"A few Solutions & notes"

Math 155 Chapter 9 Practice Test

#2

(b) $\left\{ \dfrac{3n^2-n+4}{2n^2+1} \right\}$ Do this sequence diverge or converge?

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n^2-n+4}{2n^2+1}$$

$$= \lim_{n\to\infty} \left[\frac{\frac{3n^2-n+4}{1}}{\frac{2n^2+1}{1}} \right] \left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}} \right)$$

$$= \lim_{n\to\infty} \frac{3 - \frac{1}{n} + \frac{4}{n^2}}{2 + \frac{1}{n^2}}$$

$$= \frac{3}{2}$$

, So, the sequence $\left\{ \dfrac{3n^2-n+4}{2n^2+1} \right\}$ converges.

#3

Remember, page 612 is really page 614 in our 9^{th} Edition. So, try to do #31, #33.
And, page 636 is really 638, so do #7 on page 638.

#33 pg 614: Consider $\displaystyle\sum_{n=1}^{\infty} 2(0.9)^{n-1} = \sum_{k=0}^{\infty} 2\left(\frac{9}{10}\right)^k$

a.

n	5	10	20	50	100
S_n	8.1902	13.0264	17.5685	19.8969	19.9995

use: sum(seq($2*(0.9)^k$, k, 1, $\square$))

5, 10, 20, 50, 100

"A few Solutions and notes"

Math 155 Chapter 9 Practice Test

#33 pg 614

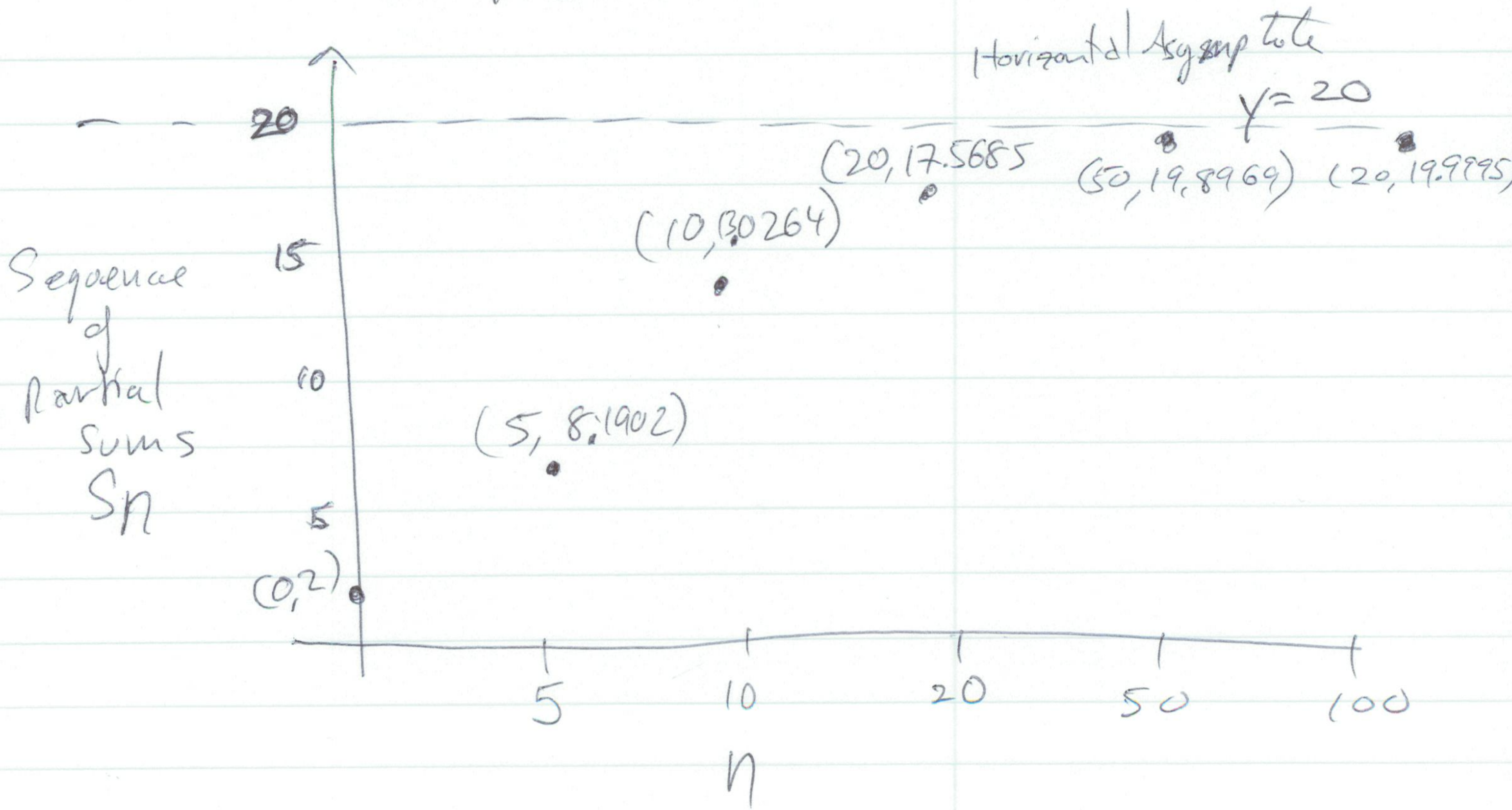
$\sum_{k=0}^{\infty} 2\left(\frac{9}{10}\right)^k$ is a geometric series

