#19 $r(\theta) = 1 + \sin(\theta)$

arclength $= S = \int_a^b \sqrt{(r(\theta))^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$[r(\theta)]^2 = [1+\sin\theta]^2$

$= 1 + 2\sin\theta + \sin^2$

$\frac{dr}{d\theta} = 0 + \cos\theta$

$\left(\frac{dr}{d\theta}\right)^2 = \cos^2\theta$

$S = 2\int_{\pi/2}^{3\pi/2} \sqrt{[1+2\sin\theta]^2 + (\cos\theta)^2}\, d\theta$

$S = \int_{\pi/2}^{3\pi/2} \sqrt{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}\, d\theta$

$S = \int_{\pi/2}^{3\pi/2} \sqrt{2(1+\sin\theta)}\, d\theta$ $\quad \left(\frac{\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}\right)$

$S = \sqrt{2}\int_{\pi/2}^{3\pi/2} \frac{\sqrt{1-\sin^2\theta}}{\sqrt{1-\sin\theta}}\, d\theta \Rightarrow 1 - \sin^2\theta = \cos^2\theta$

$\sqrt{1-\sin^2\theta} = \cos\theta$

$S = -2\sqrt{2}\int_{\pi/2 \to \pi/2}^{3\pi/2 \to 3\pi/2} \frac{\cos\theta}{\sqrt{1-\sin\theta}}\, d\theta$ $\quad$ let $u = 1 - \sin\theta$, $du = -\cos\theta\, d\theta$

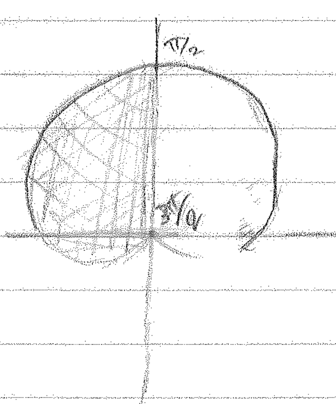
$= -2\sqrt{2}\int_0^2 \frac{\cos\theta}{\sqrt{u}} \frac{du}{-\cos\theta}$ $\quad \theta = \frac{3\pi}{2}$ $\quad \theta = \frac{\pi}{2}$

$u = 2 \quad u = 0$

$= 2\sqrt{2}\left[2(u)^{1/2}\right]_0^2$

$= 4\sqrt{2}(\sqrt{2} - \sqrt{0})$

$= 8$



#18 $x = 2t - t^2$

$y = 2t^{3/2}$

$\frac{dx}{dt} = 2-2t$

$\frac{dy}{dt} = 3t^{1/2}$

arc leng = $S = \int_1^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$S = \int_1^2 \sqrt{(2-2t)^2 + (3t^{1/2})^2}$$

$$S = \int_1^2 \sqrt{4-4t+4t^2+9t}\, dt$$

$$S \int_1^2 \sqrt{4t^2+5t+4}\, dt$$

$S \approx 4.5322$

$S \approx 4.5$

#9. $x = t + 1$
$y = t^2 + 3t$

(b) $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2t+3}{1} = \boxed{2t+3}$

(c) $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \dfrac{\frac{d}{dt}\left[2t+3\right]}{1} = \dfrac{2}{1} = 2$

(d) $\left.\dfrac{dy}{dx}\right|_{t=-1} = 2(-1) + 3 = -2 + 3 = 1$

(e) $\left.\dfrac{d^2y}{dx^2}\right|_{t=-1} = 2$ concave up

#10 $x = 2\cos(\theta)$
$y = 2\sin(\theta)$

(b) $\dfrac{dy}{dx} = \dfrac{d}{d\theta}\left(\dfrac{dy/d\theta}{dx/d\theta}\right) = \dfrac{\cancel{2}\cos\theta}{-\cancel{2}\sin\theta} = -\cot\theta$

(c) $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}} = \dfrac{\frac{d}{d\theta}(-\cot(\theta))}{-2\sin\theta} = \dfrac{-\csc^2\theta}{2\sin\theta} = -\dfrac{1}{2}\left(\dfrac{1}{\sin^3\theta}\right)$ or $-\frac{1}{2}$cs

