability of drawing a 7.
    b. The probability of drawing a 25.
    c. The probability of drawing a 7 or a number less than 5.
    d. The probability of drawing a 7 and a number less than 5.
    e. The probability of drawing two numbers, a 14 and a 9 (in order), with replacement.
    f. The probability of drawing two numbers, a 14 and a 9 (in order), without replacement.
    g. The probability of drawing two numbers, a 14 and a 9 (in any order), with replacement.
    h. The probability of drawing two numbers, a 14 and a 9 (in any order), without replacement.
    i. The probability of drawing the numbers 4-3-2-1 in order, without replacement.
    j. The probability of drawing the numbers 4-3-2-1 in order, with replacement.
    k. The probability of drawing 10 numbers in a row, with replacement, and getting at least one 14.
    l. The probability of drawing two numbers where the numbers add up to 7 (with replacement).

11. In statistics courses, problems like True-False tests, flipping a coin, and others like winning/losing or boys/girls have a way to distribute the probabilities. They are referred to as **binomial distributions**. Look up the word binomial and explain why this name makes sense.

12. In a group of 25 random people,
    a. what is the probability that at least one person shares your birthday?
    b. what is the probability that at least two people in the group share a birthday?

13. There is a game where the host rolls one 20 sided die to establish the prize number. Then each contestant will come up and roll the same die to try to meet that value. If they roll the same number, then that's
    a. What is the probability that out of the next 10 contestants, at least one person wins?
    b. What is the probability that out of the next 20 contestants, at least one person wins?
    c. What is the probability that out of the next 50 contestants, at least one person wins?

14. In your first roll of Yahtzee, you end with 5-5-5-2-1. You are going for either Yahtzee or 4 of a kind.
    a. What is the probability that you will get Yahtzee within the next two rolls?
    b. What is the probability that you will get 4 of a kind within the next two rolls?
    c. What is the probability that you won't get 4 of a kind or Yahtzee?

Wrap-up and look back:

15. It is much more likely that at least 2 people in a group share a birthday than at least 1 person in a group shares your birthday (or one specific birthday). Explain.

16. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.9: Odds and Probability

Vocabulary and symbols – write out what the following mean:

- Odds in favor
- Odds against

Concept questions:

1. How do odds relate to probability?

2. If there are 20 equally likely ways that an event can happen and 35 equally likely ways that it can't, what are the odds against? What are the odds in favor?

3. If the odds in favor are 2:5, is this the same as 20:50?

4. How does FLOF relate to the odds? Can we 'simplify' the odds using FLOF? Explain.

5. If $O(E) = 4:1$, is the event *E* likely to happen or not? Explain.

6. If $O(\overline{E}) = 7:2$, is the event *E* likely to happen or not? Explain.

7. After winning Super Bowl 48, barely losing Super Bowl 49, the Seattle Seahawks were given 5-1 odds to win Super Bowl. The Tennessee Titans were given 250 – 1 odds. Explain whether you think this website is reporting odds against or odds in favor?

Exercises:

8. Fill out the rest of the table:

	$P(E)$	$P(\overline{E})$	$O(E)$	$O(\overline{E})$
a.	$\frac{3}{5}$			
b.		$\frac{7}{20}$		
c.			5 : 9	
d.				11 : 3
e.				8 : 912
f.			12 : 17	
g.		0.73		
h.	0.6			

9. If the probability of losing a game is $\frac{15}{23}$. How can you find the odds against winning?

10. In a race, the odds against a person winning are listed. Determine which person has the best chance of winning.
    a. Nacho Libre (8 : 1)
    b. The Dude (14 : 3)
    c. Heisenberg (10 : 7)
    d. Khalessi (53 : 9)
    e. Bruce Wayne (23 : 5)
    f. Stephen Colbert (11: 2)

Wrap-up and look back:

11. The odds against winning Blackjack in a casino vary based on the house rules and the number of decks used to play. If the odds in favor of winning are 127:129. What is the probability of winning?

12. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.10: Expected Value and Gambling

Vocabulary and symbols – write out what the following mean:

- Expected value
- Payout Odds
- Roulette
- Keno
- Lottery
- Craps

Concept questions:

1. What are the similarities and differences between odds and payout odds? How do these relate to probability?

2. How many total numbers are there for Keno, or does it vary?

3. How many total numbers are there for Lotteries, or does it vary?

4. If the actual odds matched the payout odds, what would be the result to the casino?

5. What are the options for a roulette wheel in terms of number of spaces on the wheel?

Exercises:

6. The expected value formula shown was involving winning and losing amounts. If there was a 25% chance to win \$10 and a 75% chance to lose \$5, the formula would be: $EV = (10)\left(\frac{1}{4}\right) + (-5)\left(\frac{3}{4}\right) = -1.25$.
   Could the formula be reworked and used with the "walk away amount" instead of "win/lose amount." For \$5 coming into the game, you'll either walk away with \$15 or walk away with \$0. Compute the expected value using these amounts and compare to the other formula.

7. If you bring \$500 to a casino and…
   a. Walk out with \$300, how much did you win?
   b. You win \$200, how much do you walk away with?

8. If you put a \$25 chip down on a roulette game, with a 1:1 payout, how much will you walk away with…
   a. If you win?
   b. If you lose?

9. If you buy \$25 in lottery tickets, describe your winnings if…
   a. Your tickets are cashed in for \$50,000?
   b. Your tickets are cashed in for \$10?
   c. Your tickets are cashed in for \$0?
   d. Your tickets are cashed in for \$23?
10. An insurance company charges you \$38.85 per year for a policy that will pay out \$150,000 if you die.
    a. Will they expect to make a profit if the probability that you die is 0.0003?
    b. Will they expect to make a profit if the probability that you die is 0.00008?
    c. How does the chance of something happening to you affect your premium?
    d. Explain why people who are in poor health are charged more for an insurance policy.

ROULETTE (standard 0-00 wheel, 38 total numbers):

11. Calculate the expected value of…
    a. the double-street bet (6 number line) that pays out at 5:1.
    b. the street bet (3 number line) that pays out at 11:1.
    c. the high bet (18 numbers) that pay out at 1:1.
    d. Is there a bet in standard roulette that is worse than the others?
    e. What is the "house edge" in standard roulette?

ROULETTE (standard 0 only wheel, only 37 numbers):

12. Calculate the expected value of…
    a. the double-street bet (6 number line) that pays out at 5:1.
    b. the street bet (3 number line) that pays out at 11:1.
    c. the basket bet (5 number line) that pays out at 6:1.
    d. the split bet (2 number) that pays out at 17:1.
    e. a straight bet (1 number) that pays out at 35:1.
    f. the even bet (18 numbers) that pay out at 1:1.
    g. is there a bet in 'single-0' roulette that is worse than the others?
    h. What is the "house edge" in 'single-0' roulette?

ROULETTE (standard 0 only wheel, only 37 numbers – Euro payouts):

13. Some casinos in Vegas (and a few in California) have a single wheel with modified "euro" payouts. When you bet on a 1:1 bet, if you lose, then you can keep half of your bet (but lose the other half). http://wizardofvegas.com/guides/roulette-survey/
    a. the street bet (3 number line) that pays out at 11:1.
    b. the square bet (4 number line) that pays out at 8:1.
    c. the basket bet (5 number line) that pays out at 6:1.
    d. the split bet (2 number) that pays out at 17:1.
    e. the even bet (18 numbers) that pay out at 1:1.
    f. the high bet (18 numbers) that payout at 1:1.
    g. is there a bet in 'single-0 with Euro payouts' roulette that is worse than the others?
    h. are there bets in 'single-0 with Euro payouts' roulette that are better than the others?
    i. What is the "house edge" in 'single-0 with Euro payouts' roulette?

ROULETTE – wheel and payout options.

14. Consider this site: http://wizardofvegas.com/guides/roulette-survey/
    a. What do you notice about standard roulette casinos in terms of the min and max bets?
    b. What do you notice about 'single-0' roulette casinos in terms of the min and max bets?
    c. What do you notice about 'single-0 w/ Euro payouts' roulette casinos in terms of the min/max bets?
    d. Why is the betting structure different?
    e. What is the smallest min bet for Euro? For single-0 betting? For standard 00 betting?
    f. What is the ratio of betting limits (max:min) – where is that ratio the highest and where is it the lowest?

ROULETTE – payouts.