and $\frac{a}{1-r} = \sum_{k=0}^{\infty} 2\left(\frac{9}{10}\right)^k$, when $a=2$, $r=\frac{9}{10}$ and $|r|<1$.

$$\frac{2}{1-\frac{9}{10}} = \sum_{k=0}^{\infty} 2\left(\frac{9}{10}\right)^k$$

$$\frac{2}{\frac{1}{10}} = \sum_{k=0}^{\infty} 2\left(\frac{9}{10}\right)^k$$

$$20 = \sum_{k=0}^{\infty} 2\left(\frac{9}{10}\right)^k$$



Math 155

#4

(a) $\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$ Find the sum of the series.

$\leftarrow$ Can be written as a "telescoping" series.

$$\frac{3}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$ $\Leftarrow$ "use partial fractions decomposition"

$$3 = A(n+2) + Bn$$

choose $n=0$

$$3 = 2A$$

$$\frac{3}{2} = A$$

choose $n=-2$

$$3 = -2B$$

$$\frac{3}{2} = B$$

$$\sum_{n=1}^{\infty} \frac{3}{n(n+2)} = \sum_{n=1}^{\infty} \left[\frac{3}{2}\cdot\frac{1}{n} - \frac{3}{2}\cdot\frac{1}{n+2}\right]$$

$$= \frac{3}{2}\cdot \sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{1}{n+2}\right]$$

So, $S_n = \frac{3}{2}\cdot\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)\right.$

$$\left.+\left(\frac{1}{6}-\frac{1}{8}\right)+\cdots+\left(\frac{1}{n-2}-\frac{1}{n}\right)+\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$$

And $S_n = \frac{3}{2}\left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$

So, $\sum_{n=1}^{\infty} \frac{3}{n(n+2)} = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{3}{2} + \frac{3}{4} + 0 + 0 = \frac{6}{4} + \frac{3}{4} = \frac{9}{4}$$ ✓

"A few Solutions & notes"

Math 155 Chapter 9 Practice Test

$\boxed{\#5}$

(b) $0.\overline{81} = 0.8181818181818181\ldots\ldots$

$= 0.81 + 0.0081 + 0.000081 + 0.00000081 + \cdots +$

$= \frac{81}{100} + \frac{81}{10,000} + \frac{81}{1,000,000} + \frac{81}{100,000,000} + \cdots +$

$= \sum_{n=0}^{\infty} \frac{81}{100} \cdot \left[\frac{1}{100}\right]^n = \frac{81}{100}\left[1 + \frac{1}{100} + \frac{1}{10,000} + \frac{1}{1,000,000} + \cdots +\right]$

$= \frac{a}{1-r}$ ⇐ Sum of Geometric Series, $a = \frac{81}{100}$, $r = \frac{1}{100}$

$= \frac{\left(\frac{81}{100}\right)}{1 - \left(\frac{1}{100}\right)}$

$= \frac{\frac{81}{100}}{\frac{100}{100} - \frac{1}{100}}$

$= \frac{\frac{81}{100}}{\frac{99}{100}}$

$= \frac{81}{99}$ ✓

$\boxed{\#6}$

(b) $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$, consider $\int_1^{\infty} \frac{\arctan(x)}{x^2+1}\,dx$

where $f(x) = \frac{\arctan(x)}{x^2+1}$ is positive for $x \geq 1$, and

continuous for $x \geq 1$.

$f'(x) = \frac{(x^2+1)\cdot\frac{d}{dx}[\arctan(x)] - (\arctan(x))\cdot\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$

$f'(x) = \frac{(x^2+1)\cdot\left[\frac{1}{x^2+1}\right] - [\arctan(x)]\cdot(2x)}{(x^2+1)^2}$

$f'(x) = \frac{1 - 2x\arctan(x)}{(x^2+1)^2}$, which is negative for $x \geq 1$

"A few Solutions & notes"

Math 155 Chapter 9 Practice Test

more 6 (b)

Since $f'(x) < 0$ for $x \geq 1$, $f(x) = \frac{\arctan(x)}{x^2+1}$ is decreasing on $(1, \infty)$.

So, we can use the Integral Test to see whether or not $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$ converges or diverges.

Let $u = \arctan(x)$

$\frac{du}{dx} = \frac{1}{x^2+1}$

$(x^2+1) \cdot du = dx$

$x = 1$, $u = \arctan(1)$, $u = \pi/4$

$x = b$, $u = \arctan(b)$

$$\int_1^{\infty} \frac{\arctan(x)}{x^2+1}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{\arctan(x)}{x^2+1}\,dx$$

$$= \lim_{b\to\infty} \int_{\pi/4}^{\arctan(b)} \frac{u}{\cancel{(x^2+1)}} \cdot \left[\left(\frac{\cancel{x^2+1}}{1}\right) du\right]$$

$$= \lim_{b\to\infty} \int_{\pi/4}^{\arctan(b)} u\,du$$

$$= \lim_{b\to\infty} \left[\frac{u^2}{2}\right]_{\pi/4}^{\arctan(b)}$$

$$= \lim_{b\to\infty} \frac{(\arctan(b))^2}{2} - \frac{\left(\frac{\pi}{4}\right)^2}{2}$$

$$= \lim_{b\to\infty} \frac{[\arctan(b)]^2}{2} - \lim_{b\to\infty} \frac{\pi^2}{32}$$

$$= \frac{1}{2} \cdot \left[\lim_{b\to\infty} \frac{\arctan(b)}{1}\right]^2 - \frac{\pi^2}{32}$$

$$= \frac{1}{2}\left[\frac{\pi}{2}\right]^2 - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32}$$

$$= \frac{4}{4} \cdot \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{4\pi^2 - \pi^2}{32}$$

$$= \frac{3\pi^2}{32}$$

Therefore, $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$ converges by the Integral Test, since $\int_1^{\infty} \frac{\arctan(x)}{x^2+1}\,dx$ converges.

"A few notes & Solutions"

Math 155 Practice Test – Chapter 9

$\boxed{\#6}$

(f) $\sum_{n=1}^{\infty} \frac{1}{3n^2-4n+5}$, consider $\sum_{n=1}^{\infty} \frac{1}{n^2}$ which is convergent p-series with $p=2$.

Use the Limit Comparison Test:

for $n \geq 1$, $\frac{1}{3n^2-4n+5} > 0$.

and for $n \geq 1$, $\frac{1}{n^2} > 0$, Let $a_n = \frac{1}{3n^2-4n+5}$ and

$b_n = \frac{1}{n^2}$.

Consider $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \lim_{n\to\infty} \left[\frac{\frac{1}{3n^2-4n+5}}{\frac{1}{n^2}}\right]$

$= \lim_{n\to\infty} \left[\frac{n^2}{3n^2-4n+5}\right] \cdot \left[\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right]$

$= \lim_{n\to\infty} \frac{1}{3-\frac{4}{n}+\frac{5}{n^2}}$

$= \frac{1}{3}$. Since this limit is finite and positive, the Limit Comparison Test says that both series converge.

"A few notes & Solutions"

Math 155 Chapter 9 Practice Test

MM $\boxed{\#6}$

c

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$, consider $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ which is a divergent p-series with $p = 1/2$.

use the Direct Comparison Test:

for $n \geq 2$, $\frac{1}{\sqrt{n}-1} > 0$, and for $n \geq 2$, $\frac{1}{\sqrt{n}} > 0$.

Term-by-term comparison yields

$$\sqrt{n} \geq \sqrt{n} - 1 \quad \text{for } n \geq 2$$

and

$$\frac{1}{\sqrt{n}-1} \geq \frac{1}{\sqrt{n}}$$, so,

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1} \geq \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}.$$

By the Direct Comparison Test, since $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges, we know that $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges too.

