Name **KEY**

Use Algebraic Notation AND Show All of Your Work

Find all numbers for which each rational expression is undefined. If the rational expression is defined for all real numbers, so state.
[8 pts]

denominators can't be zero

1. $\dfrac{7x+1}{4x-16}$

$4x - 16 = 0$

$16 + 4x - 16 = 0 + 16$

$\frac{1}{4} \cdot \frac{4x}{1} = \frac{16}{1} \cdot \frac{1}{4}$

$x = 4$ ← excluded value is 4.

The expression is undefined when the x-value is 4.

Ans:_____

[12 pts]

2. $\dfrac{x-4}{2x^2+x-3}$

$2x^2 + x - 3 = 0$

$(2x + 3)(x - 1) = 0$

Either

$2x + 3 = 0$, or $x - 1 = 0$

$-3 + 2x + 3 = -3 + 0$

$2x = -3$

$\boxed{x = -3/2}$ $\boxed{x = 1}$ ← Exclude values are -3/2 and 1.

The expression is undefined when the x-value is -3/2 and 1.

Ans:_____

Simplify each rational expression.
[12 pts]

3. $\dfrac{y^2+3y}{y^2-y-12} = \dfrac{y(y+3)}{(y-4)(y+3)}$

$\begin{array}{c|c} 12 \\ \hline 1, 12 \\ 2, 6 \\ \hline 3, 4 \end{array}$

$= \dfrac{y}{y-4}$

Ans: $\dfrac{y}{y-4}$

[14 pts]

4. $\dfrac{x^2-2x-15}{x^2-9} = \dfrac{(x-5)(x+3)}{(x-3)(x+3)}$

$\begin{array}{c|c} 15 \\ \hline 1, 15 \\ 3, 5 \end{array}$

$= \dfrac{x-5}{x-3}$

Ans: $\dfrac{x-5}{x-3}$

[14 pts]

$$\frac{3x^2-2x}{8-12x} = \text{\Big|}\begin{array}{l} 3x^2-2x = x(3x-2) \\ -12x+8 = -4(3x-2) \end{array}$$

5. $\dfrac{3x^2-2x}{8-12x} = \dfrac{x(3x-2)}{-12x+8}$

$$= \frac{x(3x-2)}{-4(3x-2)}$$

$$= -\frac{x}{4}$$

Ans: $\quad -\dfrac{x}{4}$

Perform the indicated operations. Simplify your answer if possible.

[21 pts]

6. $\dfrac{x^2-8x+16}{2x-8} \cdot \dfrac{x^2-2x-24}{x^2-16}$

$$= \frac{(x-4)(x-4)\cdot(x-6)(x+4)}{2(x-4)\cdot(x+4)(x-4)}$$

$$= \frac{x-6}{2}$$

SDWK

$x^2-8x+16 = (x-4)(x-4)$

$x^2-2x-24 = (x-6)(x+4)$

$x^2-16 = (x-4)(x+4)$

$$\begin{array}{l}16\\1,16\\2,8\\4,4\end{array}\qquad\begin{array}{l}24\\1,24\\2,12\\3,8\\4,6\end{array}$$

Ans: $\quad \dfrac{x-6}{2}$

[24 pts]

7. $\dfrac{x^2-3xy+2y^2}{2xy+2y^2} \div \dfrac{x^2+5xy-14y^2}{8x+8y}$

$$= \frac{x^2-3xy+2y^2}{2xy+2y^2} \cdot \frac{8x+8y}{x^2+5xy-14y^2}$$

$$= \frac{(x-2y)(x-y)\cdot 2\cdot 2\cdot 2\cdot(x+y)}{2y(x+y)\cdot(x+7y)(x-2y)}$$

$$= \frac{4(x-y)}{y(x+7y)}$$

SDWK

$x^2-3xy+2y^2 = (x-2y)(x-y)$

$2xy+2y^2 = 2y(x+y)$

$8x+8y = 8(x+y)$
$\qquad = 2\cdot2\cdot2(x+y)$

$x^2+5xy-14y^2 = (x+7y)(x-2y)$

$$\begin{array}{c}8\\ \wedge\\2\ \ 4\\ \wedge\\ \ \ 2\ 2\end{array}\qquad\begin{array}{l}14\\1,14\\2,7\end{array}$$

Ans: $\quad \dfrac{4(x-y)}{y(x+7y)}$

[16 pts]

8. $\dfrac{3y+5}{4y-8} - \dfrac{5-y}{4y-8} = \dfrac{3y+5-(5-y)}{4y-8}$

$$= \frac{3y+5-5+y}{4y-8}$$

$$= \frac{4y}{4y-8}$$

$$= \frac{2\cdot2\cdot y}{2\cdot2(y-2)}$$

$$= \frac{y}{y-2}$$

SDWK

$4y = 2\cdot2y$

$4y-8 = 4\cdot(y-2)$
$\qquad = 2\cdot2(y-2)$

Ans: $\quad \dfrac{y}{y-2}$

[16 pts]

9. $\dfrac{9x-1}{3x-2}+\dfrac{6x-2}{2-3x}$

$=\dfrac{9x-1}{3x-2}-\dfrac{6x-2}{3x-2}$

$=\dfrac{9x-1-(6x-2)}{3x-2}$

$=\dfrac{9x-1-6x+2}{3x-2}$

$\Rightarrow=\dfrac{3x+1}{3x-2}$

Ans: $\dfrac{3x+1}{3x-2}$

[24 pts]

10. $\dfrac{y^2-16}{y^2+9y+18}-\dfrac{y-4}{y+6}$

$=\dfrac{y^2-16}{(y+6)(y+3)}-\dfrac{(y-4)(y+3)}{(y+6)(y+3)}$

$=\dfrac{y^2-16-[y^2+3y-4y-12]}{(y+6)(y+3)}$

$=\dfrac{y^2-16-(y^2-y-12)}{(y+6)(y+3)}$

$=\dfrac{y^2-16-y^2+y+12}{(y+6)(y+3)}$

$\Rightarrow=\dfrac{y-4}{(y+6)(y+3)}$

Ans: $\dfrac{y-4}{(y+6)(y+3)}$

[28 pts]

11. $\dfrac{x+1}{3x^2-5x-2}-\dfrac{x-1}{2x^2-5x+2}$

$=\dfrac{(x+1)}{(3x+1)(x-2)}\cdot\dfrac{(2x-1)}{(2x-1)}-\dfrac{(x-1)}{(2x-1)(x-2)}\cdot\dfrac{(3x+1)}{(3x+1)}$

$=\dfrac{(2x^2-x+2x-1)-(3x^2+x-3x-1)}{(3x+1)(x-2)(2x-1)}$

$=\dfrac{2x^2+x-1-(3x^2-2x-1)}{(3x+1)(x-2)(2x-1)}$

$=\dfrac{2x^2+x-1-3x^2+2x+1}{(3x+1)(x-2)(2x-1)}$

$=\dfrac{-x^2+3x}{(3x+1)(x-2)(2x-1)}=\dfrac{-x(x-3)}{(3x+1)(x-2)(2x-1)}$

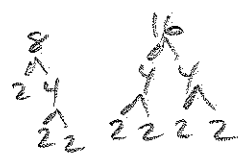Ans: $\dfrac{-x(x-3)}{(3x+1)(x-2)(2x-1)}$

[16 pts]

12. $\dfrac{\dfrac{3}{8}+\dfrac{1}{16}}{\dfrac{1}{2}-\dfrac{1}{4}} = \dfrac{\left(\dfrac{3}{8}+\dfrac{1}{16}\right)\cdot\dfrac{16}{1}}{\left(\dfrac{1}{2}-\dfrac{1}{4}\right)\cdot\dfrac{16}{1}}$ $\Rightarrow = \dfrac{7}{4}$

$\begin{array}{l} 8 = 2\cdot2\cdot2 \\ 16 = 2\cdot2\cdot2\cdot2 \\ 4 = 2\cdot2 \\ 2 = 2 \end{array}$

$\dfrac{8}{2\overline{)4}}$ $\dfrac{16}{2\overline{)4}}$
$\quad 2\overline{)2}$ $\quad 2\,2\,2\,2$

$= \dfrac{\dfrac{3}{8}\cdot\dfrac{16}{1}+\dfrac{1}{16}\cdot\dfrac{16}{1}}{\dfrac{1}{2}\cdot\dfrac{16}{1}-\dfrac{1}{4}\cdot\dfrac{16}{1}}$

$= \dfrac{3\cdot2+1}{8-4}$

$= \dfrac{6+1}{4}$

Ans: $\dfrac{7}{4}$

[20 pts]

13. $\dfrac{\dfrac{12}{x}+3}{1-\dfrac{16}{x^2}} = \dfrac{\left(\dfrac{12}{x}+\dfrac{3}{1}\right)\cdot\dfrac{x^2}{1}}{\left(\dfrac{1}{1}-\dfrac{16}{x^2}\right)\cdot\dfrac{x^2}{1}}$

SD wk
$\begin{array}{l} x = x \\ x^2 = x\cdot x \\ \hline LCD = x^2 \end{array}$

$x^2-16 = (x+4)(x-4)$

$12x+3x^2 = 3x^2+12x$
$\qquad\qquad = 3x(x+4)$

$= \dfrac{\dfrac{12}{x}\cdot\dfrac{x^2}{1}+\dfrac{3}{1}\cdot\dfrac{x^2}{1}}{\dfrac{1}{1}\cdot\dfrac{x^2}{1}-\dfrac{16}{x^2}\cdot\dfrac{x^2}{1}}$

$\Rightarrow = \dfrac{3x(x+4)}{(x+4)(x-4)}$

$= \dfrac{3x}{x-4}$

$= \dfrac{12x+3x^2}{x^2-16}$

Ans: $\dfrac{3x}{x-4}$

Solve each equation. State the solution set.
[24, 3 pts]

14. $\dfrac{x-3}{x} = \dfrac{5}{x+4}$

$\left[\dfrac{(x)(x+4)}{1}\right]\cdot\left(\dfrac{x-3}{x}\right) = \left[\dfrac{(x)(x+4)}{1}\right]\cdot\left(\dfrac{5}{x+4}\right)$

$(x+4)(x-3) = 5\cdot x$

$x^2-3x+4x-12 = 5x$

$5x-1 \quad x^2+x-12 = 5x+(-5x)$

$x^2-4x-12 = 0$

$(x-6)(x+2) = 0$

Either
$x-6 = 0,$ or $x+2 = 0$
$\quad x = 6 \qquad x = -2$

check
$\dfrac{(6)-3}{6} = \dfrac{5}{(6)+4}$ $\qquad$ $\dfrac{(-2)-3}{(-2)} = \dfrac{5}{(-2)+4}$

$\dfrac{3}{6} = \dfrac{5}{10}$ $\qquad\qquad$ $\dfrac{-5}{-2} = \dfrac{5}{2}$

$3\cdot10 = 5\cdot6$ $\qquad\qquad$ $\dfrac{5}{2} = \dfrac{5}{2}$

$30 = 30$ $\qquad\qquad\qquad$ TRUE!
TRUE!

Solution Set: $\{6, -2\}$

[27, 3 pts]

15. $\dfrac{6}{x+4} - \dfrac{5}{x+2} = \dfrac{-20}{x^2+6x+8}$

$\left[\dfrac{(x+4)(x+2)}{1}\right]\cdot\left[\dfrac{6}{x+4} - \dfrac{5}{x+2}\right] = \left[\dfrac{(x+4)(x+2)}{1}\right]\cdot\left[\dfrac{-20}{(x+4)(x+2)}\right]$

$\dfrac{(x+4)(x+2)}{1}\cdot\dfrac{6}{(x+4)} - \dfrac{(x+4)(x+2)}{1}\cdot\dfrac{5}{(x+2)} = -20$

$6(x+2) - 5(x+4) = -20$

$6x+12 - 5x - 20 = -20$

$x - 8 = -20$

$8 + x - 8 = -20 + 8$

$x = -12$

check

$\dfrac{6}{(-12)+4} - \dfrac{5}{(-12)+2} = \dfrac{-20}{(-12)^2+6(-12)+8}$

$\dfrac{6}{-8} - \dfrac{5}{-10} = \dfrac{-20}{144-72+8}$

$-\dfrac{3}{4} + \dfrac{1}{2} = \dfrac{-20}{80}$

$-\dfrac{3}{4} + \dfrac{1}{2}\cdot\dfrac{2}{2} = -\dfrac{1}{4}$

$-\dfrac{3}{4} + \dfrac{2}{4} = -\dfrac{1}{4}$

$-\dfrac{1}{4} = -\dfrac{1}{4}$ TRUE!

Solution Set: $\{-12\}$

[27, 3 pts]

16. $\dfrac{1}{y-1} + \dfrac{2}{y} = \dfrac{y}{y-1}$

$\left[\dfrac{y\cdot(y-1)}{1}\right]\cdot\left[\dfrac{1}{y-1} + \dfrac{2}{y}\right] = \left[\dfrac{y(y-1)}{1}\right]\cdot\left(\dfrac{y}{y-1}\right)$

$\dfrac{y(y-1)}{1}\cdot\dfrac{1}{(y-1)} + \dfrac{y(y-1)}{1}\cdot\dfrac{2}{y} = y^2$

$y + 2(y-1) = y^2$

$y + 2y - 2 = y^2$

$(-3y+2) + (3y-2) = (-3y+2) + y^2$

$0 = y^2 - 3y + 2$

$0 = (y-2)(y-1)$

Either

$y - 2 = 0$, or $y - 1 = 0$

$y = 2$ $y = 1$ ← Restricted Value

check

$\dfrac{1}{(2)-1} + \dfrac{2}{(2)} = \dfrac{(2)}{(2)-1}$

$\dfrac{1}{1} + \dfrac{1}{1} = \dfrac{2}{1}$

$2 = 2$ TRUE!

$\dfrac{1}{(1)-1} + \dfrac{2}{(1)} = \dfrac{(1)}{(1)-1}$

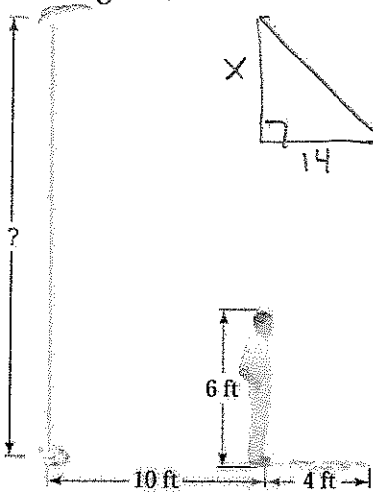$\dfrac{1}{0} + 2 = \dfrac{1}{0}$

↑ undefined   ↑

1 is a restricted value.

Solution Set: $\{2\}$

17. A person who is 6 feet tall is standing 10 feet from the base of a lamppost. The person's shadow has a length of 4 feet How tall is the lamppost? *(Define a variable, create an equation, solve using algebra, and answer in a sentence.)*
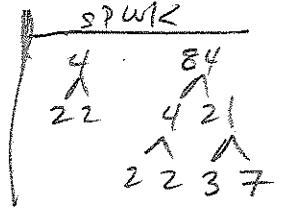
let x = height of lamppost

spwk

$$\frac{x}{14} = \frac{6}{4}$$

$$4 \cdot x = 6 \cdot 14$$

$$4x = 84$$

$$\frac{1}{4} \cdot \frac{4x}{1} = \frac{1}{4} \cdot \frac{84}{1}$$

$$x = 21$$

Ans: The lamppost is 21 feet tall.

18. In still water, a boat averages 20 miles per hour. It takes the boat the <u>same amount</u> of time to travel 72 miles downstream, with the current, as 48 miles upstream, against the current. What is the rate of the river's current? *(Set-up an organizational chart structure, define a variable, create an equation, solve using algebra, and answer in a sentence.)* Let x = rate of the river's current

$D = R \cdot T$ $\frac{D}{R} = T$

Trip	Distance	Rate	Time
upstream	48	20−x	$\frac{48}{20-x}$
downstream	72	20+x	$\frac{72}{20+x}$

$$\frac{48}{20-x} = \frac{72}{20+x}$$

$$48 \cdot (20+x) = 72(20-x)$$

$$960 + 48x = 1,440 - 72x$$

$$72x + 960 + 48x = 72x + 1,440$$

$$120x + 960 = 1,440$$

$$-960 + 120x + 960 = -960 + 1,440$$

$$120x = 480$$

$$\frac{1}{120} \cdot \frac{120x}{1} = \frac{1}{120} \cdot \frac{480}{1}$$

$$x = 4$$

check

$$\frac{48}{20-(4)} = \frac{72}{20+(4)}$$

$$\frac{48}{16} = \frac{72}{24}$$

$$3 = 3$$

TRUE!

Ans: The rate of the river's current is 4 miles per hour.

[7, 2, 9, 16, 2 pts]

19. A painter can paint a fence around a house in 6 hours. Working alone, the painter's apprentice can paint the same fence in 12 hours. How many hours would it take them to do the same job if they worked together? *(Set-up an organizational chart structure, define a variable, create an equation, solve using algebra, and answer in a sentence.)*

Let x = time to complete job working together

WORKER	Time to complete job working alone	Rate of work	Time Working Together	Fractional Amount of work done
painter	6 hours	$\frac{1}{6}$	x hours	$\frac{x}{6}$
apprentice	12 hours	$\frac{1}{12}$	x hours	$\frac{x}{12}$

LCD = 12

$$\frac{x}{6} + \frac{x}{12} = 1$$

$$\frac{12}{1} \cdot \left[\frac{x}{6} + \frac{x}{12}\right] = \frac{12}{1} \cdot 1$$

$$\frac{12}{1} \cdot \frac{x}{6} + \frac{12}{1} \cdot \frac{x}{12} = 12$$

$$2x + x = 12$$

$$3x = 12$$

$$\frac{1}{3} \cdot \frac{3x}{1} = \frac{1}{3} \cdot \frac{12}{1}$$

$$x = 4$$

check

$$\frac{(4)}{6} + \frac{(4)}{12} = 1$$

$$\frac{2}{3} + \frac{1}{3} = 1$$

$$\frac{3}{3} = 1$$

$$1 = 1$$

TRUE!

Ans: It will take 4 hours for the painter and the apprentice to paint the fence working together.

[2, 9, 13, 2 pts]

20. To determine the number of trout in a lake, a conservationist catches 112 trout, tags them, and returns them to the lake. Later, 82 trout are caught and 32 of them are found to be tagged. How many trout are in the lake? *(Define a variable, create an equation, solve using algebra, and answer in a sentence.)*

Let x = number of trout in the lake

$$\frac{112 \text{ trout tagged}}{x \text{ trout in the lake}} = \frac{32 \text{ trout tagged & caught}}{82 \text{ trout caught}}$$

$$\frac{112}{x} = \frac{32}{82}$$

$$112 \cdot 82 = 32 \cdot x$$

$$\frac{1}{32} \cdot \frac{9184}{1} = \frac{1}{32} \cdot 32x$$

$$287 = x$$

Check

$$\frac{112}{287} = \frac{32}{82}$$

$$112 \cdot 82 = 32 \cdot 287$$

$$9184 = 9184 \checkmark$$

TRUE!

Ans: There are 287 trout in the lake.