Math Fundamentals for Statistics I (Math 52)

Key

Unit 2: Number Line and Ordering

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"The 'How' and 'Whys' Guys"

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2.1: Place Values

We just looked at graphing ordered pairs using two number lines, now we need to look at the patterns with just one number. Since there is only one, we won't use coordinates or ordered pairs. Instead, we can think about the values represented visually on a single number line.

Some of what we will do connects to previous concepts about arithmetic sequences. We will deal with number lines that may not have all the labels, and it's up to us to find the missing pieces using what we know about common differences.

Example: Finish labeling the number line.

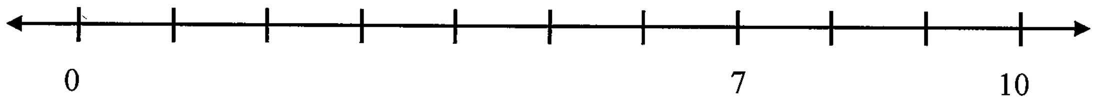
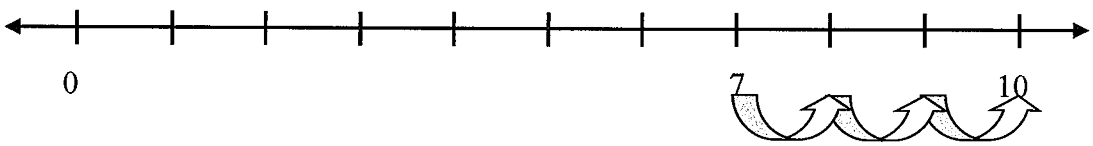


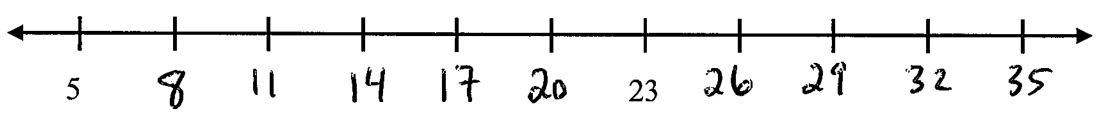
Figure out how far apart the numbers are by picking two numbers and counting the number of steps necessary to get to the next number. Count from the 7 to the 10.



It took 3 steps to get from 7 to 10, and the distance from 7 to 10 is 3 units. So we can divide to find out the common difference: $3 \div 3 = 1$, so we are counting by 1's. You can start at 0 and label the rest of the numbers counting by 1's.

Before we move on, let's try that one more time.

Interactive Example 2: Find the missing numbers on the number line.



A) What is the distance from 5 to 23 (found by doing 23 – 5)? 18

B) How many steps are there from 5 to 23? 6

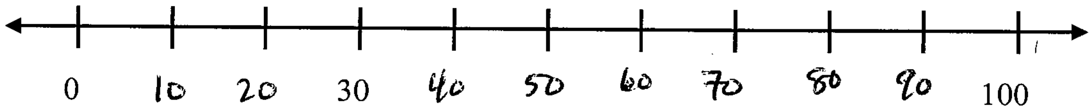
C) To find the common difference, it is part (A) divided by part (B). The difference is: 3

D) Now write in the rest of the numbers on the number line.

E) If it took 12 steps to get from the number 8 to the number 500, what is the common difference? $\frac{500-8}{12} = \frac{492}{12} = 41.$

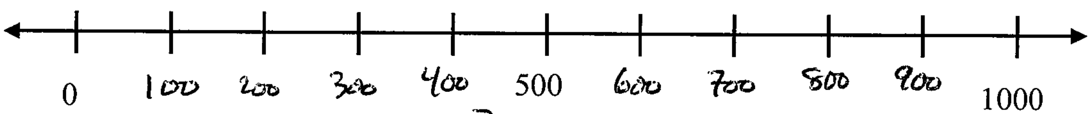
EXPLORE! Finish labeling the number line.

A) **



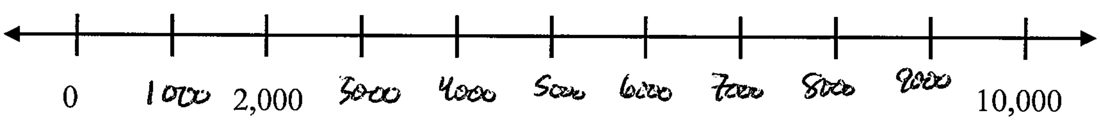
up 30 in 3 steps... $\frac{30}{3} = 10$ per step.

B)



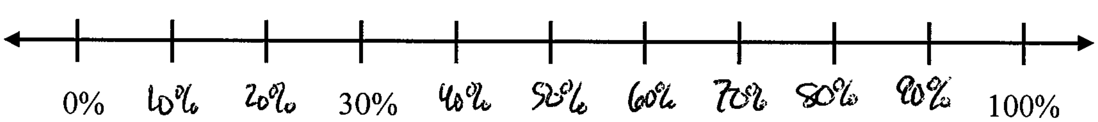
up 500 in 5 steps! $\frac{500}{5} = 100$ per step

C)



This one we can just <u>see</u> is 1000 per step.

D)



up 30% in 3 steps... $\frac{30\%}{3} = 10\%$ per step.

To understand the number line we need to be able to read numbers correctly, and to read numbers correctly, we need to know place value. The system of writing numbers in the way that we do is based on ten symbols. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are the **digits** that are used to form any number and the order listed here shows the order of the digits from smallest to largest. The location of the digits changes the value of the number. The **place value** is the value of a position (or place) of a digit in a number.

When we look at the number 5,372, the numeral 2 is in the ones place, 7 is in the ten place, 3 is in the hundreds place, and 5 is in the thousands place. The place values go up in value by a power of ten (times ten) for every place you move to the left.

Example: In the number 12,345, the 2 is in the thousands place.

We can also name numbers that are smaller than one by writing them as decimals. The **decimal point** divides the numbers that are greater than 1 from the numbers that are less than one. As an example, the number 0.5918 has 5 in the tenths place, 9 in the hundredths place, 1 in the thousandths place, and 8 in the ten thousandths place.

EXPLORE! Fill in the digit that is in the given place values:

38,045.6

A) Tens: 4

B) Thousands: 8

C) Ones: 5

D) Millions: 0

E) Tenths: 6

F) Hundreds: 0

G) Ten Thousands: 3

H) Hundreds: 0

↑

In some areas of science, decimals to the right are not listed as a 0 because it would imply more accuracy. In those classes, the answer would be N/A, or not applicable.

EXPLORE! Fill in the blank spaces for the Place Value and the Written Form.

Billions			Millions			Thousands			Ones				Decimals			
Place Value	10,000,000,000		100,000,000	10,000,000	1,000,000	100,000	10,000	1,000	100	10	1	.	0.1	0.01	0.001	0.0001
Written Form	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths	Ten Thousandths
	2	5	9	6	1	7	0	3	4	8	3	.	4	3	2	7

Interactive Example:

A) What are the words that come to mind when you think of the number 0?

Nada, zilch, nothing, zip

B) Does a number change in value if you put a 0 into the number?

It could; depends on where the 0 is put.

C) Consider the number 753. Would this number have the same value as 0753? How about 000000753?

Yes, all have the same value.

D) What if I put the zero somewhere else in the number, say 7053 or 7530, is that still the same value as the number 753? Why?

No, the place values are different.
753 has 7 hundreds, but the others
have 7 thousands.

EXPLORE! Create a rule for when you can put zeros into a number and not change its value.

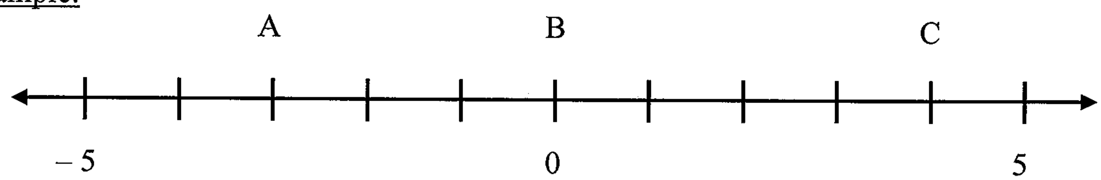
The value will not change if you put a 0…
past the last digit to the right of the decimal point OR
past the last digit to the left of the decimal point.

For Love of the Math: *In many areas of science, there actually is a difference between 4.00 and 4.0. Looking at the differences is an area known as* ***significant figures*** *and it measures not just the value of a number, but how accurate the tools were when measuring. For example, if you had a scale that measured to the nearest pound only, then 4 pounds would be all you could write. The actual weight might have been 4.1 or 4.2, or even 3.9, but you just can't tell. However, if you had a scale accurate to the nearest hundredth of a pound, then writing 4.00 means you are confident of the weight out to 2 decimal places. You may see significant figures (or significant digits) in biology, chemistry, and other sciences.*

2.2: Comparing Numbers

The **value** of a number is its position on the number line. Remember that the digits 0, 1, 2, …, 9 were in order from smallest to largest.

Example:



For the graph above, the value of A is – 3 because it is 3 units to the left of 0. The value of B is 0. The value of C is 4 because it is 4 units to the right of 0.

EXPLORE!

A) Could we write the value of A as just 3? Why or why not?

No, 3 implies it is positive (right of zero).

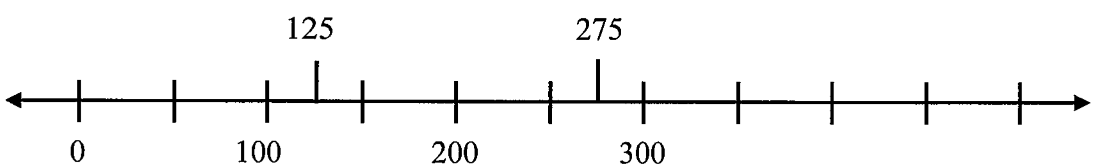
B) Could we write the value of C as just 4? Why or why not?

Yes, 4 implies positive (right of zero).

The **larger** of two numbers on the number line is the number located to the right, and the smaller number is to the left.

Example #1: Compare 275 and 125.

On the number line, 275 is to the right of 125, so it is the bigger number.



If the numbers are not on a number line, we determine the larger of two numbers by comparing the digits in the place values of each number.

<u>Example #2:</u> Compare 347 and 1,253.

If we consider the numbers 347 and 1,253, we can see that 1,253 has 1 in the thousands place value and 347 has 0 in the thousands place value. Since 1 is larger than 0, then 1,253 is the larger number.

<u>Example #3:</u> Compare 256 and 301.

If we have 256 and 301, both have digits in the hundreds place value. 256 has 2 in the hundreds place value, and 301 has 3 in the hundreds place value. Since 301 has more hundreds, it is the larger number.

<u>Example #4:</u> Compare 2,347 and 2,343.

Now consider 2,347 and 2,343. Both have a digit in the thousands place value, and each is a 2 representing 2,000. When this occurs we move to the next place value, the hundreds. If these have the same digit we move to the tens, and so on. With two positive numbers, if all the place values are the same, the two numbers are **<u>equal</u>**.

For 2,347 and 2,343, we see that the thousands, hundreds, and tens are the same, leaving the ones to determine if the numbers are equal or not. So 2,347 is bigger than 2,343.

<u>Example #4 (other method):</u> Compare 2,347 and 2,343 using a different method.

Another way to see this quickly is to stack the numbers up on top of each other and compare the place values from left to right. As soon as one place value in one number is bigger, that is the bigger number.

	Step 1	**Step 2**	**Step 3**	**Step 4**
First Number	**2**,347	2,**3**47	2,3**4**7	2,34**7**
Second Number	**2**,343	2,**3**43	2,3**4**3	2,34**3**
Comparison	Same	Same	Same	Different

7 wins in the last step, since 7 is larger than 3. So we know 2,347 is the bigger number.

We can even stack the values from Example 2: 0347
1253

We can quickly see that the bottom number has a larger digit in the thousands place value, and is therefore larger. This technique shows us rewriting 347 as 0347 so that it looks different, but has the same value.

<u>For Love of the Math:</u> *While doing math, mathematicians often change the way a number looks, without changing its value, to make a task easier. This is an excellent technique to learn as we continue through this course – keep the value, but change the way it looks!*

Interactive Example:

Which of the following two numbers are larger? Explain why using the words "place value" and "digit."

3,847,025
3,847,035

The digits in each place value are the same from left to right until the tens place. Since 2 is smaller than 3, the top number is smaller.

EXPLORE! Circle the larger number.

	Numbers
A)	888 and 8,352
B)	13,256 and 13,296
C)	1,473 and 1,573
D)	1,138 and 1,135
E)	1,113 and 1,111
F)	2,373 and 2,573
G)	8,452 and 8,352

Explain, in your own words, how you determine the larger of a pair of positive numbers:

Start at the largest place value and compare. If they are the same, move right to the next place value.

Create a rule for how to determine if two positive numbers are equal.

If all place values are identical, the numbers are equal.

2.3: Equality and Inequality

We've now been working with numbers that are larger or smaller than other numbers. In mathematics, we tend to write symbols to represent the concepts without using words! When we talk about **less than**, **greater than**, or **equal to**, each term has a specific symbol.

	Symbol	Example	Meaning
A)	$<$	$5 < 32$	5 is less than 32
B)	$>$	$356 > 316$	356 is greater than 316
C)	$=$	$437 = 437$	437 is equal to 437

Interactive Examples: Place the correct symbol ($<$, $>$, or $=$) between the following numbers:

A)	15	$<$	23
B)	$43{,}866 - 1$	$=$	43,865
C)	$53 + 6$	$>$	$37 + 20$

NOTE: *The pointy end of the symbol points to the smaller number.*

EXPLORE (1)! Place the correct symbol ($<$, $>$, or $=$) between the following numbers:

A)	34	$<$	36
B)	1,465	$<$	1,467
C)	$27 - 2$	$>$	$27 - 3$
D)	4,365	$>$	3,456
E)	933	$>$	399
F)	467	$=$	$466 + 1$

NOTE: The symbols used here can be combined to form new symbols: $\leq$ and $\geq$. Putting an equality with each inequality can increase the usefulness.

EXPLORE (2)! Place all correct symbols ($<$, $>$, $\leq$, $\geq$, or $=$) between the following numbers:

A)	34	$\leq$ or $<$	36
B)	1,465	$\leq$ or $<$	1,467
C)	$27 - 2$	$=$ or $\leq$ or $\geq$	$28 - 3$
D)	4,365	$\geq$ or $>$	3,456

2.4: Sorting Numbers

There are different ways to sort numbers. An easy way is to sort by finding the smallest number first, then the next smallest, and so on. (This is how most computers sort numbers)

Example: Sort the following numbers from smallest to largest.

1,357; 1,428; 1,345; 1,388; 1,401

In our technique, we will start with a number and compare it to the rest in order, swapping if necessary!