(d) $\left.\dfrac{dy}{dx}\right|_{t=\pi/4} = -\cot\theta$
$= -\cot\left(\dfrac{\pi}{4}\right)$
$= \dfrac{\cos(\pi/4)}{\sin(\pi/4)} = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{-1}$

(e) $\left.\dfrac{d^2y}{dx^2}\right|_{t=\pi/4} = -\frac{1}{2}\csc^3(\pi/4) = -\frac{1}{2}\left(\dfrac{2}{\sqrt{2}}\right)^3 = -\sqrt{2}$ Concave down

#4. $9x^2+36x+4y^2-24y=-36$

$9(x^2+4x+4)+4(y^2-6y+9)=-36+36+36$

$[\,9(x+2)^2+4(y-3)^2=36\,]\ 1/36$

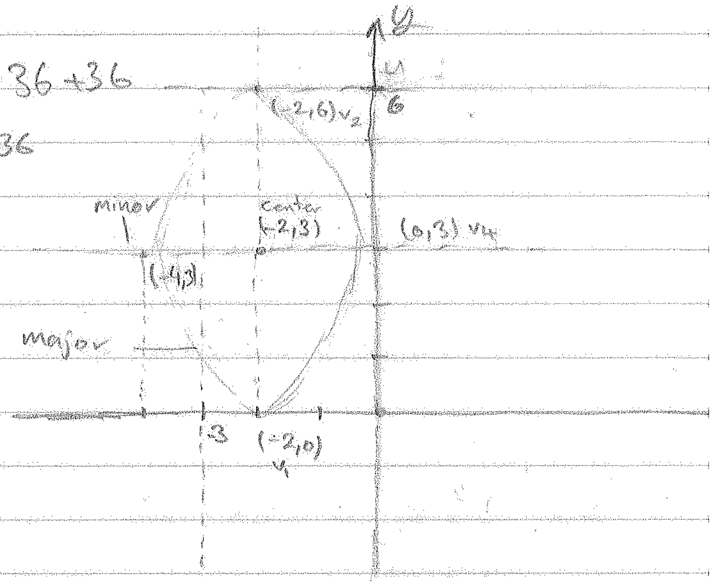
$\boxed{\dfrac{(x+2)^2}{4}+\dfrac{(y-3)^2}{9}=1}$

$a^2=9 \Rightarrow a=3$

$b^2=4 \Rightarrow b=2$

center $(h,k)=(-2,3)$

foci $(-2,\ 3\pm\sqrt{5})$



#5. $3x^2-6x+(-2y^2-12y)=27$

$3(x^2-2x)-2(y^2+6y)=27$

$3(x^2-2x+1)-2(y^2+6y+9)=27+3-18$

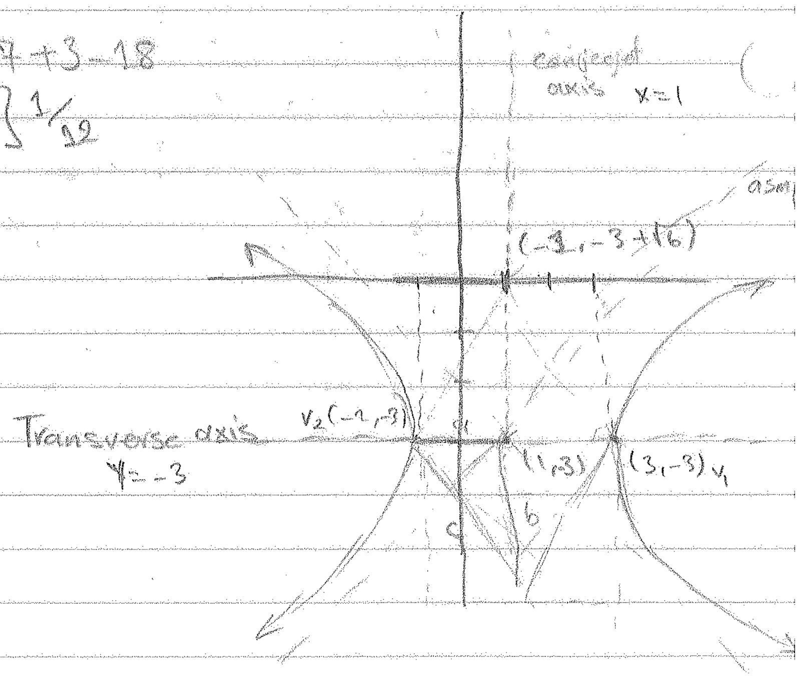
$[\,3(x-1)^2-2(y+3)^2=12\,]\ 1/12$

$\dfrac{(x-1)^2}{4}-\dfrac{(y+3)^2}{6}=1$

$a^2=4 \Rightarrow a=2$

$b^2=6 \Rightarrow b=\sqrt{6}$

center $(h,k)=(1,-3)$



vertix$_1$ $(3,-3)$

vertix$_2$ $(-1,-3)$

foci $(1\pm\sqrt{10},\ -3)$

$y_1-y=m(x-x_1)$

$y-(-3)=\frac{-\sqrt{6}}{2}(x-1)$

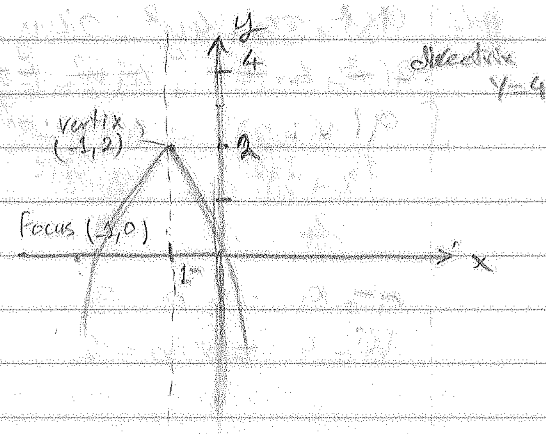
$c^2=a^2+b^2$

$c=\sqrt{4+6}$

$c=\sqrt{10}$

Ch 10. Test sample

#1. vertix $(-1, 2)$
focus $(-1, 0)$

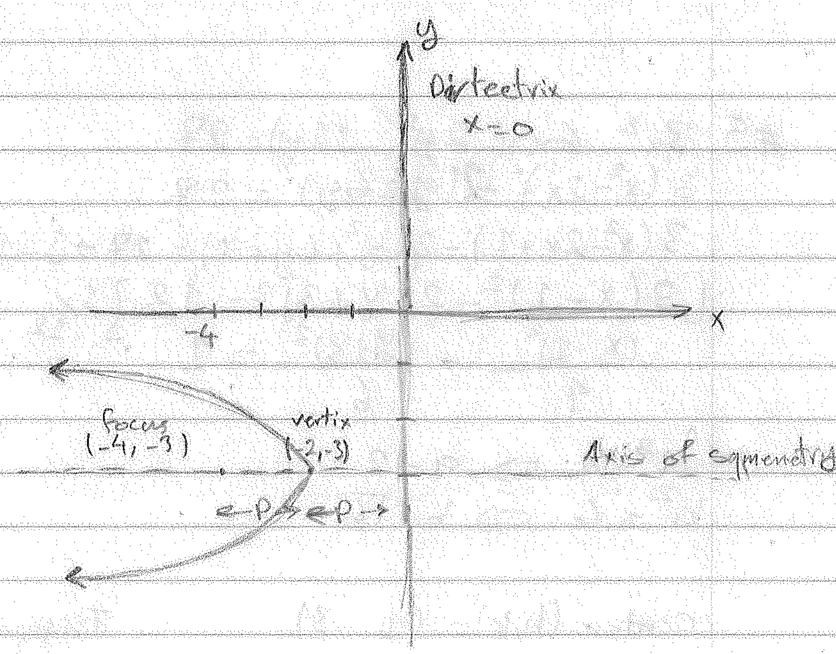


$(x-h)^2 = 4p(y-k)$
$[x-(-1)]^2 = 4(-2)[y-(2)]$
$(x+1)^2 = -8(y-2)$

#2 $y^2+6y = -8x-25$

$\boxed{\begin{array}{l}[\frac{1}{2}(6)] \\ =(3)^2 \\ =9\end{array}}$

$y^2+6y+9 = -8x-25+9$
$(y+3)^2 = -8x-16$
$(y+3)^2 = -8(x+2)$
$-8 = 4p$
$-8/4 = p$
$-2 = p$
$(h,k) = (-2,-3)$



