Friday's Notes

$\boxed{10.5}$

(47) $r = 1+\sin\theta, \quad 0 \leq \theta \leq 2\pi$

$\frac{d}{d\theta}(r) = \frac{d}{d\theta}(1+\sin\theta)$

$\frac{dr}{d\theta} = \cos\theta$

$S = \int_0^{2\pi} \sqrt{(r'(\theta))^2+(r(\theta))^2}\, d\theta$

$= \int_0^{2\pi} \sqrt{\cos^2\theta+(1+\sin\theta)^2}\, d\theta$

$= \int_0^{2\pi} \sqrt{\cos^2\theta+1+2\sin\theta+\sin^2\theta}\, d\theta$

$= \int_0^{2\pi} \sqrt{2+2\sin\theta}\, d\theta$

$= \sqrt{2}\int_0^{2\pi} \sqrt{1+\sin\theta}\, d\theta$

$= \sqrt{2}\int_0^{2\pi} \frac{\sqrt{1+\sin\theta}}{1}\cdot\left(\frac{\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}\right) d\theta$

$= \sqrt{2}\int_0^{2\pi} \frac{\sqrt{1-\sin^2\theta}}{\sqrt{1-\sin\theta}}\, d\theta$

$= \sqrt{2}\int$ Since the identity $\sqrt{1-\sin^2\theta}=\cos\theta$ doesn't hold over the whole interval, must split up the interval. <u>BUT</u> it is apparent from the graph that there is symmetry.

$= -2\sqrt{2}\int_{\pi/2}^{3\pi/2} \frac{\cos\theta}{\sqrt{1-\sin\theta}}\, d\theta$

Let $u = 1-\sin\theta$ $du = -\cos\theta\, d\theta$ $d\theta = \frac{-du}{\cos\theta}$	$u = 1-\sin\frac{3\pi}{2}$ $u = 2$	$u = 1-\sin\frac{\pi}{2}$ $u = 0$

$= +2\sqrt{2}\int_{u=0}^{u=2} \frac{\cos\theta}{u^{1/2}}\left(\frac{du}{\cos\theta}\right)$

$= 2\sqrt{2}\int_{u=0}^{u=2} u^{-1/2}\, du$

$= \left[2\sqrt{2}\,(2\sqrt{u})\right]_0^2$

$= 4\sqrt{2}(\sqrt{2}) = \boxed{8}$

(9) Area $= \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$

$A_{\text{one petal}} = \frac{1}{2}\int_0^{\pi/4} (\cos 2\theta)^2\, d\theta$

$= \int_0^{\pi/4} \cos^2 2\theta\, d\theta$

$= \int_0^{\pi/4} \frac{1+\cos 4\theta}{2}\, d\theta$

$= \int_0^{\pi/4} \frac{1}{2}\, d\theta + \int_0^{\pi/4} \frac{\cos 4\theta}{2}\, d\theta$

$= \left[\frac{1}{2}\theta + \frac{1}{8}\sin 4\theta\right]_0^{\pi/4} = \boxed{\frac{\pi}{8}}$

10.5

(13) Inner loop of $r = 1+2\cos\theta$

$r = 1+2\cos\theta$

solve: $0 = 1+2\cos\theta$

$\theta = \frac{2\pi}{3}$

takes care of the issue of the area being negative

$|1+2\cos\theta| \Rightarrow$ same shape but with location of loop on other side of y-axis

$$A = 2\cdot\left[\tfrac{1}{2}\int_{\frac{2\pi}{3}}^{\pi} (|1+2\cos\theta|)^2 d\theta\right.$$

$$A = \int_{2\pi/3}^{\pi} (1+2\cos\theta)^2 d\theta$$

$$= \int_{\frac{2\pi}{3}}^{\pi} (1+4\cos\theta+4\cos^2\theta)d\theta$$

$$= \int_{2\pi/3}^{\pi} d\theta + 4\int_{2\pi/3}^{\pi}\cos\theta + 4\int_{2\pi/3}^{\pi}\cos^2\theta\, d\theta$$

$$= [\theta + 4\sin\theta]_{2\pi/3}^{\pi} + 4\int_{2\pi/3}^{\pi} \frac{1+\cos 2\theta}{2} d\theta$$

$$= [\theta + 4\sin\theta + 2\theta + \sin 2\theta]_{2\pi/3}^{\pi}$$

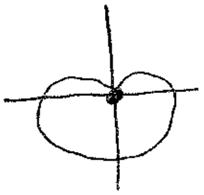
$$= (3\pi + 0 + 0) - \left(2\pi + 4\left(\tfrac{\sqrt{3}}{2}\right) + \left(-\tfrac{\sqrt{3}}{2}\right)\right)$$

$$= \pi - \frac{3\sqrt{3}}{2}$$

$$\boxed{A = \frac{2\pi - 3\sqrt{3}}{2}}$$

10.4

(75) $r(\theta) = 2-2\sin\theta$ find tangents at pole



thm: $r(\theta)=0$, $r'(\theta)\neq 0$

$r'(0) = \frac{d}{d\theta}(2-2\sin\theta)$

$= -2\cos\theta$

solve for $r(\theta)=0$

$0 = 2-2\sin\theta$

$\theta = \frac{\pi}{2}$

plug into $r'(\theta) = -2\cos\theta$

$r'(\frac{\pi}{2}) = -2\cos(\frac{\pi}{2})$

$r'(\frac{\pi}{2}) = 0 \leftarrow$ so why vertical tangency? look at graph of