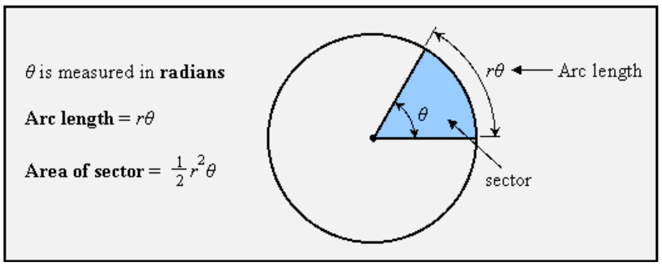
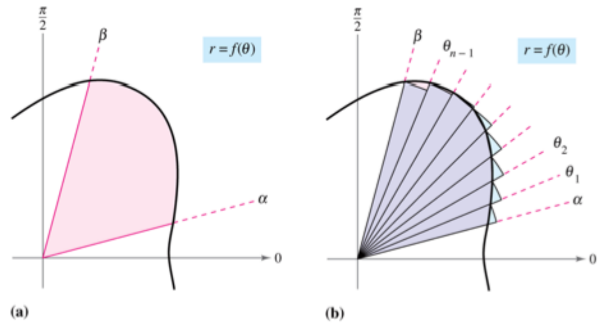
Math 155, ***Lecture Notes- Bonds*** **Name____________**

Section 10.5 *Area of a Polar Region & Arc Length in Polar Coordinates*

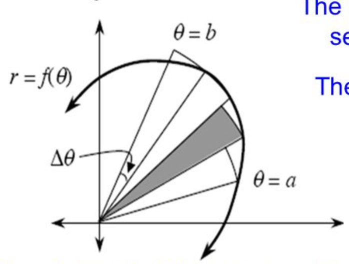
The development of a formula for the area of a polar region parallels that for the area of a region on the rectangular coordinate system, but uses sectors of circles instead of rectangles as the basic element of area.





Area Enclosed by Polar Curves

Similar to Cartesian equations, we can find the exact area of the polar region using an integral. The only exception is that we are using sectors to approximate the area, not rectangles.



The area of an sector is: $A = \frac{\theta}{2\pi}\pi r^2$

The area the highlighted sector is:

$$A = \frac{\Delta\theta}{2\pi}\pi f\left(\theta\right)^2$$
$$= \frac{1}{2} f\left(\theta\right)^2 \Delta\theta$$

If we integrate this area over the entire interval, it represents the total area bounded by the curve.

THEOREM 10.13 Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, $0 < \beta - \alpha \leq 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2\, d\theta$$
$$= \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta. \qquad 0 < \beta - \alpha \leq 2\pi$$

Note: The function f cannot change sign on the interval $[\alpha, \beta]$, since the sectors must be adjacent to each other as we sum their areas.

Ex. 1: Find the area of one petal of the curve defined by $r = 6\sin(2\theta)$.

More Ex. 1:

Ex. 2: Find the area between the "loops" of the curve defined by $r = 2\left[1 + 2\sin\left(\theta\right)\right]$.

More Ex. 2:

More Ex. 2:

THEOREM 10.14 Arc Length of a Polar Curve

Let f be a function whose derivative is continuous on an interval $\alpha \leq \theta \leq \beta$. The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

Ex. 3: Find the length of the curve defined by $r = 8[1 + \cos(\theta)]$, over the interval $[0, 2\pi]$.

More Ex. 3: