

## Prep for Exam on Chapter 9

① Carefully define each of the following convergence tests/theorems. Write the conditions for which each test/theorem holds.

(a) Geometric Series

(b)  $p$ -series

(c) Integral test

(d) Direct Comparison

(e) Limit Comparison

(f) Alternating Series

(g) Ratio Test

(h) Root Test

(i)  $n^{\text{th}}$ -Term

② Determine the convergence or divergence of the following sequences. If the sequence converges find its limit.

(a)  $\left\{ \frac{n^3}{3^n} \right\}$

(b)  $\left\{ \frac{3n^2 - n + 4}{2n^2 + 1} \right\}$

(c)  $\left\{ \frac{(n+1)!}{n!} \right\}$

(d)  $\left\{ \frac{(-1)^n}{n!} \right\}$

③ page 614, #31 & #33 AND Page 638 #7

④ Find the sum of the following series if it converges. If it diverges, so state and explain.

(a)  $\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$

(b)  $\sum_{n=0}^{\infty} \left( \frac{1}{2^n} - \frac{1}{3^n} \right)$

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(5) Express the repeating decimals as fractions. (Make use of series.)

(a)  $0.\overline{15}$

(b)  $0.\overline{81}$

(6) Determine the convergence or divergence of the following series using any appropriate test. Classify the convergence if necessary (conditional convergence or absolute convergence). Be sure to show that your series meets the conditions for any test that you use.

(a)  $\sum_{n=1}^{\infty} e^{-n}$

(b)  $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$

(c)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

(d)  $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(e)  $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}^\pi}$

(f)  $\sum_{n=1}^{\infty} \frac{1}{3n^2-4n+5}$

(g)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$

(h)  $\sum_{n=1}^{\infty} \frac{n^2-10}{4n^2+n^3}$

(i)  $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

(j)  $\sum_{n=1}^{\infty} \frac{4^n}{3^n-1}$

(k)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

(l)  $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$

(m)  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sqrt{n}}{n+2}$

(n)  $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

(o)  $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n2^n}$

(p)  $\sum_{n=0}^{\infty} \frac{4^n}{3^n+1}$

(q)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n+3)}{n+10}$

(r)  $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

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#6

$$(S) \sum_{n=1}^{\infty} \left( \frac{n}{2n+1} \right)^n$$

$$(T) \sum_{n=1}^{\infty} (2\sqrt{n+1})^n$$

#7 Page 639 #37, #41, #47  
#33, #35, #45

8 Approximate the sum of the series by using the first six terms.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3}{n^2}$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4}{\ln(n+1)}$$

9 (a) Find the Maclaurin polynomial of degree 4 for  $f(x) = \frac{1}{1+x}$ .

(b) Use this polynomial to approximate  $f(0.1)$ .

(c) Use Taylor's Theorem to obtain an upper bound for the error of the approximation.

10 (a) Find the Maclaurin polynomial of degree 5 for  $g(x) = \sin x$ .

(b) Use this polynomial to approximate  $g(0.1)$ .

(c) Use Taylor's Theorem to obtain an upper bound for the error of the approximation.

11 Find the 4<sup>th</sup> degree Taylor polynomials centered at  $c=1$  for  $f(x) = \sqrt{x}$  & for  $g(x) = \ln(x)$ .

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- (12) For each power series below, find the interval of convergence. Be sure to include a check for convergence at the endpoints of each interval, (if necessary).

$$(a) \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$$

$$(b) \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-5)^n}{n 5^n}$$

$$(d) \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

- (13) Given  $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ , find the following series:

(a)  $\int f(x) dx$  & (b)  $f'(x)$ . Find interval of convergence for each series. (Don't forget to check the endpoint convergence for each interval).

- (14) Find a power series each of the following functions centered at  $c$ , and determine the interval of convergence. (Endpoints?)

$$(a) f(x) = \frac{1}{2x-5}, \quad c = -3$$

$$(b) g(x) = \frac{4}{x+2}, \quad c = 0$$