"A few solutions & notes"

Math 155 Chapter 9 Practice Test

more #6

m $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$ is an Alternating Series.

Show: ① $\lim_{n\to\infty} a_n = 0$ and ② $0 < a_{n+1} \le a_n$, for $a_n = \frac{\sqrt{n}}{n+2}$.

① $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}}{n+2}$

$= \lim_{n\to\infty} \left[\frac{n^{1/2}}{n+2}\right]\left[\frac{\frac{1}{n^{1/2}}}{\frac{1}{n^{1/2}}}\right]$

$= \lim_{n\to\infty} \frac{1}{n^{1/2} + \frac{2}{n^{1/2}}} \to 0$

$= 0$

② $a_{n+1} = \frac{\sqrt{n+1}}{(n+1)+2}$

$a_{n+1} = \frac{\sqrt{n+1}}{n+3}$

$a_n = \frac{\sqrt{n}}{n+2} > 0$, for $n \ge 1$

and

$a_{n+1} = \frac{\sqrt{n+1}}{n+3} > 0$, for $n \ge 1$

$n^3 + 6n^2 + 9n \ge n^3 + 5n^2 + 8n + 4$, for $n \ge 1$

$n(n^2 + 6n + 9) \ge (n+1)(n^2 + 4n + 4)$

$n(n+3)^2 \ge (n+1)(n+2)^2$

$\sqrt{n}\cdot(n+3) \ge \sqrt{n+1}\cdot(n+2)$

$\frac{\sqrt{n}}{n+2} \ge \frac{\sqrt{n+1}}{n+3}$, and

$a_n \ge a_{n+1}$, for $n \ge 1$.

Hence, by the Alternating Series Test,

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$ converges.

(a) $\left\{\frac{n^3}{3^n}\right\}$ Sequence Converges

$\lim_{n\to\infty} \frac{n^3}{3^n}$ $\frac{\infty}{\infty}$ l'hopital Rule

$\lim_{n\to\infty} \frac{\frac{d}{dn}(n^3)}{\frac{d}{dn}(3^n)}$

$\lim_{n\to\infty} \frac{3n^2}{\ln 3 . 3^n} \rightarrow \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{3 \frac{d}{dn} n^2}{\ln 3 \frac{d}{dn}(3^n)}$

$\lim_{n\to\infty} \frac{3 . 2n}{\ln 3 . \ln 3 . 3^n}$

$\lim_{n\to\infty} \frac{3 . 2 \frac{d}{dn} n}{(\ln 3)^2 \frac{d}{dn} 3^n}$

$\lim_{n\to\infty} \frac{6}{(\ln 3)^3 3^n} = 0$

(b) $\left\{\frac{(n+1)!}{n!}\right\}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(n+1)!}{n!}$

$\lim_{n\to\infty} \frac{(n+1) \cancel{n} . \cancel{(n-1)} . \cancel{(n-2)} \cdots \cancel{3 . 2 . 1}}{\cancel{n} . \cancel{(n-1)} \cancel{(n-2)} \cdots \cancel{3 . 2 . 1}}$

$\lim_{n \to \infty} n+1$

$= \infty$ the sequence diverges

614

$\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$ doing on calculator

$S_5 = \sum_{n=1}^{5} \frac{6}{n(n+3)} \approx 2.79$ → Factored form → Telescoping series

$S_{10} = \sum_{n=1}^{10} \frac{6}{n^2+3n} \approx 3.1643$

$S_{20} = \sum_{n=1}^{20} \frac{6}{n^2+3n} = 3.3956$

$S_{50} = \sum_{n=1}^{50} \frac{6}{n^2+3n} = 3.5513$

$S_{100} = \sum_{n=1}^{100} \frac{6}{n^2+3n} = 3.607$

finding the sum:

$$\frac{6}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}$$

$$6 = A(n+3) + Bn$$

choose $n=0$	choose $n=-3$
$6 = 3A$	$6 = -3B$
$2 = A$	$-2 = B$

$$\frac{6}{n(n+3)} = \frac{2}{n} - \frac{2}{n+3}$$

$$S_N = \sum_{n=1}^{N} \frac{2}{n} - \frac{2}{n+3}$$

$$2\sum_{n=1}^{N}\left(\frac{1}{n}-\frac{1}{n+3}\right)$$

$$2\left[\left(\frac{1}{1}-\frac{1}{4}\right)+\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{3}-\frac{1}{6}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)-\,-\right.$$