#6. $x = t^3$

$y = \frac{1}{2}t^2$

Solve for t

$x = t^3$

$\sqrt[3]{x} = \sqrt[3]{t^3}$

$\sqrt[3]{x} = t$

$\boxed{y = \frac{1}{2}\left(\sqrt[3]{x}\right)^2}$ or $\boxed{y = \frac{1}{2}x^{2/3}}$

#7. $x = 4 + 2\cos(\theta)$

$y = -1 + 4\sin(\theta)$

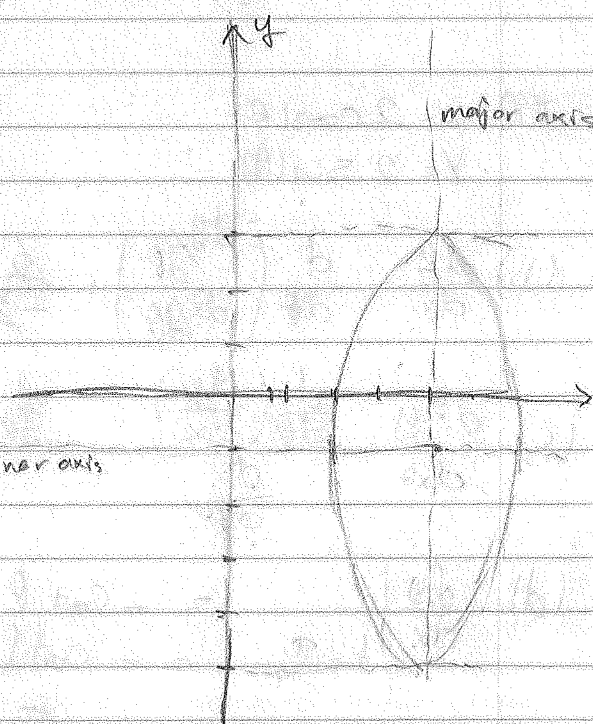
$\frac{x-4}{2} = \cos(\theta)$

$y + 1 = 4\sin(\theta)$

$\frac{y+1}{4} = \sin(\theta)$

$\left(\frac{x-4}{2}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1$

$\frac{(x-4)^2}{4} + \frac{(y+1)^2}{16} = 1$



center $(4, -1)$

$a^2 = 16 \Rightarrow a = 4$

$b^2 = 4 \Rightarrow b = 2$

vertix$_1$ $(4, 3)$

vertix$_2$ $(4, -5)$

foci $(4, -1 \pm 2\sqrt{3})$

$c^2 + b^2 = a^2$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{12} = 2\sqrt{3}$

#11. $x = t^2 - t - 2$

$y = t^3 - 3t$

$$\frac{dy}{dx} = \frac{d}{dt}\left(\frac{dy/dt}{dx/dt}\right) = \frac{3t^2-3}{2t-1}$$

$\frac{dx}{dt} = 0$

$2t - 1 = 0$

$2t = 1$

$t = 1/2$

$\frac{dy}{dt} = 0$

$3t^2 - 3 = 0$

$3t^2 = 3$

$t^2 = \frac{3}{3} = 1$

$t = \pm 1$

Horizontal tangency $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$

Vertical tangency $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

Vertical tangent at $t = 1/2$

Horizontal tangent at $t = \pm 1$

#17 $r(\theta) = 6\sin(2\theta)$ $\qquad 0 \leq \theta \leq \pi$

Area of one petal $= 2 \cdot \left[\frac{1}{2} \int_0^{\pi/4} [6\sin(2\theta)]^2 \, d\theta \right]$

$$= 36 \int_0^{\pi/4} \sin^2(2\theta) \, d\theta$$

$$\boxed{\begin{aligned} \sin^2(u) &= \frac{1-\cos(2u)}{2} \\ \sin^2(2\theta) &= \frac{1-\cos(4\theta)}{2} \end{aligned}}$$

$$= 36 \int_0^{\pi/4} \frac{1-\cos(4\theta)}{2} \, d\theta$$

$$= 18 \int_0^{\pi/4} 1 \, d\theta - 18 \int_0^{\pi/4} \cos(4\theta) \, d\theta$$

$$= 18 \left[\theta\right]_0^{\pi/4} - 18 \left[\frac{\sin(4\theta)}{4}\right]_0^{\pi/4}$$