Step 1: Find the smallest number. Start by assuming the smallest number is the first number: 1,357.

Step 2: Compare 1,357 to the next number in the list. If it is smaller, then use the new number as the smallest and continue. $1{,}357 < 1{,}428$ so our smallest is still 1,357.

Step 3: Compare 1,357 to the next number in the list. If it is smaller, then use the new number as the smallest and continue. $1{,}357 > 1{,}345$ so our smallest is now 1,345.

Step 4: Compare 1,345 to the next number in the list. If it is smaller, then use the new number as the smallest and continue. $1{,}345 < 1{,}388$ so our smallest is still 1,345.

Step 5: Compare 1,345 to the next number in the list. If it is smaller, then use the new number as the smallest and continue. $1{,}345 < 1{,}388$ so our smallest is still 1,345.

Step	Remaining Numbers	Smallest (so far)
Step 1	**1,357**; 1,428; 1,345; 1,388; 1,401	1,357
Step 2	1,357; **1,428**; 1,345; 1,388; 1,401	1,357
Step 3	1,357; 1,428; **1,345**; 1,388; 1,401	1,345
Step 4	1,357; 1,428; 1,345; **1,388**; 1,401	1,345
Step 5	1,357; 1,428; 1,345; 1,388; **1,401**	1,345

Another way to sort the numbers is to split them (mentally) into groups and compare quickly.

In the list 1,357; 1,428; 1,345; 1,388; 1,401, we immediately rule out any of the numbers starting with "14" because they are bigger than all of the "13" numbers.

This narrows focus to: 1,357; 1,345; 1,388. Now look at the tens place for the smallest, which is "134" and you've got the smallest.

1,345 (smallest) and the next two are 1,357 and 1,388 (from our narrowed list). Comparing the last two "14" numbers is quick giving us 1,401 and 1,428 in order.

So the ordered list is: 1,345; 1,357; 1,388; 1,401; 1,428.

EXPLORE!

Sort the following numbers from smallest to largest.

A) ** 1,325 1,294 1,249 1,311 1,289

$1{,}249 < 1{,}289 < 1{,}294 < 1{,}311 < 1{,}325$

B) **(L)** 211 194 192 1,134 187

$187 < 192 < 194 < 211 < 1{,}134$

C) **(R)** 4,567 5,467 4,657 4,756 4,357

$4{,}357 < 4{,}567 < 4{,}657 < 4{,}756 < 5{,}467$

D) 2,213,496 2,213,596 2,212,497

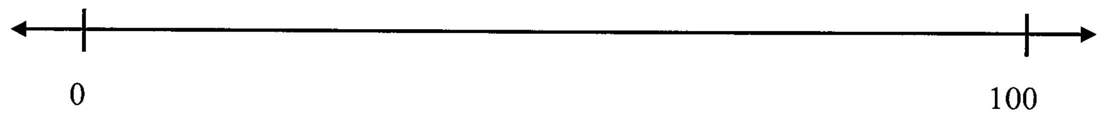
$2{,}212{,}497 < 2{,}213{,}496 < 2{,}213{,}596$

2.5: Placing integers on a number line

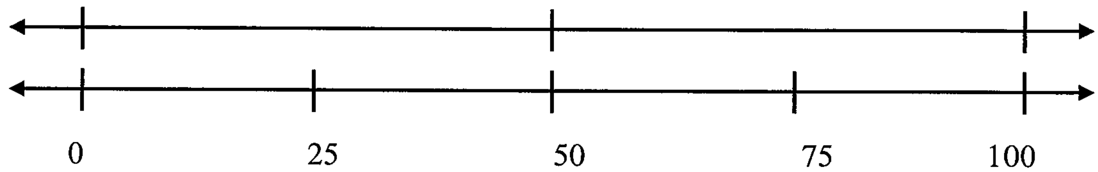
Number lines show the value of a number, and being able to visually see sizes of numbers is important going forward.

Interactive Example:

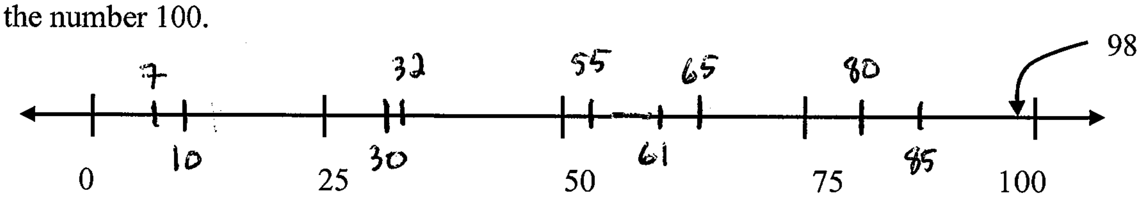
Place the number in the approximate position on the number line (as shown):
98, 10, 80, 30, 55, 32, 85, 7, 65, 61



It is a really good idea to have an idea of some number sizes, and we recommend splitting up the number line quickly. A fast way is to cut the line in half, then cut those pieces in half. Label these to make it easier to find numbers quickly.

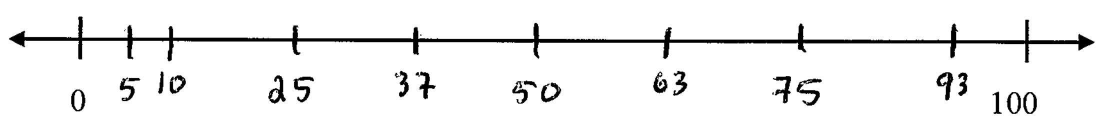


One way of approximating the numbers is to judge which numbers above is to ask which tens value it is closer to. For example: we know 98 is between 75 and 100, but which number is it closer to? 98 is 23 away from 75 and only 2 away from 100, so it's closer to 100. This is why the 98 is labeled close to the number 100.

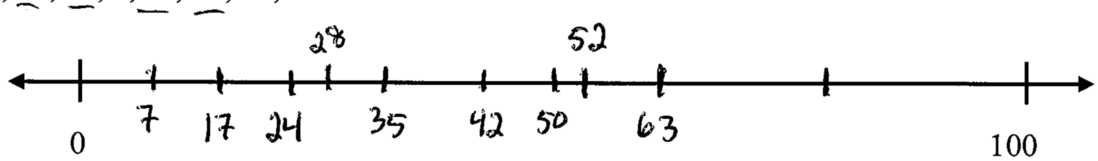


EXPLORE!

A) Place the number in the approximate position on the number line (same as above):
50, 25, 75, 37, 5, 63, 93, 10,



B) Place the number in the approximate position on the number line (same as above):
17, 35, 42, 7, 52, 24, 63, 28

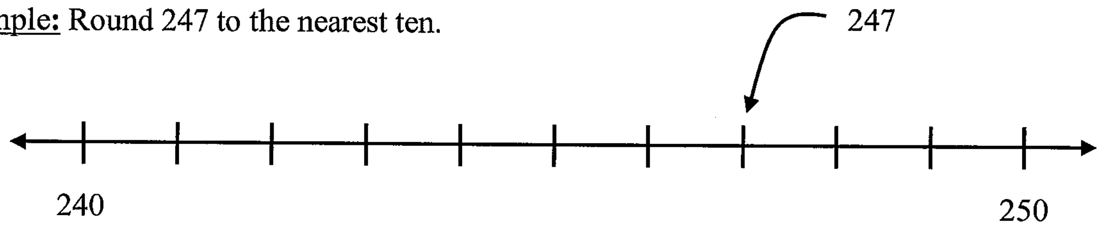


Pay attention to the ordering as well as the position. However, in this class, if the position is off a bit, that's not a big problem. But if the order is off, where you've written a smaller number so that it looks larger, that's a really big problem!