$$\left.2\left[\left(1+\frac{1}{2}+\frac{1}{3}\right)-\left(\frac{1}{N+1}+\frac{1}{N+2}+\frac{1}{N+3}\right)\right]\right.$$

$$\lim_{N\to\infty} S_N = \sum_{n=1}^{\infty}\frac{6}{n^2+3n}$$

$$\lim_{N\to\infty} 2\left[\left(1+\frac{1}{2}+\frac{1}{3}\right)-\left(\frac{1}{N+1}+\frac{1}{N+2}+\frac{1}{N+3}\right)\right]$$

$$=2\left(1+\frac{1}{2}+\frac{1}{3}\right)=2\cdot\frac{11}{\cancel{36}}=\frac{11}{3}$$

4/22 Fri

$$f(x) = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad |r| < 1$$

(14) let $z = x+3$

$x = z-3$

$f(x) = \frac{1}{2x-5}$ simplify

$$f(z-3) = \frac{1}{2(z-3)-5} = \frac{1}{2z-6-5} = \frac{1}{2z-11} = \frac{1}{-11+2z}$$

$$= \left(\frac{1}{-11+2z}\right)\left(\frac{-\frac{1}{11}}{-\frac{1}{11}}\right) = \frac{-\frac{1}{11}}{1-\frac{2}{11}z}$$

so, $a = -\frac{1}{11}$ $r = \frac{2}{11}z$

$$f(x) = \sum_{n=0}^{\infty} ar^n, \quad a = -\frac{1}{11} \text{ and } r = \frac{2}{11}z$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{11}\right)\left(\frac{2}{11}z\right)^n$$

$$= -\frac{1}{11}\sum_{n=0}^{\infty} \left(\frac{2}{11}\right)^n (z)^n = -\frac{1}{11}\sum_{n=0}^{\infty} \left(\frac{2}{11}\right)^n (x+3)^n$$

$$\boxed{\text{so } f(x) = -\frac{1}{11}\sum_{n=0}^{\infty} \left(\frac{2}{11}\right)^n (x+3)^n}$$ power series representation

* we used geometric formula instead of using Taylor's series

Now find the intervals of convergence: use $|r| < 1$

* (not in the book)

$$|r| < 1$$

$$\left|\frac{2}{11}z\right| < 1$$

$$\left|\frac{2}{11}(x+3)\right| < 1$$

$$\frac{2}{11}|x+3| < 1$$

center is $c = -3$

$R = \frac{11}{2}$

$$|x+3| < \frac{11}{2}$$

$$-\frac{11}{2} < x+3 < \frac{11}{2}$$ so, $-\frac{17}{2} < x < \frac{5}{2}$

Check the endpoints → $\left(-\frac{17}{2}, \frac{5}{2}\right)$ (we dont have to)

$x = -\frac{11}{2}$

$$f\left(-\frac{11}{2}\right) = -\frac{1}{11}\sum_{n=0}^{\infty}\left(\frac{2}{11}\right)^n\left(-\frac{17}{2}+3\right)^n$$

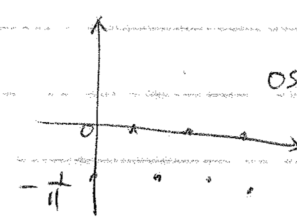
$$= -\frac{1}{11}\sum_{n=0}^{\infty}\left(\frac{2}{11}\right)^n\left(-\frac{11}{2}\right)^n$$

$$= -\frac{1}{11}\sum_{n=0}^{\infty}(-1)^n$$ Alternating Series

$a_n = 1$

$\lim_{n\to\infty} a_n = 0$ fails

Diverges by AST



oscillating

we want

$$\boxed{f(x)=\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \ |r|<1}$$

(14) 1b) $g(x)=\frac{4}{x+2}$, $c=0$

Since $c=0$, no need for z

$g(x)=\frac{4}{x+2}$

$a=2$
$r=-\frac{x}{2}$

want $\ominus$

$=\left(\frac{4}{2+x}\right)\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) = \frac{2}{1+\frac{x}{2}} = \frac{2}{1-(-\frac{x}{2})} = \frac{a}{1-r}$

so $g(x)=\sum_{n=0}^{\infty} ar^n = \sum_{n=0}^{\infty} (2)(-\frac{x}{2})^n$

our Series $\rightarrow$ $\boxed{g(x)=2\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^n}}$

