MATH 135 FORMULA SHEET

Cube Factor Formula

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Distance Formula

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Point-Slope Equation of a Line

$$y - y_1 = m(x - x_1)$$

Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-Intercept Equation of a Line

$$y = mx + b$$

Standard Equation of Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Quadratic Formula

If $ax^2 + bx + c = 0, \quad a \neq 0$

$$then \;\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Vertex = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a} \right) \right)$$

$$Axis \;\; of \;\; symmetry: x = \frac{-b}{2a}$$

Logarithms

$$\log_b(MN) = \log_b M + \log_b N$$

$$\log_b(\tfrac{M}{N}) = \log_b M - \log_b N$$

$$\log_b M^r = r \cdot \log_b M$$

$$\log_b M = \frac{\log_a M}{\log_a b}$$

$$\log_b b = 1 \qquad \log_b(b^x) = x$$

$$\log_b 1 = 0 \qquad b^{\log_b(x)} = x$$

If $|x| = a \iff x = a \;\; or \;\; x = -a$ [where $a \geq 0$]

If $|x| \prec a \iff -a \prec x \prec a$

If $|x| \succ a \iff x \prec -a \;\; or \;\; x \succ a$

$$I = PRT \qquad A = P(1 + \tfrac{r}{n})^{nt}$$

$$A = Pe^{rt} \qquad N(t) = N_0 e^{kt}$$

$$P(t) = \frac{c}{1 + ae^{-bt}}$$

Discriminant = $b^2 - 4ac$

If > 0, there are two unequal real solutions.
If = 0, there is a repeated real solution.
If < 0, there are 2 complex, non-real solutions.

Remainder Theorem: If a polynomial function *f(x)* is divided by $x - c$, then the remainder is *f(c).*

Factor Theorem: $x - c$ is a factor of *f(x)* if and only if *f(c)* = 0.

Rational Zeros Theorem

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, with $a_n \neq 0$ and $a_0 \neq 0$, where each coefficient is an integer. If $\frac{p}{q}$, in lowest terms, is a rational zero of f, then p must be a factor of a_0, and q must be factor of a_n.

Bounds on Zeros Theorem:

Let $f(x) = x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$. A number M bound on the zeros of *f* is the <u>smaller</u> of the two numbers: $Max\left\{1, |a_0| + |a_1| + ... + |a_{n-1}|\right\} \;\; or \;\; 1 + Max\left\{|a_0|, |a_1|, ..., |a_{n-1}|\right\}$

Intermediate Value Theorem:

Let *f* denote a continuous function. If *a*<*b* and if *f(a)* and *f(b)* are of opposite sign, then *f* has at least one zero between *a* and *b*.

MATH 135 FORMULA SHEET

Graphing Ellipse & Hyperbola

Note: a = distance from center to vertex
b = cross on major axis
c = distance from center to foci

Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

Center = *(h, k)*	Center = *(h, k)*
$a^2-c^2=b^2$	$a^2-c^2=b^2$
Foci = $(h \pm c,\ k)$	Foci = $(h,\ k \pm c)$
Vertices:	Vertices:
$(h \pm a,\ k), (h,\ k \pm b),$	$(h,\ k \pm a), (h \pm b,\ k)$

Hyperbola

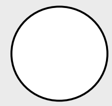
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Center = *(h, k)*	Center = *(h, k)*
$a^2+b^2=c^2$	$a^2+b^2=c^2$
Foci = $(h \pm c,\ k),$	Foci: $(h,\ k \pm c)$
Vertices = $(h \pm a,\ k)$	Vertices = $(h,\ k \pm a)$
Asymptotes:	Asymptotes:
$y-k=\pm\frac{b}{a}(x-h)$	$y-k=\pm\frac{a}{b}(x-h)$

Parabola

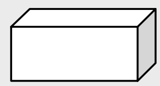
$$(y-k)^2=4a(x-h) \quad \text{or} \quad (x-h)^2=4a(y-k)$$

Vertex = *(h, k)*	Vertex = *(h, k)*
Focus = (h + a, k)	Focus = (h, k + a)
Directrix	Directrix
x = h – a	y = k – a



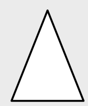
$A=\pi r^2, \quad C=2\pi r$

$V=\frac{4}{3}\pi r^3, \quad S=4\pi r^2$

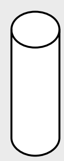


$A=LW, \quad P=2L+2W$

$V=LWH$



$A=\frac{1}{2}bh$



$V=\pi r^2 h$

Matrix

1. Interchange any two rows.
2. Replace a row by a non-zero multiple of that row.
3. Replace a row by the sum of that row and a constant non-zero multiple of some other row

$$Goal\ Matrix=\begin{bmatrix} 1 & a & b & : & c \\ 0 & 1 & d & : & e \\ 0 & 0 & 1 & : & f \end{bmatrix}$$

$$\begin{aligned} a_{11}x+a_{12}y+a_{13}z&=C_1 \\ a_{21}x+a_{22}y+a_{23}z&=C_2 \\ a_{31}x+a_{32}y+a_{33}z&=C_3 \end{aligned} \quad \Rightarrow \quad D=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad Dx=\begin{pmatrix} C_1 & a_{12} & a_{13} \\ C_2 & a_{22} & a_{23} \\ C_3 & a_{32} & a_{33} \end{pmatrix}$$

$$D=\begin{pmatrix} a & b \\ c & d \end{pmatrix}=ad-bc \qquad x=\frac{Dx}{D},\ y=\frac{Dy}{D},\ z=\frac{Dz}{D}\ \ (if\ D\neq 0)$$