2.6: Rounding

We round numbers to estimate the value of the number and to make the value easier to work with when the exact value isn't needed. We typically **round** a number to a specific place value which means that we determine which value it is closest to.

Example: Round 247 to the nearest ten.



We're asking if 247 is closer to 240 or 250. From this picture we can see that 247 is closer to 250, so we say: 247 rounded to the nearest ten is 250.

Interactive Example: What about 240, 241, 242, 243 and 244? Which group of ten they closer to: 240 or 250?

These are closer to 240 than 250.

Interactive Example: Which of the numbers from 241 to 250 are closer to 250?

246, 247, 248, 249, and 250 are closer to 250 than 240.

What about 245? 245 is the same distance from 240 and 250. In this class, when a number is the same distance from the end points they are rounding to, we will always round them up; so 245 would round to 250.

For Love of the Math: *This method of rounding is sometimes known as rounding the 5 up, and while it is a common method, it is not the only way to round. There are other methods of rounding that will round the 5 up sometimes and down sometimes. Remember that both 240 and 250 are equal distances from 245, so based on our concept of rounding, either 240 or 250 would be correct for an answer. Having multiple correct rounding values can create problems, so mathematicians often agree on one answer that will be known as* ***conventional rounding****.*

You've noticed that the numbers from 240 to 250, rounded to the tens place, round to different numbers based on the ones place value.

- If the ones place value is 1, 2, 3, or 4, the number rounds to 240. Because the size of the new number is less than the original, we call this **rounding down**.
- If the ones place value is 5, 6, 7, 8, or 9 the number rounds to 250. Because the size of the new number is more than the original, we call this **rounding up**.
- On the ends, 240 rounds to 240 and 250 rounds to 250 so there is no need to round, because they are already whole groups of ten!

For the following, use a number line if necessary to determine the correct answer.

EXPLORE (1)! Round the following numbers to the nearest ten:

A) ** 549
550

B) 623
620

C) 3256
3,260

D) 195
200

EXPLORE (2)! Round the following to the nearest hundreds:

A) ** 2,551
2,600

B) 9,648
9,600

C) 27
0

D) 450
500 ← Convention
or
400

EXPLORE (3)! Round 7 to the nearest:

A) ** ten
10

B) hundred
0

C) thousand
0

EXPLORE (4)! Round 4,795 to the nearest:

A) ten 4,790 or 4,800 ↑ Convention

B) hundred
4,800

C) thousand 5,000

D) ten thousand
0

EXPLORE (5)! Round 4,734 …

A) up to the ten
4,740

B) down to the hundred
4,700

C) down to the thousand
4,000

D) up to the ten thousand
10,000

2.7: Decimals

We will look at other numbers later, but now we're now going to explore the set of positive decimal numbers. Let's look again at the decimals system that we use to write many of our numbers.

Billions			Millions			Thousands			Ones				Decimals				
Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	← Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousands	Hundred-thousandths
	6	8	2	4	1	0	3	9	0	0	5	.	2	1	7		

You have been using the left side of the decimal point so far, now we'll use the whole system.

- 0.3 is read as 3 tenths
- 0.27 is read as 27 hundredths
- 0.432 is read as 432 thousandths
- 0.006 is read as 6 thousandths
- 3.26 is read as 3 and 26 hundredths

Interactive Example: There are no "oneths." Can you explain why?

Tenths are a part of a whole number, but oneths would still be the whole number.

Tenths are 10 times bigger than hundredths, and hundredths are 10 times bigger than thousandths and so on. This is very similar to the whole place values that are greater than 0 because tens are 10 times bigger than ones, hundreds are 10 times bigger than tens, and so on.

When we read decimals we read the number as though there was no decimal place then we say the place value of the non-zero number furthest to the right of the decimal point.

Example:

2.35 tenths hundredths thousandths ten thousandths

2 and 35 hundredths

EXPLORE (1)! Circle the proper units for the decimal number, and then write the number in words. Refer to the place value chart on the previous page if needed.

A) ** 9.236 tenths hundredths thousandths ten thousandths

9 and 236 thousandths

B) 0.35264 thousandths ten thousandths hundred thousandths millionths

35,264 hundred thousandths

C) **(L)** 15.000007 thousandths ten thousandths hundred thousandths millionths

15 and 7 millionths

D) **(R)** 3.462 thousandths ten thousandths hundred thousandths millionths

3 and 462 thousandths

EXPLORE (2)! Determine if these students are correct in their answer and in their reasoning.

A) Marcy says that the 6 in 3.462 is "tenths" because when you count you do ones then tens. Because it's a decimal, you just add the 'ths' at the end.

Not correct, the 6 is hundredths.

B) Jacken says that the 3 in 3.462 is thousands because it is 4 digits from the start of the number.

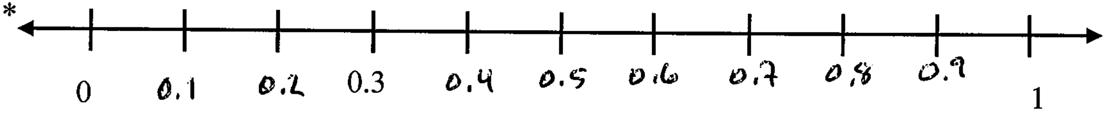
Not correct, the 3 is ones. We start at the decimal point... not the right.

C) Jalissa says that the 2 in 3.462 is thousandths because it is 3 digits to the right of the decimal point.

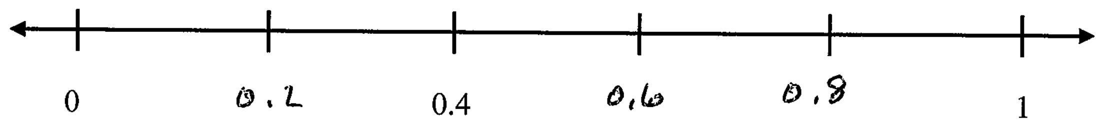
Yes, this is correct!

EXPLORE (1)! Finish labeling the number line.

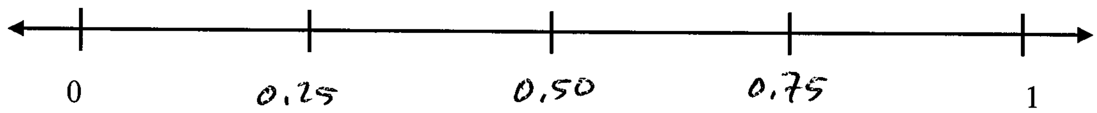
A) **



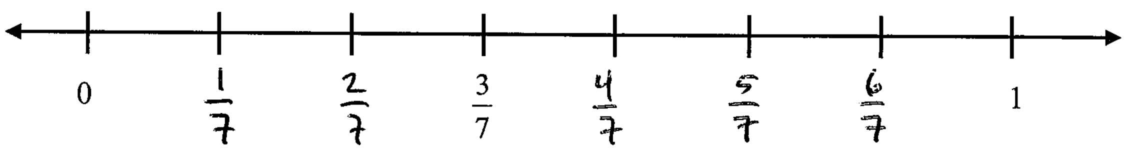
B)



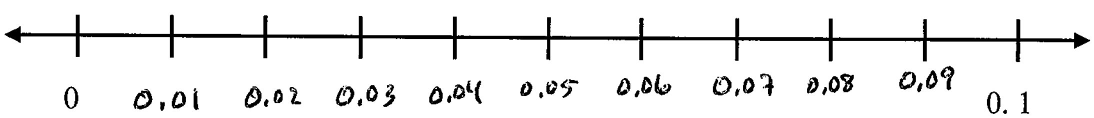
C)



D)



E)



EXPLORE (2)! Round 4.795314 to the nearest:

A) tenth 4.8

B) hundred 0

C) thousandth 4.795

D) ten thousandth 4.7953

E) hundred thousandth 4.79531

F) hundredth 4.80

The rules of ordering decimal numbers is nearly the same as we've done before, but now there is a decimal point.