Interval of convergence

$|r|<1$

$|-\frac{x}{2}|<1$

$2\cdot\frac{|x|}{2} < 2\cdot 1$

$|x|<2$

$-2<x<2$

$(-2,2)$

(13) $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n} = \frac{x^1}{1}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots$

$f'(x)=\sum_{n=1}^{\infty}\frac{\cancel{n}x^{n-1}}{\cancel{n}} = x^0+x^1+x^2+\ldots$

Same

$f'(x)=\sum_{n=0}^{\infty}x^n = x^0+x^1+x^2+\ldots$

$\int f(x)\,dx = \int\left(\sum_{n=1}^{\infty}\frac{x^n}{n}\right)dx=\int\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}\cdots\right)dx$

$= \frac{x^2}{2}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1}{3}\cdot\frac{x^4}{4}+\ldots$

$= \sum_{n=2}^{\infty}\frac{x^n}{(n-1)n} = \sum_{n=1}^{\infty}\frac{x^{n+1}}{n(n+1)}$

Ratio Test

* must say the condition *

$\boxed{\frac{x^{n+1}}{n(n+1)} \neq 0 \quad \text{when } x\neq 0}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1+1}}{\cancel{(n+1)}(n+1+1)}\cdot\frac{n\cancel{(n+1)}}{x^{n+1}}\right|$

$=\lim_{n\to\infty}\left|\frac{x\,n}{(n+2)}\right|$

$= |x|\lim_{n\to\infty}\left|\frac{n}{n+2}\right|$

$=|x|$

Ratio Test gives convergence for $|x|<1$

$-1<x<1$

$R=1$

$(-1,1)$ ← check endpoints

check $x=1$

$$\sum_{n=1}^{\infty} \frac{(1)^{n+1}}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

lim Comparison Test

* let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^2+n}$ $\quad \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$

$a_n = \frac{1}{n^2+n} > 0$ for all $n \geq 1$

$b_n = \frac{1}{n^2} > 0$ for all $n \geq 1$

consider

$$\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \lim_{n\to\infty} \left(\frac{1}{n^2+n} \cdot n^2\right)$$

$$= \lim_{n\to\infty} \left(\frac{n^2}{n^2+n}\right)\left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{1+\underbrace{\left(\frac{1}{n}\right)}_{\to 0}}\right)$$

$= 1$ Finite, positive

therefore $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges

Check $x=-1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+n} = \sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{1}{n^2+n}\right)$$ is an Alternating Series, with $a_n = \frac{1}{n^2+n}$

① show $\lim_{n\to\infty} a_n = 0$	① $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n^2+n} = 0$
② $0 < a_{n+1} \leq a_n$	② $a_n = \frac{1}{n^2+n} > 0$ for all $n \geq 1$
	$a_{n+1} = \frac{1}{(n+1)^2+(n+1)} > 0$ for all $n \geq 1$

To show $a_{n+1} \le a_n$

$$n+1 \geq n \quad \text{for all } n \geq 1$$
$$(n+1)^2 + n+1 \geq n^2 + n \quad \text{for all } n \geq 1$$
$$\frac{1}{n^2+n} \geq \frac{1}{(n+1)^2+n+1}$$
$$a_n \geq a_{n+1}$$

* this method (Algebra) preferred (shorter easier)

By AST $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+n}$ converges

Or to show decreasing

calculus way

$$g(n) = \frac{1}{n^2+n} = (n^2+n)^{-1}$$
$$g'(n) = -1(n^2+n)^{-2}(2n+1)$$
$$= \frac{-(2n+1)}{(n^2+n)^2}$$

Critical number $\quad g'(n)$ is undefined

I $\cancel{\theta}$ $2n+1=0$, $\quad$ II $n^2+n=0$

$2n=-1 \qquad n(n+1)=0$

$\cancel{n=-\frac{1}{2}} \qquad n=0$ or $\cancel{n=-1}$

Test $n=1$ $\quad g'(1) = -\frac{3}{4} < 0 \qquad$ (Test ~~the right of 0~~)

by AST $\rightarrow [-1, 1] \leftarrow$ by LC

(12) (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n\,5^n}$ Interval of convergence