TRIGONOMETRY HANDOUT

Pythagorean Identities

$$\sin^2 a + \cos^2 a = 1$$
$$1 + \tan^2 a = \sec^2 a$$
$$1 + \cot^2 a = \csc^2 a$$

Even-Odd Identities

$$\sin(-a) = -\sin a$$
$$\cos(-a) = \cos a$$
$$\tan(-a) = -\tan a$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$
$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$
$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

Sum & Difference Identities

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$
$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Double Angle Identities

$$\sin 2a = 2\sin a \cos a$$
$$\cos 2a = 2\cos^2 a - 1 = 1 - 2\sin^2 a = \cos^2 a - \sin^2 a$$
$$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$$

Half-Angle Identities

$$\sin\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}} \qquad \cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$$
$$\tan\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{1+\cos a}} = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

Sum to Product Identities

$$\sin a + \sin b = 2\sin\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$
$$\sin a - \sin b = 2\cos\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$
$$\cos a + \cos b = 2\cos\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$
$$\cos a - \cos b = -2\sin\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

Product to Sum Identities

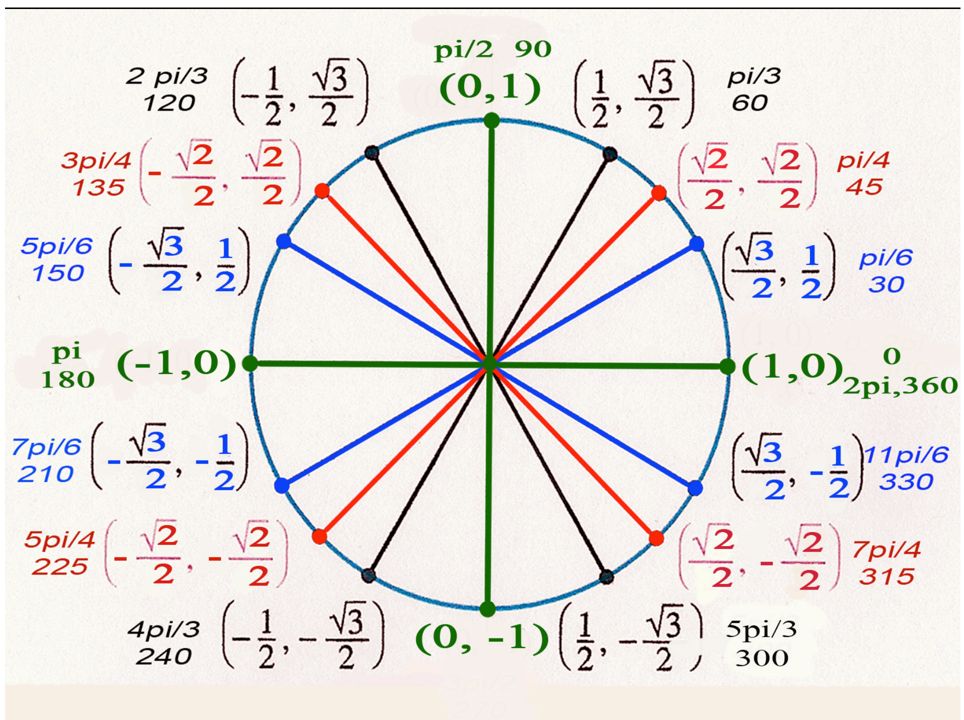
$$\sin a \sin b = \tfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\cos a \cos b = \tfrac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\cos a \sin b = \tfrac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$$
$$\sin a \cos b = \tfrac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

For the graphs of $y = A\sin(\omega x - \phi)$ *or* $y = A\cos(\omega x - \phi)$, *with* $\omega \succ 0$,

Amplitude $= |A|$, *Period* $= \dfrac{2\pi}{\omega}$, *Phase shift* $= \dfrac{\phi}{\omega}$ (*Phase shift is to left if* $\phi < 0$ *and to right if* $\phi > 0$.)

If θ is an angle in standard position and *P(x,y)* is the point where the terminal side of the angle meets the unit circle, then the six trigonometric functions of θ are defined as follows.

Sin $\theta = y$	Csc $\theta = 1/y$	$(y \neq 0)$
Cos $\theta = x$	Sec $\theta = 1/x$	$(x \neq 0)$
Tan $\theta = y/x$ $(x \neq 0)$	Cot $\theta = x/y$	$(y \neq 0)$



Fundamental Identities

Sin θ = 1/csc θ
Cos θ = 1/sec θ
Tan θ = 1/cot θ = sin θ/cos θ

Csc θ = 1/sin θ
Sec θ = 1/cos θ
Cot θ = 1/tan θ = cos θ/sin θ