$$= 18\left[\frac{\pi}{4}\right] - \frac{9}{2}\left[\sin\pi - \sin 0\right]$$

$$= \frac{9}{2}\pi - \frac{9}{2}(0)$$

$$\boxed{= \frac{9\pi}{2}}$$

#18. $r(\theta) = 3\cos(3\theta)$

area of one petal $= S = \cancel{2}\left[\frac{1}{\cancel{2}}\int_0^{\pi/6}\left[3\cos(3\theta)\right]^2 d\theta\right]$

$$S = \int_0^{\pi/6} 9\cos^2(3\theta)\, d\theta$$

$$S = 9\int_0^{\pi/6} \frac{1+\cos(6\theta)}{2}\, d\theta$$

$$S = \frac{9}{2}\int_0^{\pi/6}\left[1+\cos(6\theta)\right] d\theta$$

$$S = \frac{9}{2}\int_0^{\pi/6} 1\, d\theta + \frac{9}{2}\int_0^{\pi/6}\cos(6\theta)\, d\theta$$

let $u = 6\theta$

$\frac{du}{d\theta} = 6$

$\frac{du}{6} = d\theta$

$\theta = 0$	$\theta = \pi/6$
$u = 6(0)$	$u = 6(\pi/6)$
$u = 0$	$u = \pi$

$$= \frac{9}{2}\left[\theta\right]_0^{\pi/6} + \frac{9}{2}\int_0^{\pi}\cos(u)\left(\frac{du}{6}\right)$$

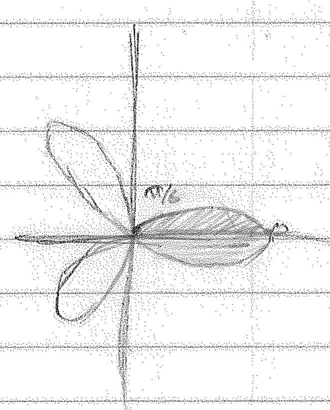
$$= \frac{9}{2}\left[\frac{\pi}{6} - 0\right] + \frac{9}{12}\int_0^{\pi}\cos(u)\, du$$

$$= \frac{\cancel{9}^{3}\pi}{4\cancel{12}} + \frac{\cancel{9}^{3}}{4\cancel{12}}\left[\sin(u)\right]_0^{\pi}$$

$$= \frac{3\pi}{4} + \frac{3}{4}\left[\sin(\pi) - \sin(0)\right]$$

$$= \frac{3\pi}{4} + \frac{3}{4}\left[0 - 0\right]$$

$$= \frac{3\pi}{4}$$



20. (a) $\frac{dr}{d\theta} = 2\sin\theta$

$$S = \int_0^{2\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{4 - 8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{8 - 8\cos\theta}\, d\theta$$

$$= \int_0^{2\pi} 2\sqrt{2}\sqrt{1-\cos\theta}\, d\theta$$

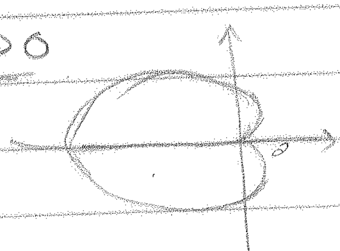
$$= 2\sqrt{2}\int_0^{2\pi} \sqrt{1-\cos\theta}\, d\theta \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= 2\sqrt{2}\int_0^{2\pi} \sqrt{2\sin^2\frac{\theta}{2}}\, d\theta \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= 2\sqrt{2}\sqrt{2}\int_0^{2\pi} \sin\frac{\theta}{2}\, d\theta \qquad 2\sin^2\theta = 1-\cos 2\theta$$

$$= 4\int_0^{2\pi} \left|\sin\left(\frac{\theta}{2}\right)\right| d\theta \qquad 2\sin^2\frac{\theta}{2} = 1-\cos\theta$$

on $(0,\pi)$ $\sin\frac{\theta}{2} > 0$



$u = \frac{\theta}{2}$

$\frac{du}{d\theta} = \frac{1}{2}$

$2\,du = d\theta$

$$= 2\cdot 4\int_0^{\pi} \sin\frac{\theta}{2}\, d\theta$$

$$= 16\left[-\cos\frac{\theta}{2}\right]_0^{\pi}$$

$$= 16\left(-\cos\frac{\pi}{2} + \cos 0\right)$$

$$= 16(-0+1)$$

$$= 16$$