Ratio Test ① $\frac{(-1)^{n+1}(x-5)^n}{n\,5^n} = 0$ if $x=5$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x-5)^{n+2}}{(n+1)\,5^{n+1}} \cdot \frac{n\,5^n}{(-1)^{n+1}(x-5)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-5)\,n}{(n+1)\,5}\right|$$

$$= \frac{|x-5|}{5}\lim_{n\to\infty}\frac{n}{n+1}$$

$$= \frac{|x-5|}{5}$$

Solve $\frac{|x-5|}{5} < 1$

$$5\cdot\frac{|x-5|}{5} < 5\cdot 1$$

$$|x-5| < 5$$

$$-5 < x-5 < 5$$

$$0 < x < 10$$ check end points

By Harmonic Diverges $(0, 10]$

$R=5$

check $x=0$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(0-5)^n}{n\,5^n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-5)^n}{n\,5^n} \qquad (AB)^n = A^n B^n$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n\cancel{5^n}}{n\,\cancel{5^n}}$$

always odd

$$= \sum_{n=1}^{\infty}\frac{(-1)^{2n+1}}{n} = \sum_{n=1}^{\infty}\frac{-1}{n}$$ Diverges as harmonic series (no test necessary)

Check $x=10$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(10-5)^n}{n\,5^n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,5^n}{n\,5^n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$ converges by AST

① $\lim_{n\to\infty} a_n = 0$

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

② $0 < a_{n+1} \le a_n$

show $a_{n+1} = \frac{1}{n+1} > 0$ for all $n \geq 1$ ✓

$a_n = \frac{1}{n} > 0$ for all $n \geq 1$ ✓

$n+1 \geq n$ for all $n \geq 1$

$\frac{1}{n} \geq \frac{1}{n+1}$

$a_n \geq a_{n+1}$ ✓

So, $\sum \frac{(-1)^{n+1}}{n}$ converges by AST

⑫ (b) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

<u>Ratio Test</u>

$\frac{x^n}{n!} = 0$ ~~for all $x=0$~~

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right| = \lim_{n\to\infty}\left|\frac{x}{n+1}\right| = |x| \lim_{n\to\infty}\left|\frac{1}{n+1}\right|$$

$= 0 \to R = \infty$

Interval of Convergence

$(-\infty, \infty)$

(11) $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $\quad f(x) \approx f(c) + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3$

$c = 1 \quad f(1) = 1 \quad + \frac{f^{IV}(x)}{4!}(x-c)^4 = P_4(x)$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \qquad f'(1) = \frac{1}{2}\cdot 1 = \frac{1}{2}$

$f''(x) = -\frac{1}{4}x^{-3/2} \qquad f''(1) = -\frac{1}{4}\cdot 1 = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}x^{-5/2} \qquad f'''(1) = \frac{3}{8}\cdot 1 = \frac{3}{8}$

$f^{IV}(x) = -\frac{15}{16}x^{-7/2} \qquad f^{IV}(x) = -\frac{15}{16}\cdot 1 = -\frac{15}{16}$

$P_4 = 1 + \frac{1}{2}\left(\frac{1}{1!}\right)(x-1) + \left(-\frac{1}{4}\right)\left(\frac{1}{2!}\right)(x-1)^2 + \frac{3}{8}\left(\frac{1}{3!}\right)(x-1)^3 + \left(-\frac{15}{16}\right)\left(\frac{1}{4!}\right)(x-1)^4$

$P_4 = 1 + \frac{1}{2}(x-1) - \frac{1}{4\cdot 2}(x-1)^2 + \frac{3}{8}\cdot\frac{1}{2\cdot 3}(x-1)^3 - \frac{15}{16}\cdot\frac{1}{2\cdot 3\cdot 4}(x-1)^4$

$P_4 = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$

$g(x) = \ln(x) \qquad c = 1$

$g(1) = \ln(1) = 0$

$g'(x) = \frac{1}{x} = x^{-1} \qquad g'(1) = 1$

$g''(x) = -1x^{-2} \qquad g''(1) = -1$

$g'''(x) = 2x^{-3} \qquad g'''(1) = 2$

$g^{IV}(x) = -6x^{-4} \qquad g^{IV}(1) = -6$

$g(x) \approx g(1) + \frac{g'(1)}{1!}(x-1) + \frac{g''(1)}{2!}(x-1)^2 + \frac{g'''(1)}{3!}(x-1)^3 + \frac{g^{IV}(1)}{4!}(x-1)^4$

$P_4 = 0 + \frac{(1)}{1!}(x-1) + \frac{(-1)}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{(-6)}{4!}(x-1)^4$

