Math 155 Practice Test - Chapter 8

8.1

#1. $\int \frac{1}{\cos\theta - 1}\,d\theta = \int \frac{1}{(\cos\theta-1)}\left(\frac{\cos\theta+1}{\cos\theta+1}\right)d\theta$

$= \int \frac{\cos\theta+1}{\cos^2\theta-1}\,d\theta$ — use $\sin^2\theta+\cos^2\theta=1$, $\cos^2\theta-1=-\sin^2\theta$

$= \int \frac{\cos\theta+1}{-\sin^2\theta}\,d\theta$

$= -\int \frac{\cos\theta}{\sin^2\theta}\,d\theta - \int \frac{1}{\sin^2\theta}\,d\theta$ — let $u=\sin\theta$, $\frac{du}{d\theta}=\cos\theta$, $\frac{du}{\cos\theta}=d\theta$

$= -\int \frac{\cos\theta}{u^2}\left(\frac{du}{\cos\theta}\right) - \int \csc^2\theta\,d\theta$

$= -\int u^{-2}\,du - (-\cot\theta) + c$

$= -\left[-1\cdot u^{-1}\right] + \cot\theta + c$

$= \frac{1}{u} + \cot\theta + c$

$= \frac{1}{\sin\theta} + \cot\theta + c$

$= \underline{\csc\theta + \cot\theta + c}$

8.2

#2. $\int e^x\cos(x)\,dx$ — let $u=e^x$, $\frac{du}{dx}=e^x$, $du=e^x dx$; $\frac{dv}{dx}=\cos(x)$, $dv=\cos(x)dx$, $\int dv=\int\cos(x)dx$, $v=\sin(x)$

$= (e^x)(\sin x) - \int(\sin x)(e^x dx)$

$= e^x\sin x - \left[(e^x)(-\cos x) - \int(-\cos x)(e^x dx)\right]$

$\int u\,dv = uv - \int v\,du$

$= e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$ ← look

let $u=e^x$, $du=e^x dx$; $dv=\sin(x)dx$, $v=-\cos(x)$

$\int e^x\cos(x)dx = e^x\sin x + e^x\cos x - \int e^x\cos(x)dx$

$+\ \int e^x\cos(x)dx = \qquad\qquad \int e^x\cos(x)dx$

$2\int e^x\cos(x)dx = e^x\sin(x) + e^x\cos(x)$ →

#2. continued

$$\frac{1}{2}\left[2\int e^{x}\cos(x)\,dx\right] = \frac{1}{2}\left[e^{x}\sin(x) + e^{x}\cos(x)\right] + C$$

$$\int e^{x}\cos(x)\,dx = \frac{1}{2}\left(e^{x}\sin(x) + e^{x}\cos(x)\right) + C$$

$\boxed{8.2}$

#3. $$\int_0^{\pi/4} e^{-x}\cos(x)\,dx$$

let $u = e^{-x}$	$\frac{dv}{dx} = \cos(x)$
$\frac{du}{dx} = -e^{-x}$	$\int dv = \int \cos(x)\,dx$
$du = -e^{-x}\,dx$	$v = \sin(x)$

$$= \left[(e^{-x})(\sin x)\right]_0^{\pi/4} - \int_0^{\pi/4}(\sin(x))(-e^{-x}\,dx)$$

$$= \left[e^{-(\pi/4)}\cdot\sin(\pi/4) - e^{-(0)}\cdot\sin(0)\right] + \int_0^{\pi/4} e^{-x}\sin(x)\,dx \qquad \int u\,dv = uv - \int v\,du$$

$$= \left[\left(\frac{1}{e^{\pi/4}}\right)\left(\frac{\sqrt{2}}{2}\right) - (1)\cdot 0\right] + \left\{\left[(e^{-x})(-\cos(x))\right]_0^{\pi/4} - \int_0^{\pi/4}(-\cos(x))(-e^{-x}\,dx)\right\}$$

let $u = e^{-x}$	$\frac{dv}{dx} = \sin(x)$
$\frac{du}{dx} = -e^{-x}$	$\int dv = \int \sin(x)\,dx$
$du = -e^{-x}\,dx$	$v = -\cos(x)$

$$= \frac{\sqrt{2}}{2e^{\pi/4}} + \left[-e^{-(\pi/4)}\cdot\cos(\pi/4) + e^{-(0)}\cdot\cos(0)\right] - \int_0^{\pi/4} e^{-x}\cos(x)\,dx$$

$$\int_0^{\pi/4} e^{-x}\cos(x)\,dx = \frac{\sqrt{2}}{2e^{\pi/4}} - \left(\frac{1}{e^{\pi/4}}\right)\left(\frac{\sqrt{2}}{2}\right) + (1)(1) - \int_0^{\pi/4} e^{-x}\cos(x)\,dx$$

$$+ \int_0^{\pi/4} e^{-x}\cos(x)\,dx = \qquad \int_0^{\pi/4} e^{-x}\cos(x)\,dx$$

$$2\int_0^{\pi/4} e^{-x}\cos(x)\,dx = 1$$

$$\frac{1}{2}\cdot 2\int_0^{\pi/4} e^{-x}\cos(x)\,dx = \frac{1}{2}\cdot 1$$

$$\int_0^{\pi/4} e^{-x}\cos(x)\,dx = \frac{1}{2}$$

$\boxed{8.3}$

#4. $\int \cos(2x)\cos(3x)\,dx$

use $\cos(mx)\cos(nx) = \frac{1}{2}\{\cos[(m-n)x] + \cos[(m+n)x]\}$

$= \frac{1}{2}\int [\cos(x) + \cos(5x)]\,dx$

let $u = 5x$

$\frac{du}{dx} = 5$

$\frac{du}{5} = dx$

$= \frac{1}{2}\int \cos(x)\,dx + \frac{1}{2}\int \cos(5x)\,dx$

$= \frac{1}{2}[\sin(x)] + \frac{1}{2}\int \cos(u)\left(\frac{du}{5}\right) + C$

$= \frac{1}{2}\sin(x) + \frac{1}{10}\int \cos(u)\,du + C$

$= \frac{1}{2}\sin(x) + \frac{1}{10}(\sin(u)) + C$

$= \frac{1}{2}\sin(x) + \frac{1}{10}\sin(5x) + C$

$\boxed{8.3}$

#5

$\int \sin^2(x)\cos^2(x)\,dx$

use $\sin^2(x) = \frac{1-\cos(2x)}{2}$

$\cos^2(x) = \frac{1+\cos(2x)}{2}$

$\Downarrow$

$\cos^2(2x) = \frac{1+\cos(4x)}{2}$

$= \int \left[\frac{1-\cos(2x)}{2}\right]\left[\frac{1+\cos(2x)}{2}\right]dx$

$= \frac{1}{4}\int [1 - \cos^2(2x)]\,dx$

$= \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2(2x)\,dx$

$= \frac{1}{4}x - \frac{1}{4}\int\left[\frac{1+\cos(4x)}{2}\right]dx + C$

let $u = 4x$

$\frac{du}{dx} = 4$

$\frac{du}{4} = dx$

$= \frac{1}{4}x - \frac{1}{8}\int dx - \frac{1}{8}\int \cos(4x)\,dx + C$

$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{8}\int \cos(u)\left(\frac{du}{4}\right) + C$

$= \frac{2}{8}x - \frac{1}{8}x - \frac{1}{32}\int \cos(u)\,du + C$

$= \frac{1}{8}x - \frac{1}{32}[\sin(u)] + C$

$= \frac{1}{8}x - \frac{1}{32}\sin(4x) + C$

8.4

#6 $\displaystyle\int_0^3 \frac{x^3}{\sqrt{x^2+9}}\,dx$

$= \displaystyle\int_0^3 \left(\frac{x}{\sqrt{x^2+9}}\right)\left(x^2\right)dx$

$= \displaystyle\int_{\theta=0}^{\theta=\pi/4} (\sin\theta)(3\tan\theta)^2(3\sec^2 d\theta)$

$= 27\displaystyle\int_0^{\pi/4}\left(\sin\theta \cdot \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\cos^2\theta}\right)d\theta$

$= 27\displaystyle\int_0^{\pi/4} \frac{\sin\theta(1-\cos^2\theta)}{\cos^4(\theta)}\,d\theta$

$= 27\displaystyle\int_0^{\pi/4}\left(\frac{\sin\theta}{\cos^4\theta} - \frac{\cos^2\theta\cdot\sin\theta}{\cos^4\theta}\right)d\theta$

$= 27\displaystyle\int_0^{\pi/4}\frac{\sin\theta}{\cos^4\theta}\,d\theta - 27\int_0^{\pi/4}\frac{\sin\theta}{\cos^2\theta}\,d\theta$