15. Roulette is often listed as a quick way to raise money.
    a. If you made a table max bet of $2,000 on a single number, how much could you make?
    b. If you had only $6 decide to play at the Tropicana, with a $3 min and $3,000 max …
        i. how much money could you have if you started by making a $3 bet and won 3 times in a row (on single number bets paid out at 35:1)? What is the probability that this will happen?
        ii. how much money could you have if you started by making a $3 bet on a split bet (paid 17:1) and won 4 times in a row? What is the probability that this will happen?
        iii. how much money would you have if you made $3 bets and lost twice in a row (single number bets)? What is the probability that this will happen?
        iv. to encourage betting, a 36:1 payout is sometimes put down for single number bets on 00 wheels. Determine the expected value of a $1 single number bet if the payout is upped to 36:1.

CRAPS – Expected value.

16. Compute the expected value of a $1 "don't pass" bet in Craps. [Recall that this is saying that the shooter doesn't win. All wins for the shooter become losses for you, and all losses for the shooter become wins for you. Also, rolling a 12 on the first roll is a push – you don't lose or win.]

CRAPS – Sample exercises

17. The website here allows you to play a sample game of craps with a starting bankroll of $10,000: http://wizardofvegas.com/games/craps-v2/
    a. The max bet is $5,000 so play with $500 bets on the pass line and see how you are doing after 20 bets. Note: worst case, you lose each time and your bankroll is $0. Best bet is to win 20 times and be at $20,000.
    b. Reset the browser and play again with $1,000 bets on the pass line. See how you are doing after 20 bets.
    c. Reset the browser and play again with $2,000 bets on the pass line. See how you are doing after 10 bets.
    d. Reset the browser and play again with $5,000 bets on the pass line. See how you are doing after 10 bets.

CRAPS – Expected value.

18. Compute the expected value of a $1 bet if it is a…
    a. "field" bet in Craps. This is a bet on a single roll of the dice – the person placing the bets wins even money (1:1) if a 3-4-9-10-11 comes up, wins 2:1 if a 2 comes up, and 3:1 if a 12 comes up. Everything else loses.
    b. "Any 7" bet in Craps. This is a bet on a single roll of the dice – the person placing the bets wins (4:1) if it is a 7, and loses on anything else.
    c. "Any Craps" bet in Craps. This is a bet on a single roll of the dice – the person placing the bets wins (7:1) if it is a 2, 3, or 12, and loses on anything else.
    d. Out of all the bets we've seen so far, which of the bets seems like the best for the person placing a bet? Choose from Pass, Don't Pass, Field, Any-7, or Any-Craps and explain your decision.

19. Which is a better bet for the player – a $5 bet on single number (roulette) or a $5 bet on the pass line (craps)? Explain your reasoning.

KENO – Expected value.

20. For Keno, it is often easier to compute the expected value using the "walk-away" method instead of the "winning/losing" method. Find the expected value of a $1 bet on…
    a. 1-spot Keno in Vegas (assume $3 payout for matching 1, no payout otherwise).
    b. 2-spot Keno in Vegas (assume $12 payout for matching 1, no payout otherwise).
    c. 3-spot Keno in Vegas (assume $42 payout for matching 3, $1 for matching 2, and no payout otherwise).
    d. 4-spot Keno in Vegas (assume $130 payout for matching 4, $3 for matching 3, $1 for matching 2, and no payout otherwise).
    e. 5-spot Keno in Vegas (assume $700 payout for matching 5, $15 for matching 4, $1 for matching 3, and no payout otherwise).

KENO – with a twist

21. A person taking Math 52 and 95 decides to create a new game for people who don't win very often. She uses Keno as a basis and offers the following 5-spot Keno game called TLK (Total Loser Keno): pay $1 and win $4 if you get no numbers correct.
    a. Determine the expected value for this game.
    b. Based on the expected value in part (a), is this game better for players, or worse for players, compared with traditional Keno.
    c. In order to entice more people to play, she also offers a 6-spot Keno that costs $3 to play, but will pay out $18 if the player gets no numbers correct. Determine the expected value for this version.
    d. Based on the expected value in part (c), is this game better for players, or worse for players, compared with traditional Keno.
    e. With her success, she offers a third version: 8-spot Keno that costs only $1 to play and will pay out $11 if you get no numbers correct. Determine the expected value for this version.
    f. Based on the expected value in (e), compare this version with roulette, craps, and standard Keno. Which would be the best for the player?