$P_4 = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

! correction for #7:

#7 page 679

#41, #45, #47

in the notes 9.5

P679 #41

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e} \leftarrow \text{sum}, \quad a_n = \frac{1}{n!}$$

number of terms with an error of less than 0.001

$$|R_N| \le a_{N+1}$$

$$|R_N| \le \frac{1}{(N+1)!}$$

$$\frac{1}{(N+1)!} < 0.001$$

$$\frac{1}{(N+1)!} < \frac{1}{1000}$$

(cross multiply)

$$1000 < (N+1)!$$

$$N+1 = 7$$

$$N = 6$$

$$7! = 5040$$

$$\sum_{n=0}^{6} \frac{(-1)^n}{n!} = \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \cdots + \frac{(-1)^5}{5!} + \frac{(-1)^6}{6!} \approx 0.368 \quad \frac{53}{144}$$

7 terms

#47 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$, $a_n = \frac{1}{n^3}$

$|R_N| \leq a_{N+1}$

$|R_N| \leq \frac{1}{(n+1)^3}$

$\frac{1}{(n+1)^3} < 0.001$

$\frac{1}{(n+1)^3} < \frac{1}{1000}$

$1000 < (n+1)^3$ $\quad 11^3 = 1331$

$n+1 = 11$

$n = 10$

$\sum_{n=1}^{11} \frac{(-1)^{n+1}}{n^3} = \frac{(-1)^2}{1} + \frac{(-1)^3}{2^3} + \frac{(-1)^4}{3^3} + \frac{(-1)^5}{4^3} + \dots \frac{(-1)^{10}}{9^3} + \frac{(-1)^{11}}{10^3}$

use 10 terms

8 (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3}{n^2} \approx \sum_{n=1}^{6} \frac{(-1)^{n+1} 3}{n^2}$

$\approx \frac{(-1)^2 3}{1} + \frac{(-1)^3 3}{2^2} + \frac{(-1)^4 3}{3^2} + \frac{(-1)^5 3}{4^2} + \frac{(-1)^6 3}{5^2} + \frac{(-1)^7 3}{6^2}$

$\approx 3 + \frac{-3}{4} + \frac{3}{9} + \frac{-3}{16} + \frac{3}{25} + \frac{-3}{36}$

$= 2.4325$

9. (a). $f(x) = \frac{1}{1+x} = (1+x)^{-1}$, $c=0$ "mac"?

$f'(x) = -1(1+x)^{-2}(1)$

$f''(x) = \cancel{+}2(1+x)^{-3}(1)$

$f'''(x) = -6(1+x)^{-4}(1)$

$f^{IV}(x) = 24(1+x)^{-5}(1)$

$f^{V}(x) = -120(1+x)^{-6}$

$f'(0) = -1$

$f''(0) = 2$

$f'''(0) = -6$

$f^{IV}(0) = 24$

$f(x) \approx f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3 + \frac{f^{IV}(x)}{4!}(x)^4$

$P_4(x) = 1 + \frac{(-1)}{1!}(x) + \frac{(2)}{2!}(x)^2 + \frac{(-6)}{3!}(x)^3 + \frac{(24)}{4!}(x)^4$

(a) $P_4(x) = 1 - x + x^2 - x^3 + x^4$

(b) $f(0.1) \approx P_4(0.1)$

$f(0.1) \approx 1 - (0.1) + (0.1)^2 - (0.1)^3 + (0.1)^4$

$f(0.1) \approx 0.9091$

0.90909090...

0.000001

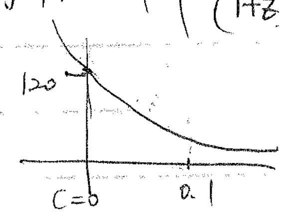
(c) $|R_N(x)| \le \frac{|x-c|^{n+1}}{(n+1)!} \text{Max} |f^{n+1}(z)|$

$n=4$

$c=0$

$x=0.1$

$|R_4(0.1)| \le \frac{|0.1-0|^5}{5!} \text{Max} |f^{V}(z)|$, $y = |f^{V}(z)| = \left| \frac{-120}{(1+z)^6} \right|$



$|R_4(0.1)| \le \frac{(0.1)^5 \; 120}{120}$

$|R_4(0.1)| \le (0.1)^5$

$|R_4(0.1)| \le \cancel{0.000} \; \frac{1}{10^5} = \frac{1}{100,000}$

0.000001