EXPLORE (1)! Circle the larger number.

	Numbers			Numbers
A) **	3.2 and 2.4		B) **	11.07 and 10.17
C)	0.735 and 0.785		D)	2.81 and 2.8
E)	7.99 and 8.0		F)	1037 and 1.037
G)	0.138 and 0.0138		H)	9.3 and 9.32

EXPLORE (2)! Place the correct symbol (<, >, or =) between the following numbers:

A)	1.99	<	2.00
B)	4.6	>	4.3
C)	14.65	<	14.67
D)	5.95	=	5.95
E)	10 – 1	<	10 – 0.5
F)	42.978	>	4.2979

EXPLORE (3)! Write a number between 0.5 and 0.6.

0.52 (answers vary)

Interactive Example: How many numbers are there between 0.5 and 0.6?

There are infinitely many!

0.52
0.521
0.5212
0.52121
0.521212
⋮

For Love of the Math: *When mathematicians look at how tightly packed together numbers are, they often ask questions like "How many numbers are between...?" Integers are interesting, but when we ask how many integers are between 3 and 5, there is only one integer: 4. But when asking how many decimal or fraction numbers are between two different decimals, we find there is always another decimal number. This can be repeated over and over to discover that there are infinitely many decimals between any two decimal numbers! Mathematicians call this the **Density Property of Rational Numbers**, and it says that between any two fractions or decimal numbers is another fraction or decimal number. The decimals are incredibly dense – pretty cool!*

EXPLORE! Use the thinking you have developed on the size of numbers to put these positive numbers in order from smallest to largest:

A) ** 2.45 3.5 7 1.234 43.525

$1.234 < 2.45 < 3.5 < 7 < 43.525$

B) **(L)** 21.1 19.4 19.2 113.4 18.7

$18.7 < 19.2 < 19.4 < 21.1 < 113.4$

C) **(R)** 0.216 0.211 0.215 0 .219 0.10

$0.10 < 0.211 < 0.215 < 0.216 < 0.219$

D) 1.11243 1.11234 1.11247 1.11311

$1.11234 < 1.11243 < 1.11247 < 1.11311$

Near the beginning of Unit 2, we saw the power of 0 (zero) and how we could write additional zeros in some place values. Depending on where the 0 was written, the value of the number could either change or not change.

Interactive Example: Determine if the two numbers have the same value or different value.

	Numbers		Same or Different?	
A) **	740	0740	Same	Different
B)	704	74	Same	Different
C)	740	740.0	Same	Different
D)	74.0	74	Same	Different

Come up with a rule to determine where a zero can be written and not change the value of a number.

To the right of a decimal, any zero past the last digit.

To the left of the decimal, any zero past the last digit.

EXPLORE! Determine if the numbers have the same value or different value.

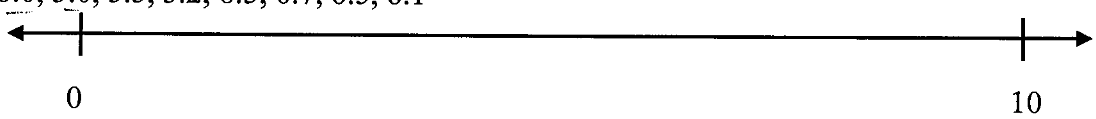
	Numbers		Same or Different?	
A)	0.37	0.037	Same	Different
B)	0.37	0.370	Same	Different
C)	0.37	.37	Same	Different
D)	10.37	1.37	Same	Different
E)	15.24	15.240	Same	Different
F)	15.024	15.24	Same	Different
G)	0027	0.027	Same	Different
H)	.0027	0.0027	Same	Different

For Love of the Math: *You might notice that a numbers like 0.45 and .45 have the same value, and that the leading 0 doesn't change the value. In this class, we use the convention of writing the 0 in front to avoid any confusion. Again, both have the same value, but we choose one way and use it.*

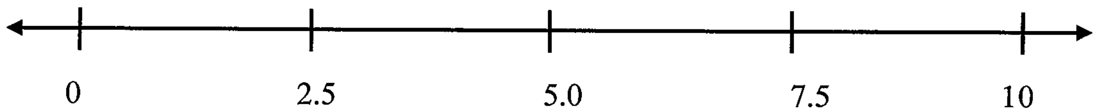
2.8: Placing decimals on a number line

Interactive Example: Place the number in the approximate position on the number line (as shown):
Hint: 6.5 is between 6 and 7, but 0.65 is between 0 and 1

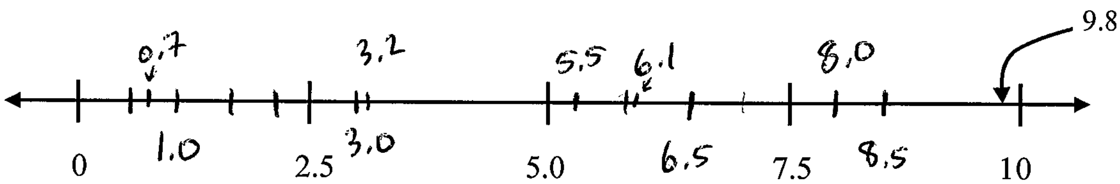
9.8, 1.0, 8.0, 3.0, 5.5, 3.2, 8.5, 0.7, 6.5, 6.1



Cut it into pieces like we've done before:



Now it is easier to put in the numbers:

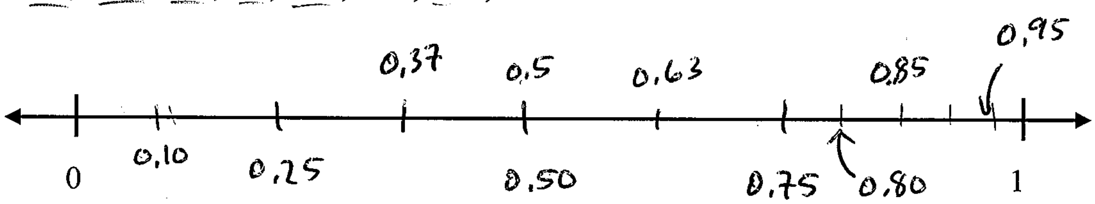


When finished, go back to section 2.5 and compare this graph with that graph. What similarities do you notice and can you explain why these are similar?

They look nearly identical... this has a decimal point while the other doesn't.