$= 27\displaystyle\int_1^{\sqrt{2}/2}\frac{\sin\theta}{w^4}\left(\frac{dw}{-\sin\theta}\right) - 27\int_1^{\sqrt{2}/2}\frac{\sin\theta}{w^2}\left(\frac{dw}{-\sin\theta}\right)$

$= -27\displaystyle\int_1^{\sqrt{2}/2} w^{-4}\,dw + 27\int_1^{\sqrt{2}/2} w^{-2}\,dw$

$= -27\left[-\frac{1}{3}w^{-3}\right]_1^{\sqrt{2}/2} + 27\left[-1\cdot w^{-1}\right]_1^{\sqrt{2}/2}$

$= 9\left[\frac{1}{w^3}\right]_1^{\sqrt{2}/2} - 27\left[\frac{1}{w}\right]_1^{\sqrt{2}/2}$

$= 9\left[\frac{1}{(\sqrt{2}/2)^3} - \frac{1}{(1)^3}\right] - 27\left[\frac{1}{(\sqrt{2}/2)} - \frac{1}{(1)}\right]$

$= 9\left[2\sqrt{2} - 1\right] - 27\left[\sqrt{2} - 1\right]$

$= 18\sqrt{2} - 9 - 27\sqrt{2} + 27$

$= \underline{18 - 9\sqrt{2}}$

Key: $\sqrt{x^2+9} = \sqrt{u^2+a^2}$

$u = x$

$a = 3$

hyp $= \sqrt{u^2+a^2} = \sqrt{x^2+9}$

opp $= u = x$

adj $= a = 3$

$\frac{u}{a} = \frac{\text{opp}}{\text{adj}}$

$\frac{x}{3} = \tan\theta$

$x = 3\tan\theta$

$\frac{dx}{d\theta} = 3\sec^2\theta$

$dx = 3\sec^2\theta\, d\theta$

$\frac{x}{\sqrt{x^2+9}} = \frac{\text{opp}}{\text{hyp}} = \sin\theta$

$x = 0$

$\frac{0}{3} = \tan\theta$

$0 = \tan\theta$, $\tan^{-1}(0) = \tan^{-1}(\tan\theta)$

$\theta = 0$

$x = 3$

$\frac{3}{3} = \tan\theta$

$1 = \tan\theta$, $\tan^{-1}(1) = \tan^{-1}(\tan\theta)$

$\theta = \pi/4$

Use $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

let $w = \cos\theta$

$\frac{dw}{d\theta} = -\sin\theta$

$\frac{dw}{-\sin\theta} = d\theta$

$\theta = \pi/4$

$w = \cos(\pi/4)$

$w = \sqrt{2}/2$

$\theta = 0$

$w = \cos(0)$

$w = 1$

$\boxed{8.4}$

#7. $\displaystyle\int \frac{\sqrt{x^2-4}}{x}\,dx$

$$= \int (\sin(\theta))(2\sec\theta\tan\theta\,d\theta)$$

$$= 2\int \sin\theta \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}\,d\theta$$

$$= 2\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$

$$= 2\int \left(\frac{1-\cos^2\theta}{\cos^2\theta}\right) d\theta$$

$$= 2\int \frac{1}{\cos^2\theta}\,d\theta - 2\int \frac{\cos^2\theta}{\cos^2\theta}\,d\theta$$

$$= 2\int \sec^2\theta\,d\theta - 2\int d\theta$$

$$= 2\left[\tan\theta\right] - 2\theta + C$$

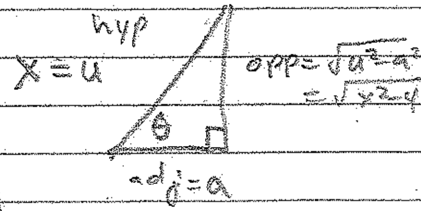
$$= 2\left[\frac{\sqrt{x^2-4}}{2}\right] - 2\left[\operatorname{arcsec}\frac{x}{2}\right] + C$$

$$= \underline{\sqrt{x^2-4} - 2\operatorname{arcsec}\left(\frac{x}{2}\right) + C}$$