KENO – analysis

22. A casino hears about the new TLK from the previous problem and begins playing 7-spot TLK, where the payout for a total loser is $5 for a $1 bet. However, every Saturday in May, the casino will double the payouts – there are 4 Saturdays in May.
    a. Determine the expected value for the original TLK game.
    b. Determine the expected value of the new (double payout) TLK game.
    c. Is it worth playing the TLK during the week? Explain.
    d. Is it worth playing the TLK on Saturdays in May? Explain.
    e. The max bet per ticket is $100, so you can play 20 games per ticket with $5 per game. If you play with $1,000 to start, and each game takes 4 minutes, you'd have to wait 80 minutes for the result of your first group of tickets to pay off. How much would you expect to get back from your $1,000?
    f. If you played the game repeatedly for the entire day, (game payouts every 80 minutes), how much would you have back in the first Saturday while putting all of your winnings back into the game? [Think of how many game sets you can get in during one full day]
    g. Repeat this for 3 more Saturdays in the month. Determine how much money you could make playing this game if there were no restrictions.

LOTTERIES – Expected value

23. Compute the expected value of the old Powerball (prior to October 2015) for a $136 million grand prize.
    a. Explain why the change was made?
    b. Would you rather play the original or the new Powerball game?

24. In 2015, the Oregon Lottery sold $1.12 billion in lottery products and of that, made payouts to winners as well as salary to a large number of employees of $525 million. That includes all other expenses too. With the money that remained, a payment of $545.9 million was made to the Oregon Economic Development Fund. On their website, they promote that the Oregon Lottery is a critical funding source for important Oregon programs. http://www.oregonlottery.org/about/
    a. http://www.theatlantic.com/business/archive/2015/05/lotteries-americas-70-billion-shame/392870/ This article shows that poor people are aggressively marketed to with lottery dollars, and that the poorest third of all households will buy 50% or more of lottery tickets.

(#24 continued)

b. Winning tickets are also subjected to federal and state income tax, so even a win of $1,000,000 would only end up being about half that amount after taxes.
c. Redo the computation for expected value on the Powerball: the grand prize is $136,000,000. This time compute the 'winnings' as the amount after taxes. The grand prize and 2nd prize would all have a 39.6% federal tax and a 10.3% state tax, with the 3rd prize having a 25% federal tax and 8.3% state tax. Remove the tax and use the new amounts to compute the expected value.
d. Nearly everyone winning the grand prize takes it as a 'lump sum' amount, which is only 60% of the listed prize. So $136,000,000 as a grand prize becomes a check for about 81,600,000. Redo the computations in part (d) with this as the new grand prize… but still tax it!
e. Compare this new EV with the original EV of about $ – 1.2147 per $2 ticket.
f. Many people say that lotteries are a tax on people who are bad a math. Explain the compounded lottery impact on poor people.

25. https://www.cmu.edu/news/archive/2008/July/july24_lottery.shtml This study showed that the poorest people often play the lottery as they see it the only way out of a bad financial situation.
    a. The survey indicates that poor people often play the lottery each week, treating it as a gamble but also a way to make money. Since they play every week, they often play when the jackpot is much smaller (starting at $40 million). Find the expected value of the Powerball with a $70 million prize as the baseline. (include the taxes and the fact that most people take the up-front lump sum).
    b. A person making less than $30,000 can spend up to 3% of their income on tickets, which is nearly $20 per week. Use this expected value to compute the expected value of playing $20 per week for one whole year?
    c. If the person had just put the money into an account with no interest, how much money would the person have?
    d. Compare (b) and (c) after 10 years. Which seems like a better tool for people with financial issues?
    e. Telling people that "everyone has an equal chance of winning the lottery" or "someone has to win it" have been shown to increase the number of lottery ticket purchases for people earning less than $30,000 per year. Explain why these slogans are often called "predatory." http://stoppredatorygambling.org/blog/category/research-center/lotteries-who-really-plays/
    f. Explain why lottery spending tends to increase when the poverty rate increase, when unemployment increases, and when people enroll in welfare programs.

Wrap-up and look back:

26. Most people say that a trip to a casino can be fun. How much money would you be willing to play with 'for fun?'

27. The New York Lottery had a slogan of "Helping Educate New York's Children Since 1967" printed on many of their tickets for decades. Explain why this is sending a confusing message.

28. If you were going to play for fun, which of the options we've learned about…
    a. offer the biggest payouts?
    b. offer the best chance of winning?
    c. offer the best expected value?
    d. Describe how the size of the payout and the expected value seem to relate.

29. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.11: Fun Probability Applications

Vocabulary and symbols – write out what the following mean:

- Blackjack

Concept questions:

1. Are there times where probability seems to be counter-intuitive? Explain while referencing an example from the text.

Exercises:

2. In the Nerf-gun example, we saw some interesting results. Compute a few more where D1 is the first daughter and D2 is the second daughter.
   a. Two darts are placed in the cylinder, but they are not side-by-side. If D1 pulls the trigger…
      i. what is the probability that a dart is fired?
      ii. if nothing happens, should D2 spin the cylinder again or just pull the trigger? Find the probability for each and explain your answer.
   b. Three darts are placed in the cylinder, and all are side-by-side. If D1 pulls the trigger…
      i. what is the probability that a dart is fired?
      ii. if nothing happens, should D2 spin the cylinder again or just pull the trigger? Find the probability for each and explain your answer.
   c. Three darts are placed in the cylinder, and none are side-by-side. If D1 pulls the trigger…
      i. what is the probability that a dart is fired?
      ii. if nothing happens, should D2 spin the cylinder again or just pull the trigger? Find the probability for each and explain your answer.
   d. Four darts are placed in the cylinder, and all are side-by-side. If D1 pulls the trigger…
      i. what is the probability that a dart is fired?
      ii. if nothing happens, should D2 spin the cylinder again or just pull the trigger? Find the probability for each and explain your answer.
   e. Five darts are placed in the cylinder, and all are side-by-side. If D1 pulls the trigger…
      i. what is the probability that a dart is fired?
      ii. if nothing happens, should D2 spin the cylinder again or just pull the trigger? Find the probability for each and explain your answer.

3. With the table minimums and maximums from this site: http://wizardofvegas.com/guides/roulette-survey/, the ratio of max:min can tell us how likely we are to lose a big bet in a Martingale (doubling bets) strategy.
   a. For the tables that are $100:$5, how many consecutive losses can you cover?
   b. What is the probability that you'll have that many losses in a row?
   c. The Cromwell offers a $3,000:$3 on a 00-wheel; how many consecutive losses can you cover?
   d. What is the probability that you'll have that many losses in a row?
   e. The Palazzo offers a $500:$100 on a 0-wheel with standard payouts; how many consecutive losses can you cover?
   f. What is the probability that you'll have that many losses in a row?
   g. The Wynn offers a $5,000:$100 on a 0-wheel with Euro payouts; how many consecutive losses can you cover?
   h. What is the probability that you'll have that many losses in a row?

4. Compare the probability of getting a Blackjack in a 4-deck shoe (4 decks of 52 are shuffled together) with the probability of getting a Yahtzee on one roll of five 6-sided dice. Which is more likely?

5. Compare the Martingale strategy (doubling bets) with standard betting of the table minimum for Roulette. If the table was $5 min and $300 max, run a computer simulation to determine which is more likely – turning a $100 bankroll into $150 or losing it all.

6. Analyze these situations on https://casino.bovada.lv/table-games. This site allows for 'practice' to begin with no email address, no real money, and $1,000 starting bankroll. The site has 500:1 table limits, which would allow for nearly 9 double-ups in a betting structure. However, that's for a $1 bet. Try the following bets in order to increase the payoffs at the lower level (especially after an early loss). 3 – 7 – 15 – 30 – 60 – 125 – 250 – 500. Once you win, you go back to the beginning again.
   a. Play American Roulette on the site until your bankroll is up to $1,100 or down below $800.
      i. Were you able to make the $1,100? Did you have a string of losses?
   b. Play European Roulette on the site until your bankroll is up to $1,100 or down below $800.
      i. Were you able to make the $1,100? Did you have a string of losers?

7. Based on this unit, determine expected casino revenue.
   a. If the casino is playing with American wheels (00) with a min bet of $10, how much would the casino bring in if there were $145,000 in bets made over the course of the day and there are 11 tables?
   b. If the casino is playing with single-0 wheels (standard payouts) with a min bet of $100, how much would the casino bring in if there were $145,000 in bets made over the course of the day and there are 11 tables?
   c. If you could bring up the min bet on the American wheel to $50, would you make more than the the single-0 wheel (standard payouts) with a min bet of $100? Explain.
   d. Which table structure would you rather have – American (low min bet) or single-0 wheel (high min bet)? Explain.
   e. Does the doubling system (Martingale) work over time?
      i. If you have 8 losses in a row on a standard 1:1 payout (00 wheel), what is the probability that you will lose 8 in a row?
      ii. If you have 8 losses in a row on a standard 1:1 payout (0 wheel), what is the probability that you will lose 8 in a row?
      iii. If you have 8 losses in a row, remember that you'll have lost nearly $1,000 with a very long road to get back any amount close to that. Would you be willing to attempt this strategy?