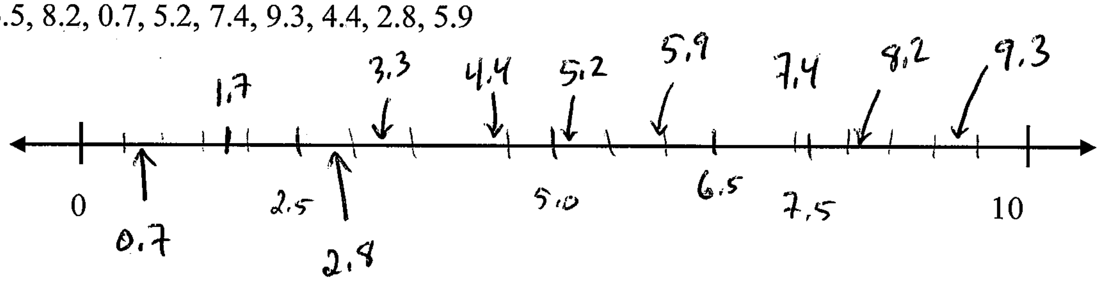
EXPLORE! Place the number in the approximate position on the number line (same as above):

0.50, 0.25, 0.75, 0.37, 0.85, 0.5, 0.63, 0.95, 0.10, 0.80



Interactive Example (2): Place the number in the approximate position on the number line:

1.7, 3.3, 6.5, 8.2, 0.7, 5.2, 7.4, 9.3, 4.4, 2.8, 5.9



2.9: Negative Integers

So far we've looked at positive numbers that are part of a group of numbers called integers. **Integers** are the numbers: . . . –5, – 4, –3, –2, –1, 0, 1, 2, 3, 4, 5, . . . The **positive integers** are: 1, 2, 3, 4, . . . and the **negative integers** are – 1, – 2, – 3, – 4, – 5, … Negative numbers have an order just like positive numbers, and this section helps show how the other side of the number line works.

EXPLORE! Finish labeling the number line.

A) **

–10   –7   –4   – 1   2   5   8   11   14   17   20

B)

–100   –80   –60   –40   – 20   0   20   40   60   80   100

C)

–1   –0,9   –0,8   –0,7   –0,6   –0,5   –0,4   –0,3   –0,2   –0,1   0

D)

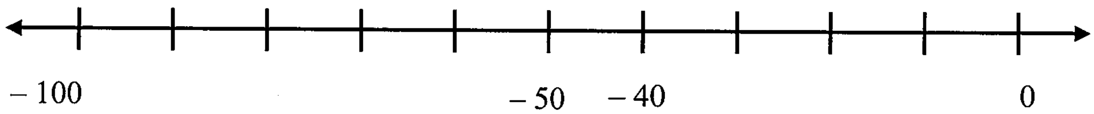
–0,09   –0,07   –0,05   –0,03   –0,01

–0.1   –0,08   –0,06   –0,04   –0,02   0

EXPLORE! Which number is larger, – 40 or – 50? Why?

–40 is larger than –50 because it is further to the right on a number line.

Recall, the definition of larger: the **larger** of two numbers on the number line is the number to the right and the smaller number is to the left.



EXPLORE! Fill in the table by circling the larger number of each pair.

	Numbers
A) **	– 7 and – 3
B) **	– 10.7 and – 15
C)	– 77 and –8
D)	– 187 and –15
E)	– 1,007 and – 50
F)	– 1,007 and 50
G)	– 2.37 and – 3.5
H)	– 93 and – 39

EXPLORE! Explain, in your own words, how you determine the larger of all pairs of negative numbers:

What is to the right on a number line, as it is the larger number.

Interactive Example: Do negative numbers follow the same pattern as positive number? Explain

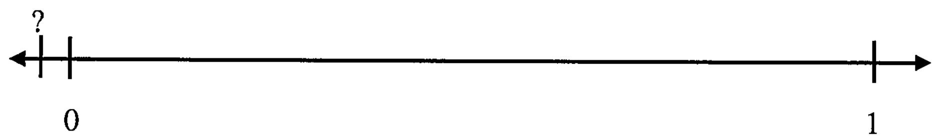
Yes, in a sense. The larger number is still to the right on the number line.

EXPLORE (1)! Place the correct symbol (<, >, or =) between the following numbers:

A) **	– 199	>	– 200
B) **	– 247	>	– 274
C)	– 4.6	<	– 4.3
D)	– 42,978	>	– 42,979
E)	– 1.28	>	– 1.45
F)	150	<	200
G)	– 150	>	– 200

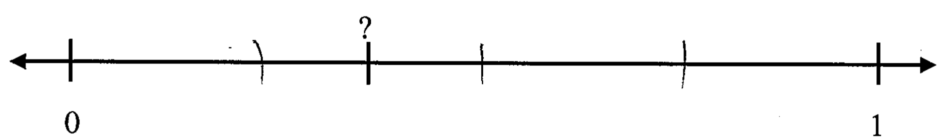
EXPLORE (2)! Approximate the value of the number using the number line.

A) **



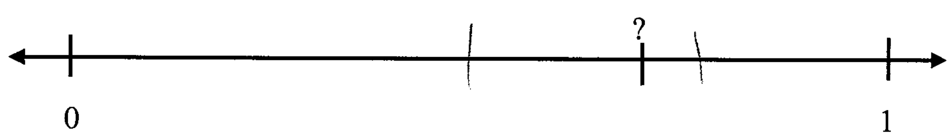
about
– 0.05

B)



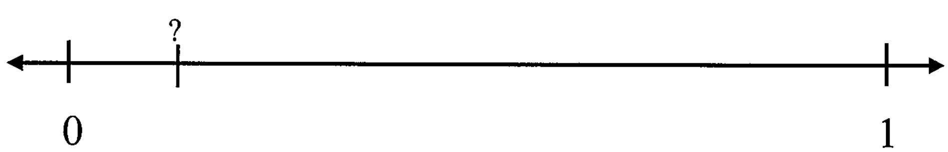
about
0.37

C)



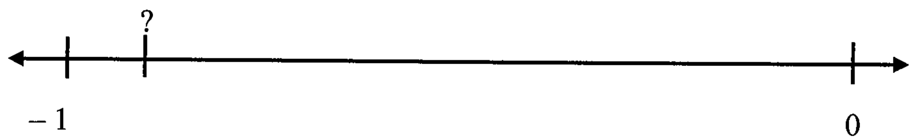
about
0.7

D)

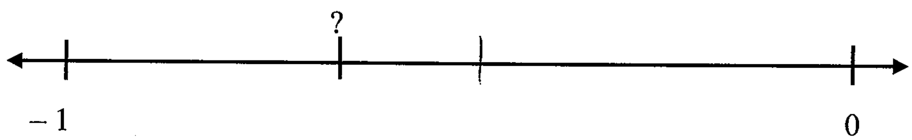


about 0.10

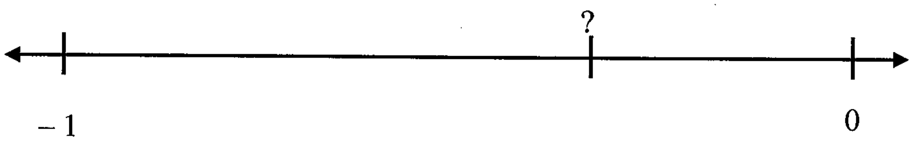
EXPLORE! Approximate the value of the number using the number line.

A) **


About −0.9

B)


About −0.7

C)


About −0.33

D)


About −0.05

Interactive Examples:

A) Write three numbers between −0.3 and −0.2.

−0.25, −0.24, −0.23 (many others)

B) How many positive numbers are there between −0.3 and −0.2?

None ... these are negative numbers!

C) How many negative numbers are there between −0.3 and −0.2?

Infinitely many:

−0.2
−0.21
−0.212
−0.2121
−0.21212
⋮

2.10: Perfect Squares

A number is a **perfect square** if it is a number multiplied by itself.

Example: Show that (A) 25 and (B) 169 are perfect squares.

A) $5 \times 5 = 25$ so 25 is a perfect square.