Key: $\sqrt{x^2-4} = \sqrt{u^2-a^2}$

$u = x$

$a = 2$



$\frac{u}{a} = \frac{hyp}{adj}$

$\frac{x}{2} = \sec\theta$

$x = 2\sec\theta$

$\frac{dx}{d\theta} = 2\sec\theta\tan\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\frac{\sqrt{x^2-4}}{x} = \frac{opp}{hyp} = \sin\theta$

use $\sin^2\theta + \cos^2\theta = 1$

$1 - \cos^2\theta = \sin^2\theta$

$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{x^2-4}}{2}$

$\frac{x}{2} = \sec\theta$

$\operatorname{arcsec}\left(\frac{x}{2}\right) = \operatorname{arcsec}(\sec\theta)$

$\operatorname{arcsec}\left(\frac{x}{2}\right) = \theta$

$\boxed{8.5}$

#8

$$\int \frac{x^3-x+3}{x^2+x-2}\,dx$$

$$\begin{array}{r} x-1 \\ x^2+x-2 \overline{\big)\, x^3+0x^2-x+3} \\ -[x^3+x^2-2x] \\ -x^2+x+3 \\ -[-x^2-x+2] \\ 2x+1 \end{array}$$

$$= \int \left[x-1+\frac{2x+1}{x^2+x-2}\right]dx$$

$$= \int x\,dx - \int 1\,dx + \int \frac{2x+1}{x^2+x-2}\,dx$$

$$\frac{2x+1}{x^2+x-2} = \frac{2x+1}{(x-1)(x+2)}$$

$$= \frac{1}{2}x^2 - x + \int \left[\frac{A}{x+2} + \frac{B}{x-1}\right]dx + C$$

$$\frac{2x+1}{(x-1)(x+2)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$= \frac{1}{2}x^2 - x + \int \frac{1}{x+2}\,dx + \int \frac{1}{x-1}\,dx + C$$

$2x+1 = A(x-1)+B(x+2)$

choose $x=1$

$2(1)+1 = A(0)+B(1+2)$

$3 = 3B$

$1 = B$

$$= \frac{1}{2}x^2 - x + \ln|x+2| + \ln|x-1| + C$$

or

$$= \frac{1}{2}x^2 - x + \ln|x^2+x-2| + C$$

choose $x=-2$

$2(-2)+1 = A((-2)-1)+B(0)$

$-3 = -3A$

$1 = A$

$\boxed{8.5}$

#9

$$\int \frac{2x-3}{(x-1)^2}\,dx$$

$$\frac{2x-3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$= \int \left[\frac{A}{x-1} + \frac{B}{(x-1)^2}\right]dx$$

$2x-3 = A(x-1)+B$

$2x-3 = Ax-A+B$

$2x = Ax$

$A=2$

$-3 = -A+B$

$-3 = -(2)+B$

$-3 = -2+B$

$-1 = B$

$$= \int \left[\frac{2}{x-1} + \frac{-1}{(x-1)^2}\right]dx$$

$$= 2\int \frac{1}{x-1}\,dx - \int \frac{1}{(x-1)^2}\,dx$$

#9 continued

| let $u = x-1$
$\frac{du}{dx} = 1$
$du = dx$

$$= 2 \cdot \ln|x-1| - \int \frac{1}{u^2}(du) + c$$

$$= 2\ln|x-1| - \int u^{-2} du + c$$

$$= 2\ln|x-1| - [-1 \cdot u^{-1}] + c$$

$$= 2\ln|x-1| + \left[\frac{1}{u}\right] + c$$

$$= 2\ln|x-1| + \frac{1}{x-1} + c$$

8.7

#10. $\lim_{x\to 1} \frac{\ln(x)}{x-1} = \frac{\ln(1)}{(1)-1} = \boxed{\frac{0}{0}}$ stop! Indeterminate Form

$$\lim_{x\to 1} \frac{\ln(x)}{x-1} = \lim_{x\to 1} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)}$$

$$= \lim_{x\to 1} \frac{\frac{1}{x}}{1}$$

$$= \lim_{x\to 1} \frac{1}{x}$$

$$= \frac{1}{1}$$

$$= \underline{\underline{1}}$$ ✓

$\boxed{8.7}$

stop!