8. With the expected value of the $136,000,000 on current structure of Powerball, the Powerball group brings in sales of about $85 million. How much will they end up making on average?

Wrap-up and look back:

9. Write in words what you learned from this first section. Did you have any questions remaining that weren't covered in class? Write them out and bring them back to class.

2.12: Wrap-up and Review

Practice Exercises:

1. What can you determine from a game with higher points on one outcome over another? (An example is the full-house vs. large straight in Yahtzee).

2. Determine all the outcomes for the following events.
   a. Rolling an 8-sided die.
   b. Rolling a 6-sided die and flipping a coin.
   c. Drawing a number from these {1, 2, 13, 24, 35, 46} and picking a colored marble from {R, B, Y}.
   d. Flipping a coin 4 times, and recording the result (in order) after each flip.

3. Are these outcomes equally likely or not?
   a. Rolling a single 12-sided die and recording the number on top.
   b. Drawing a poker chip from a bag of 30 blue, 20 brown, and 50 green chips, then recording the color.
   c. Drawing a marble from a bag with 3 green, 3 red, and 3 blue, then recording the color.
   d. Flipping a coin and recording the result.

4. If you roll two 6-sided dice and then add the results, is each option (like sum of 4 or sum of 11) equally likely? Explain.

5. There are 25 Martians applying for a job, and the company will randomly select three interviewees from a hat because they are all equally qualified. There are 8 green Martians, 16 purple Martians, and 1 Matt Damon.
   a. What is the probability that the interviewees have 1 green, 1 purple, and Matt Damon?
   b. What is the probability that the interviewees have
   c. What is the probability that there will be no green Martians selected?
   d. What is the probability that there will be at least one green Martian selected?

6. From the table below, determine the following:

	Yes (Y)	No (N)	Dunno (D)	Total
Male (M)	43	7	11	61
Female (F)	9	17	41	67
Total	52	24	52	128

   a. Find $P(M)$.
   b. Find $P(Y)$.
   c. Find $P(D)$.
   d. Find $P(M \cap D)$.
   e. Find $P(F \cup N)$.
   f. Find $P(F \mid N)$.
   g. Find $P(N \mid M)$.
   h. Find $P(D \mid F)$.
   i. Find $P(F \mid D)$.
   j. Are *F* and *N* independent or not? Are they disjoint or not? Explain.
   k. Are *M* and *F* independent or not? Are they disjoint or not? Explain.
   l. Is $P(F \mid D) = P(D \mid F)$? Does this mean anything related to dependent/independent?
   m. Is this table an example of classical or empirical probability? Explain.
   n. What will be true about $P(F \cup M)$? What type of an event is this and why?
   o. What will be true about $P(F \cap M)$? What type of an event is this and why?
   p. What will be true about $P(F) + P(\overline{F})$? Explain.

7. From a bag with 10 marbles – 2 red, 3 green, 4 blue, and 1 yellow, determine the probability of…
   a. Drawing a blue marble.
   b. Drawing a marble that is blue or green.
   c. Drawing a marble that is blue and red.
   d. Drawing a marble that is not yellow.
   e. Drawing a marble that is not blue and not red.
   f. Drawing two marbles, with replacement, and getting blue then yellow (in order.
   g. Drawing two marbles, without replacement, and getting both blue.
   h. Drawing two marbles, without replacement, and getting both yellow.

8. Thinking of rolling 2 standard 6-sided dice, determine the following:
   a. The probability of rolling a sum of 8.
   b. The probability of rolling a sum of 10 or 11.
   c. The probability of rolling a sum of 5 or a sum that is even.
   d. The probability of rolling a sum of 5 and a sum that is even.
   e. The probability of rolling a sum of 4 and a sum that is even.
   f. The probability of rolling a sum that is greater than 5 or less than 9.
   g. The probability of rolling a sum that is greater than 5 and less than 9.
   h. The probability of rolling a sum that is less than 5 and greater than 9.

9. Odds against event E are 46:7. Find…
   a. The probability that *E* occurs.
   b. Find the probability that *E* occurs twice in a row (assume that the probability doesn't change when the procedure is repeated).
   c. Find the probability that *E* occurs at least once in 7 attempts.