B) 169 is a perfect square because $13 \times 13 = 169$

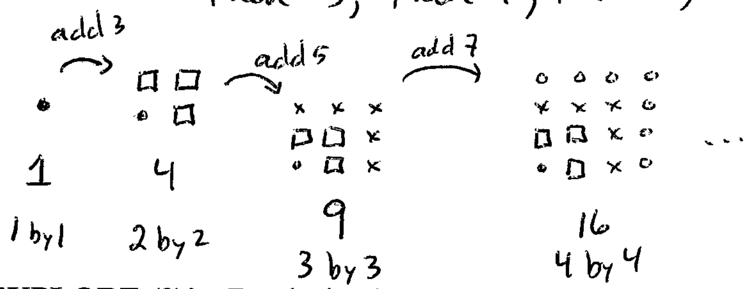
There are two different ways to write 7 squared: 7×7 and 7^2. Once again, these are the same value of 49, but are different ways to write it.

Interactive Example: Find the square for the following numbers

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100

EXPLORE (1)! Look at the numbers you wrote in. Find a pattern with these squares and explain the pattern.

The pattern seems to be that we add 3, then 5, then 7, then 9, etc.



each odd number makes a new perfect square (shape).

EXPLORE (2)! For decimal answers, round to the nearest hundredth if necessary.

Number	13	11	0.7	0.3	25	2.5	$\pi \approx$	−15
Square	169	121	0.49	0.09	625	6.25	9.87	225

approximation

Which number has the bigger square, 0.7 or 0.3? Why do you think that is?

$(0.3)^2 < (0.7)^2$

Maybe a bigger number squared is still bigger.

2.11: Square Roots

$\sqrt{x}$ is the symbol for the square root of x, where x is a number. $\sqrt{\ }$ is the square root symbol.

Examples: Find the square roots of (A) 16 and (B) 49.

A) $\sqrt{16} = 4$ because $4 \times 4 = 16$.
B) $\sqrt{49} = 7$ because $7 \times 7 = 49$.

The **square root** of a number a, denoted by n, is the non-negative number that, when multiplied by itself is equal the original number. We write this as $\sqrt{a} = n$.

EXPLORE! Find the square root of the following to 2 decimal places.

Number	1	2	3	4	5	6	7	8	9	10
Square Root	1	1.41	1.73	2	2.24	2.45	2.65	2.83	3	3.16

For Love of the Math: *We can see that both* $3^2 = 9$ *and* $(-3)^2 = 9$. *Since there are two possibilities that get us to 9, we might have two possibilities for the square root. However, the* $\sqrt{\ }$ *represents the non-negative number which is why* $\sqrt{9} = 3$ *and* $\sqrt{9} \neq -3$. *3 is called the **principal square root** of 9.*

EXPLORE! Find the value of the following (try these without a calculator):

Number	81**	25	36	49	4	100	64	9	1	16
Square Root	9	5	6	7	2	10	8	3	1	4

The main piece that we would like you to take away from square roots is an ability to estimate the relative size. In order to do this, we need the ability to find the square root of perfect squares like the ones above. Because $\sqrt{38} < \sqrt{39}$, we can use this to find whole numbers that are above or below a square root.

Example: Estimate the size of $\sqrt{38}$.

In order to estimate the size of $\sqrt{38}$, we can think of perfect squares that are above and below 38. If you can spot them quickly, do that: 36 is very close to 38 and is a perfect square. $\sqrt{36} = 6$, so the number above it must be $7 = \sqrt{49}$. This shows $6 < \sqrt{38} < 7$.

If you're not sure about what perfect squares are close to a number, pick a number and square it. Too small, go a little bigger. It may take time, but you'll get the hang of it with practice!

EXPLORE (1)! Estimate the size of the following square roots by finding whole numbers above and below them. Push yourself to not use a calculator for this part… you can do it!

A) ** $\sqrt{50}$ $7^2=49$ $8^2=64$

$\boxed{7 < \sqrt{50} < 8}$

B) $\sqrt{7}$ $2^2=4$ $3^2=9$

$\boxed{2 < \sqrt{7} < 3}$

C) $\sqrt{23}$ $4^2=16$ $5^2=25$

$\boxed{4 < \sqrt{23} < 5}$

D) $\sqrt{86}$ $9^2=81$ $10^2=100$

$\boxed{9 < \sqrt{86} < 10}$

E) $\sqrt{73}$ $8^2=64$ $9^2=81$

$\boxed{8 < \sqrt{73} < 9}$

Now use your skill to put numbers in order (without a calculator).

EXPLORE (2)! Put the following numbers in order from smallest to largest:
$\sqrt{81}$, 8.5, 3.6, $\sqrt{9}$, 5.1, $\sqrt{25}$

$\sqrt{9} < 3.6 < \sqrt{25} < 5.1 < 8.5 < \sqrt{81}$

EXPLORE (3)! Put the following numbers in order from smallest to largest. Write in $\sqrt{\ }$ form. Use your calculator if necessary:
$\sqrt{87}$, $\sqrt{32}$. $\sqrt{8}$, $\sqrt{55}$, $\sqrt{96}$, $\sqrt{69}$, $\sqrt{27}$

$\sqrt{8} < \sqrt{27} < \sqrt{32} < \sqrt{55} < \sqrt{69} < \sqrt{87} < \sqrt{96}$

Create a rule that allows you to put square root numbers in order.

If $a < b$ then $\sqrt{a} < \sqrt{b}$

Assume a and b are positive.

2.12: Approximating square roots

When using advanced calculators, we can see that $\sqrt{17} \approx 4.1231056256176605498 21\ldots$ We use the symbol $\approx$ instead of $=$ to show that this is an approximation. If we were to type all the decimal places shown on a calculator and squared it, we would get very close to 17 but wouldn't be at exactly 17.

Square roots have a decimal representation that goes forever, doesn't repeat and doesn't stop… unless it is the square root of a perfect square.

▦ **EXPLORE!** Approximate the following square roots out to 6 decimal places using the calculator. Then, with the decimal representation on screen, use the calculator to convert it to a fraction.

	Square Root	Decimal Approximation	Fraction Representation (if possible)
A) **	$\sqrt{17}$	4.123106	Not possible
B) **	$\sqrt{5{,}298}$	72.787362	Not possible
C)	$\sqrt{52.98}$	7.278736	Not Possible
D)	$\sqrt{\frac{9}{16}}$	0.75	$\frac{3}{4}$
E)	$\sqrt{0.000025}$	0.005	$\frac{1}{200}$
F)	$\sqrt{0.1364}$	0.369324	$\frac{\sqrt{341}}{50}$, but not better than this.
G)	$\sqrt{\frac{7}{9}}$	0.881917	$\frac{\sqrt{7}}{3}$, but not further.
H)	$\sqrt{\frac{169}{225}}$	0.866667	$\frac{13}{15}$

For Love of the Math: *The convert to fraction button on the calculator is pretty cool but does have limitations. Many fractions need dozens or hundreds of decimal places to be seen in order to be precise with the fraction, and the TI-30XIIS calculator has a 10 digit display, but holds a few extra digits in memory. The limitation for the fraction button is a 3-digit denominator. Try typing in 1 ÷ 999 and press enter, then press the convert to fraction button. Now try 1 ÷ 1001 and do the same thing. Because 1,001 is more than 3 digits, the calculator programming won't return the fraction form… even though it does have fraction form. Enjoy the cool feature on your calculator, but know that it is limited. The Casio fx-300ES Plus doesn't have the same drawback, but there is a limit. Where does the Casio no longer use fraction form?*

2.13: Number Line Connections

Since we've seen all types of integers and decimals, including positive and negative, let's make a number line that includes all types.