#11 $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{\infty}\right)^\infty = (1+0)^\infty = \boxed{1^\infty}$ Indeterminate form

Let $y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$\ln(y) = \ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right]$

$\ln(y) = \lim_{x\to\infty}\left\{\ln\left[\left(1+\frac{1}{x}\right)^x\right]\right\}$

$\ln(y) = \lim_{x\to\infty}\left[x\cdot\ln\left(1+\frac{1}{x}\right)\right]$

$\ln(y) = \lim_{x\to\infty}\frac{\left[x\cdot\ln\left(1+\frac{1}{x}\right)\right]}{1}\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$

$\ln(y) = \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$

Use L'Hopital's Rule

$\frac{\ln\left(1+\frac{1}{\infty}\right)}{\frac{1}{\infty}} = \frac{\ln(1+0)}{0} = \frac{\ln(1)}{0} = \boxed{\frac{0}{0}}$

$\ln(y) = \lim_{x\to\infty}\frac{\frac{d}{dx}\left[\ln\left(1+\frac{1}{x}\right)\right]}{\frac{d}{dx}\left[\frac{1}{x}\right]}$

$\ln(y) = \lim_{x\to\infty}\frac{\left[\frac{1}{1+\frac{1}{x}}\right]\cdot\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$

$\ln(y) = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}$

$\ln(y) = \frac{1}{1+0}$

$\ln(y) = 1$

$e^{\ln(y)} = e^1$

$y = e^1 = e$

So, $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

$\boxed{8.8}$

#12

$\int_0^\infty x e^{-x^2/2}\,dx = \lim_{b\to\infty}\int_0^b x e^{-\frac{x^2}{2}}\,dx$

let $u = -\frac{x^2}{2}$

$\frac{du}{dx} = -x$

$\frac{du}{-x} = dx$

$= \lim_{b\to\infty}\int_{x=0}^{x=b} x\cdot e^u\cdot\left(\frac{du}{-x}\right)$

$= -\lim_{b\to\infty}\int_{x=0}^{x=b} e^u\,du$

$= -\lim_{b\to\infty}\left[e^u\right]_{x=0}^{x=b}$

$= -\lim_{b\to\infty}\left[e^{-x^2/2}\right]_0^b = -\lim_{b\to\infty}\left[e^{-b^2/2} - e^{-(0)^2/2}\right]$

$= -\lim_{b\to\infty}\left[\underbrace{\frac{1}{e^{b^2/2}}}_{\to 0} - 1\right] = -(-1) = 1$

#13 $\displaystyle\int_0^6 \frac{4}{\sqrt{6-x}}\,dx$

let $u = 6-x$

$\frac{du}{dx} = -1$

$-du = dx$

$$= \lim_{b \to 6^-} \int_0^b \frac{4}{\sqrt{6-x}}\,dx$$

$$= \lim_{b \to 6^-} \int_{x=0}^{x=b} \frac{4}{\sqrt{u}}(-du)$$

$$= -4 \lim_{b \to 6^-} \int_{x=0}^{x=b} u^{-1/2}\,du$$

$$= -4 \lim_{b \to 6^-} \left[\frac{2}{1} u^{1/2}\right]_{x=0}^{x=b}$$

$$= -4 \lim_{b \to 6^-} \left[2\sqrt{6-x}\right]_0^b$$

$$= -4 \lim_{b \to 6^-} \left[2\sqrt{6-b} - 2\sqrt{6-0}\right]$$

$$= -4 \lim_{b \to 6^-} \left[2\sqrt{6-b} - 2\sqrt{6}\right]$$

$\sqrt{6-b} \to 0$

$$= -4\left[-2\sqrt{6}\right]$$

$$= 8\sqrt{6}$$

$\boxed{8.3}$

#14. $\int \sec^6(3x)\,dx$

use $\sec^2\theta = 1 + \tan^2\theta$

let $u = \tan 3x$

$\frac{du}{dx} = 3\sec^2(3x)$

$\frac{du}{3\sec^2 3x} = dx$

$= \int \sec^2(3x)\,\sec^2(3x)\,\sec^2(3x)\,dx$

$= \int [1+\tan^2(3x)][1+\tan^2(3x)]\,\sec^2(3x)\,dx$

$= \int [1+\tan^2(3x)]^2 \sec^2(3x)\,dx$

$= \int [1+u^2]^2 \sec^2(3x)\left(\frac{du}{3\sec^2(3x)}\right)$

$= \frac{1}{3}\int [1+u^2]^2\,du$

$= \frac{1}{3}\int (1+2u^2+u^4)\,du$

$= \frac{1}{3}\left[u + 2\cdot\frac{1}{3}u^3 + \frac{1}{5}u^5\right] + C$

$= \frac{u}{3} + \frac{2}{9}u^3 + \frac{u^5}{15} + C$

$= \frac{\tan(3x)}{3} + \frac{2}{9}\tan^3(3x) + \frac{\tan^5(3x)}{15} + C$

#15. $\int \tan^6(x)\,dx$

use $\sec^2(x) - 1 = \tan^2(x)$

let $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$\frac{du}{\sec^2(x)} = dx$