10. Find the probability of randomly arranging the letters R-E-E-L-T-T and ending with LETTER.

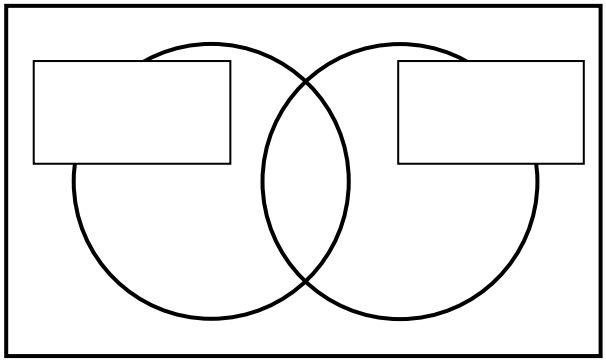
11. Annie, Bella, Christina, and Dee all applied for two jobs. There's a president and an assistant, and all qualify for the assistant position, but only Annie and Dee qualify for the president role.
   a. How many different hiring orders are possible?
   b. What is the probability that the positions are randomly selected and Annie is president and Dee is assistant?

12. There are 50 buttons from a clothing factory and 8 are defective.
   a. Find the probability of randomly selecting 6 buttons and having all defective.
   b. Find the probability of randomly selecting 6 buttons and having none defective.
   c. Find the probability of randomly selecting 6 buttons and having at least one defective.

13. Are the events "it is sunny" and "it is raining" independent? Explain.

14. On a true-false 8 question test, determine the probability of…
   a. Guessing and getting all 8 correct.
   b. Guessing and getting at least a 75%.

15. Use the Venn diagram below to help solve the problem. Assume that the probability of having a tire repair is 0.35, the probability of having an engine repair is 0.72, and the probability of neither of these is 0.11.



   a. Fill out the Venn diagram using the probabilities and label the diagram.

   b. Find the probability that both repairs are required.

   c. Are the events "tire repair" and "engine repair" disjoint? Why or why not.

   d. Find the probability that the customer needs tire repair or engine repair, but not both.

16. In a standard deck of cards, find the probability of…
   a. Drawing a spade.
   b. Drawing a spade or a diamond.
   c. Drawing a spade or an 8.
   d. Drawing a spade and an 8.
   e. Drawing two 8's in a row with replacement.
   f. Drawing two 8's in a row without replacement.
   g. Drawing three 8's in a row without replacement.
   h. Drawing three 8's in a row without replacement, given that the first card drawn was an 8.
   i. Drawing three 8's in a row without replacement, given that the first two cards drawn were both 8's.

17. Consider 15-spot Keno. Find…
   a. The number of ways to get exactly 10 correct.
   b. The probability of getting exactly 10 correct.
   c. The probability of getting no numbers correct.
   d. The probability of getting at least one number correct.

18. In craps, the "Pass Line" bet has odds of winning as 244 : 251.
   a. Find the probability of winning on the pass line.
   b. Find the probability of winning at least once in 4 bets.
   c. Find the expected value of a $20 bet on the "Pass Line" bet.
   d. What is the house edge on this bet?

19. On a standard roulette wheel (00), determine the expected value of a $1 bet on…
   a. The single number (paid 35:1).
   b. The column of 12 numbers (paid 3:1).
   c. A low bet on 18 numbers (paid 1:1)
   d. Repeat parts (a) thru (c) for a single-0 wheel with standard payouts.
   e. Repeat parts (a) thru (c) for a single-0 wheel with Euro betting (if you bet on 1:1 and a 0 comes up, then you lose only half your bet). This betting style is called "La Partage" which is French for "the sharing."
   f. What is the house edge for each of these?

20. In a group of 32 students, find the probability that…
    a. At least one student is born on July 4th. (assume all birthdays are equally likely)
    b. No one in the group shares a birthday.
    c. At least two students in the group share a birthday.

21. Jiaming is computing probability and using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Once he puts in the numbers, he has $P(A \cup B) = 0.8 + 0.6 - 0.25$. Is there an error? Explain why or why not.

22. If the expected value of a \$1 bet is \$ – 0.041, determine the expected value if…
    a. The player makes 500 \$1 bets.
    b. The player makes 50 \$10 bets.
    c. The player bets \$5000 once.