EXPLORE! Finish labeling the number line. up 30, over 10 spaces. $\frac{30}{10} = 3$

A) **

–25   –22   –19   –16   –13   –10   –7   –4   –1   2   5

up $\frac{200}{10} = 20$

B)

–150   –130   –110   –90   –70   –50   –30   –10   10   30   50

up $\frac{50}{5} = 10$

C)

–20   –10   0   10   20   30

up $\frac{4}{10} = 0.4$

D)

–3.4   –3.0   –2.6   –2.2   –1.8   –1.4   –1.0   –0.6   –0.2   0.2   0.6

<u>Interactive Example:</u> Which number is larger, – 5.3 or – 1.7? Why?

–1.7 is larger as it is further to the right on a number line.

Recall, the definition of larger: the **larger** of two numbers on the number line is the number to the right and the smaller number is to the left.

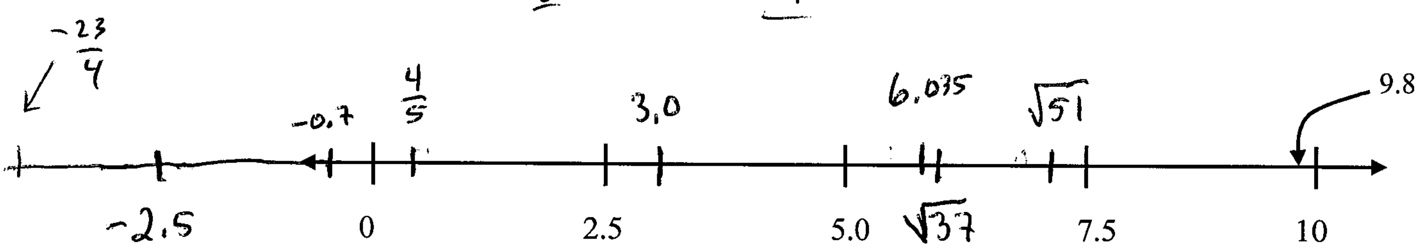
EXPLORE (1)! Circle the larger number.

	Numbers
A) **	$\sqrt{37}$ and 6.4
B) **	$-\sqrt{51}$ and $-\sqrt{37}$
C)	0.66 and $\frac{2}{3}$
D)	-187 and -159
E)	-1.007 and 0.048

$\sqrt{51} > \sqrt{37}$, but negatives are different!

Interactive Example: Place the number in the approximate position on the number line (9.8 is shown):

9.8, $\sqrt{37}$, $\sqrt{51}$, 3.0, -2.5, $\frac{4}{5}$, -0.7, 6.035, $-\frac{23}{4}$



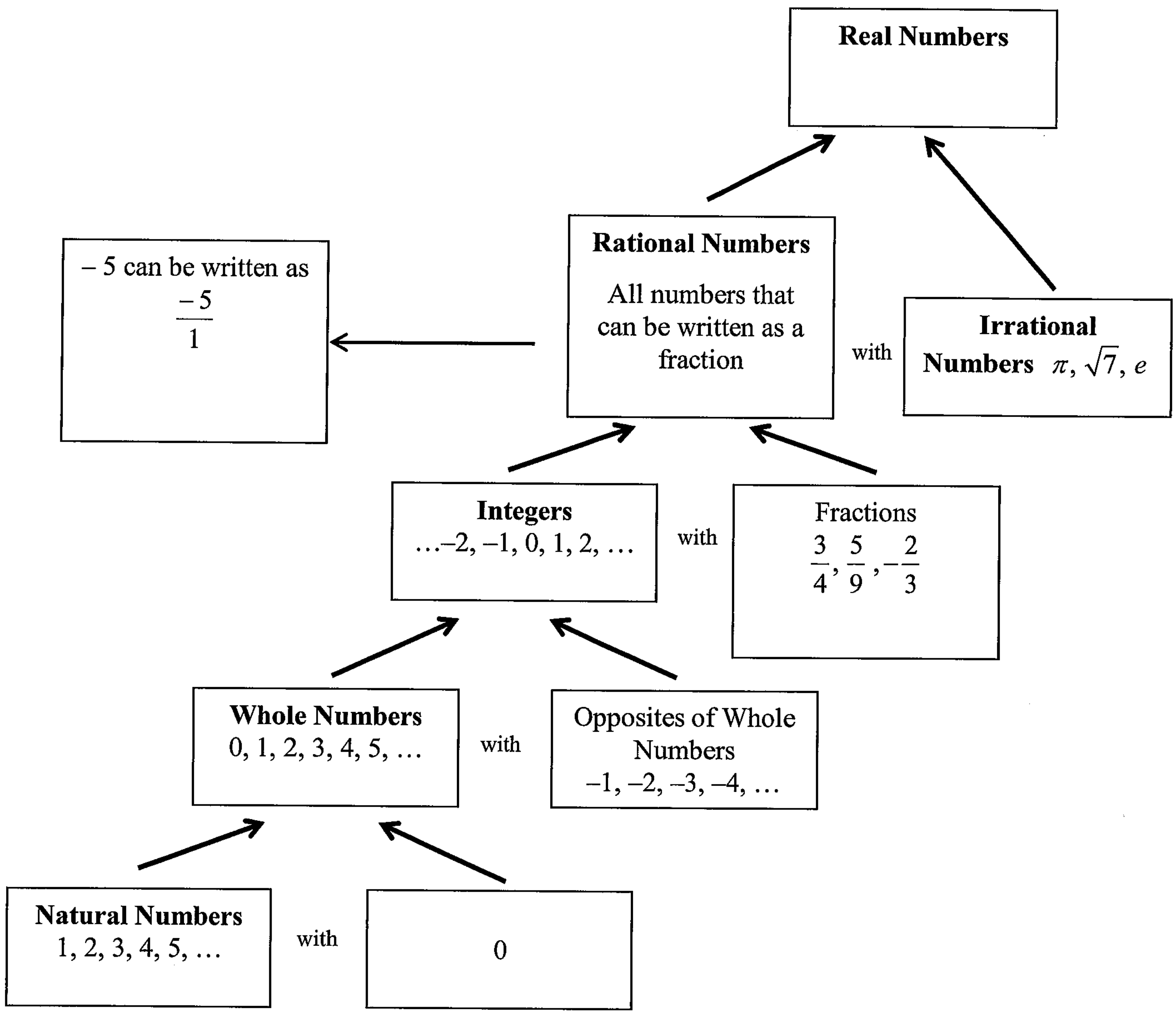
EXPLORE (2)! Use the thinking you have developed on the size of numbers to put these numbers in order from smallest to largest (without using a calculator):

A) $2\frac{3}{4}$ $\quad\sqrt{18}\quad 4 \quad 5$ ⟶ $2\frac{3}{4} < 4 < \sqrt{18} < 5$

B) $-\sqrt{27}\quad -6.25\quad -2.8\quad -5\quad -6.43$

$-6.43 < -6.25 < -\sqrt{27} < -5 < -2.8$

This is an image of the relationships between the number sets we will work with in this course. These names are what we often refer to. We've seen all of these types so far, but haven't always used their names.



<u>For Love of the Math:</u> *Mathematicians enjoy discovering different number sets, and it took thousands of years to create just the ones in the table. Our table is not complete though, and if you continue taking more math classes, you may encounter new sets. There are numbers outside of the real numbers like imaginary numbers, complex numbers, and surreal numbers (to name a few).*

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equal to.. 11
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