$= \int \tan^4(x)\tan^2(x)\,dx$

$= \int \tan^4(x)\left[\sec^2(x) - 1\right]\cdot dx$

$= \int \tan^4(x)\sec^2(x)\,dx - \int \tan^4(x)\,dx$

$= \int u^4 \sec^2(x)\left(\frac{du}{\sec^2(x)}\right) - \int \tan^2(x)\tan^2(x)\,dx$

$= \int u^4\,du - \int \tan^2(x)\left[\sec^2(x) - 1\right]dx$

$= \frac{1}{5}u^5 - \int \tan^2(x)\sec^2(x)\,dx + \int \tan^2(x)\,dx + C$

$= \frac{1}{5}\tan^5(x) - \int u^2 \sec^2(x)\left(\frac{du}{\sec^2 x}\right) + \int(\sec^2(x) - 1)\,dx + C$

$= \frac{1}{5}\tan^5(x) - \int u^2\,du + \int \sec^2(x)\,dx - \int 1\,dx + C$

$= \frac{1}{5}\tan^5(x) - \frac{1}{3}u^3 + \tan(x) - x + C$

$= \frac{1}{5}\tan^5(x) - \frac{1}{3}\tan^3(x) + \tan(x) - x + C$

$\boxed{8.3}$

#16

$$\int \tan^3\left(\frac{\pi x}{2}\right)\sec^2\left(\frac{\pi x}{2}\right)dx$$

let $u = \tan\left(\frac{\pi x}{2}\right)$

$\frac{du}{dx} = \sec^2\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}$

$\frac{2\,du}{\pi\sec^2\left(\frac{\pi x}{2}\right)} = dx$

$$= \int u^3 \sec^2\left(\frac{\pi x}{2}\right)\left(\frac{2\,du}{\pi\sec^2\left(\frac{\pi x}{2}\right)}\right)$$

$$= \frac{2}{\pi}\int u^3\,du$$

$$= \frac{2}{\pi}\left[\frac{1}{4}u^4\right] + C$$

$$= \frac{1}{2\pi}\left[\tan\left(\frac{\pi x}{2}\right)\right]^4 + C$$

$$= \frac{1}{2\pi}\tan^4\left(\frac{\pi x}{2}\right) + C$$

$\boxed{8.3}$

#17

$$\int \frac{\tan^2(x)}{\sec^5(x)}\,dx$$

use $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$= \int \left(\frac{\sin^2(x)}{\cos^2(x)}\cdot\frac{\cos^5(x)}{1}\right)dx$$

let $u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$\frac{du}{\cos(x)} = dx$

$$= \int \sin^2(x)\cdot\cos^3(x)\,dx$$

$$= \int \sin^2(x)\,(\cos^2(x))\cdot\cos(x)\,dx$$

$$= \int \sin^2(x)\,[1-\sin^2(x)]\cos(x)\,dx$$

$$= \int u^2[1-u^2]\cos(x)\left(\frac{du}{\cos(x)}\right)$$

$$= \int (u^2-u^4)\,du = \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + C$$

$\boxed{8.3}$

#18.

use $\sin(-A) = -\sin(A)$

$$\int \sin(-4x)\cos(3x)\,dx$$

use $\sin(mx)\cos(nx)$

$= \frac{1}{2}[\sin(m-n)x + \sin(m+n)x]$

$$= -\int \sin(4x)\cos(3x)\,dx$$

$$= -\int \left\{\frac{1}{2}\left[\sin(x) + \sin(7x)\right]\right\} dx$$

let $u = 7x$

$\frac{du}{dx} = 7$

$\frac{du}{7} = dx$

$$= -\frac{1}{2}\int (\sin(x) + \sin(7x))\,dx$$

$$= -\frac{1}{2}\int \sin(x)\,dx - \frac{1}{2}\int \sin(7x)\,dx$$

$$= -\frac{1}{2}\left[-\cos(x)\right] - \frac{1}{2}\int \sin(u)\left(\frac{du}{7}\right) + C$$

$$= \frac{1}{2}\cos(x) - \frac{1}{14}[-\cos(u)] + C$$

$$= \frac{1}{2}\cos(x) + \frac{1}{14}\cos(7x) + C$$