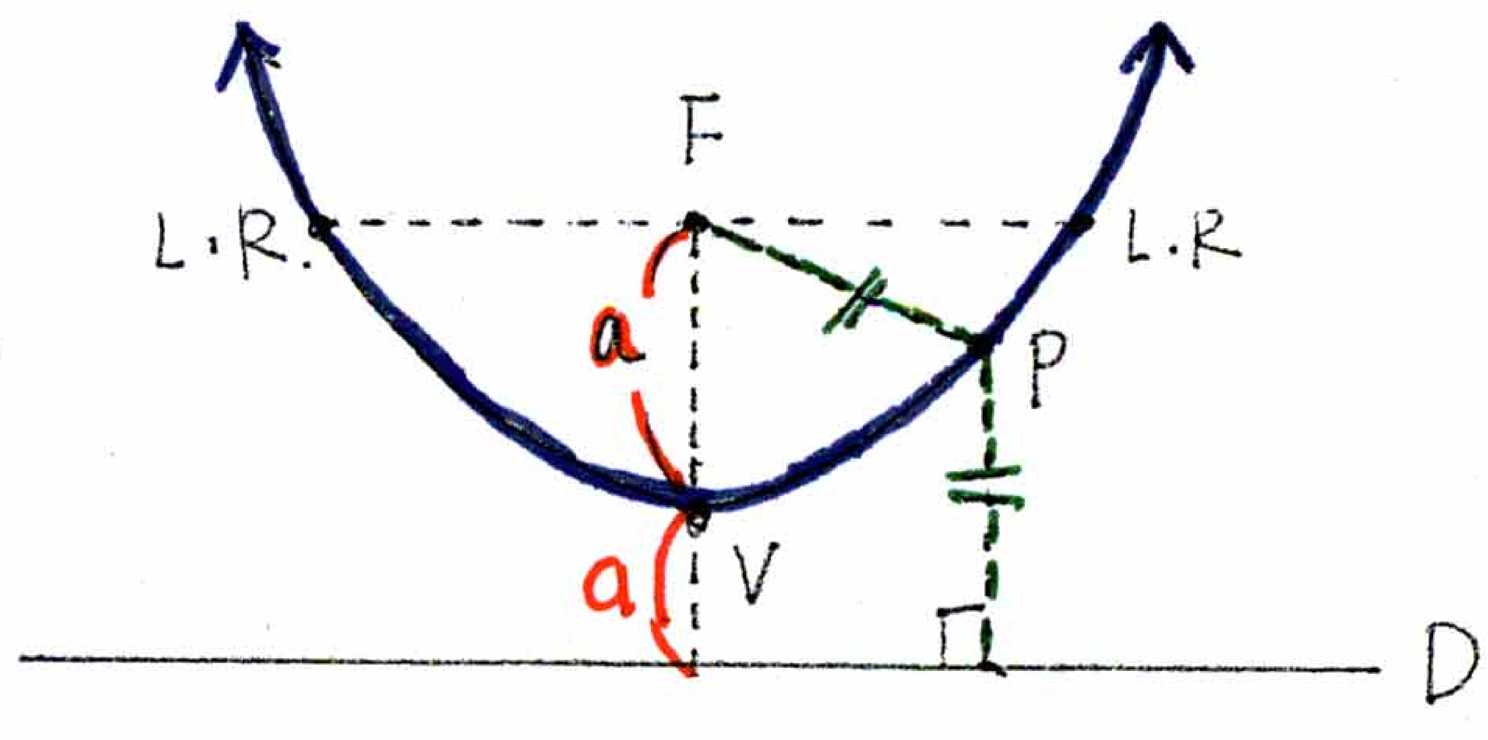
Parabola:

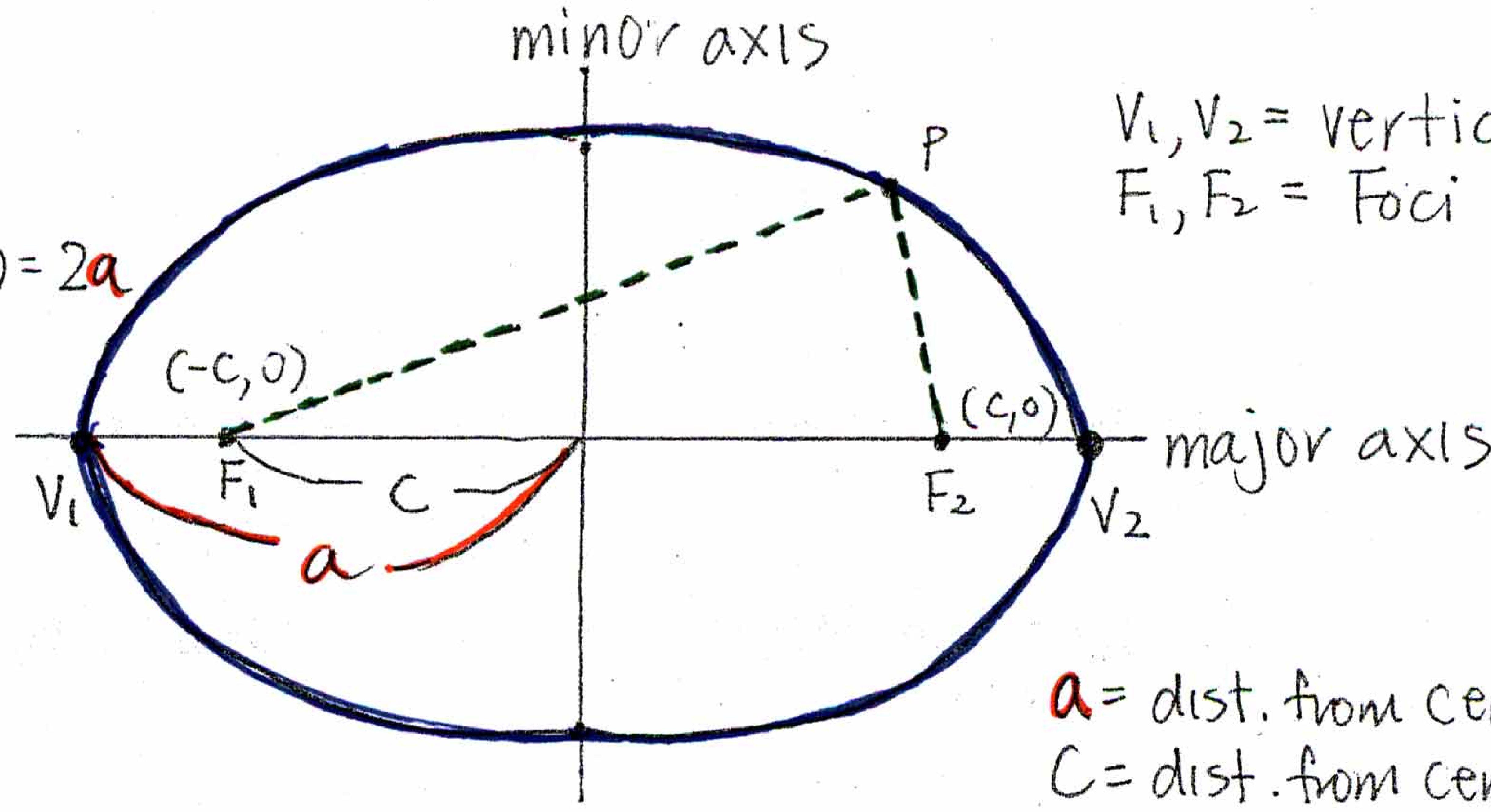
$d(F, P) = d(P, D)$



F = Focus
V = vertex
D = Directrix
L.R = Latus Rectum
P = any point on the graph.
a = distance from F to V.

Ellipse:

$d(F_1, P) + d(F_2, P) = 2a$

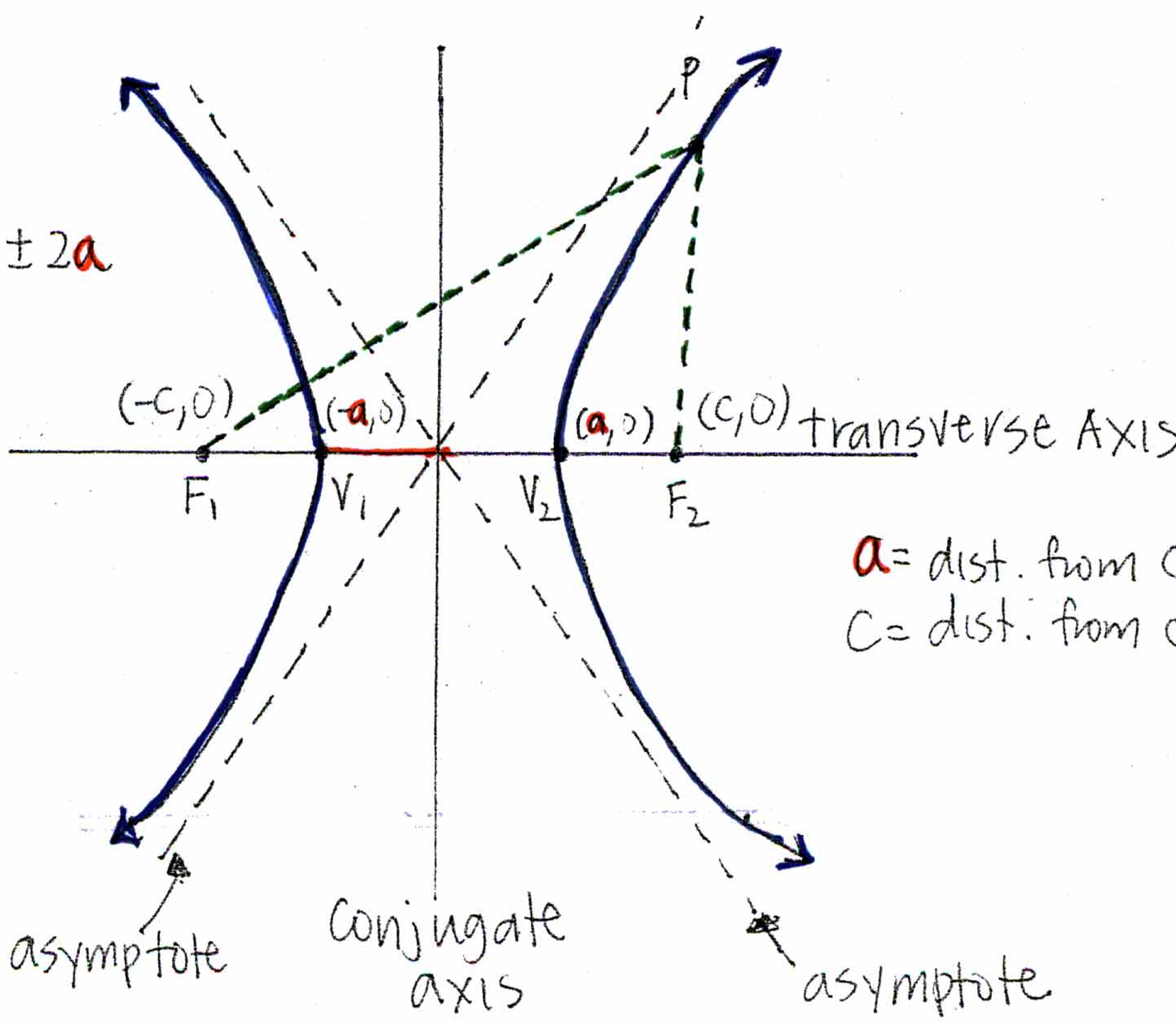


V_1, V_2 = vertices
F_1, F_2 = Foci

a = dist. from center to V.
c = dist. from center to F.

Hyperbola:

$d(F_1, P) - d(F_2, P) = \pm 2a$



a = dist. from center to V
c = dist. from center to F.

MATH 135 FORMULA SHEET

Graphing Ellipse & Hyperbola

Note: a = distance from center to vertex
b = cross on major axis
c = distance from center to foci

Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

Center = *(h, k)*
$a^2-c^2=b^2$
Foci = $(h \pm c,\ k)$
Vertices:
$(h \pm a,\ k),\ (h,\ k \pm b),$

Center = *(h, k)*
$a^2-c^2=b^2$
Foci = $(h,\ k \pm c)$
Vertices:
$(h,\ k \pm a),\ (h \pm b,\ k)$

Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Center = *(h, k)*
$a^2+b^2=c^2$
Foci = $(h \pm c,\ k),$
Vertices = $(h \pm a,\ k)$
Asymptotes:
$y-k=\pm\frac{b}{a}(x-h)$

Center = *(h, k)*
$a^2+b^2=c^2$
Foci: $(h,\ k \pm c)$
Vertices = $(h,\ k \pm a)$
Asymptotes:
$y-k=\pm\frac{a}{b}(x-h)$

Distance Formula

$$d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Midpoint Formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Point-Slope Equation of a Line

$$y-y_1=m(x-x_1)$$

Slope Formula

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Slope-Intercept Equation of a Line

$$y=mx+b$$

Standard Equation of Circle

$$(x-h)^2+(y-k)^2=r^2$$

Parabola

$(y-k)^2=4a(x-h)$ or $(x-h)^2=4a(y-k)$

Vertex = *(h, k)*
Focus = (h + a, k)
Directrix
x = h – a

Vertex = *(h, k)*
Focus = (h, k + a)
Directrix
y = k – a

Quadratic Formula

If $ax^2+bx+c=0,\quad a \neq 0$

$$\text{then } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\text{Vertex}=\left(\frac{-b}{2a},f\left(\frac{-b}{2a}\right)\right)$$

$$\text{Axis of symmetry}: x=\frac{-b}{2a}$$

$\boxed{10.1}$ Conics and Calculus

#16 Find the vertex, focus, and directrix of the parabola, and sketch its graph.

$$y^2 + 4y + 8x - 12 = 0$$

complete the square

$$y^2 + 4y = -8x + 12$$

$$y^2 + 4y + 4 = -8x + 12 + 4$$

$$\left[\tfrac{1}{2}(4)\right]^2 = (2)^2 = 4$$

$$(y+2)^2 = -8x + 16$$

$$(y+2)^2 = -8(x-2)$$

opens left

Goal:

$(y-k)^2 = 4a(x-h)$

Vertex $= (h,k)$

Focus $= (h+a, k)$

Directrix: $x = h - a$

$k = -2$, $h = 2$, $-8 = 4a$

$\underline{-2 = a}$

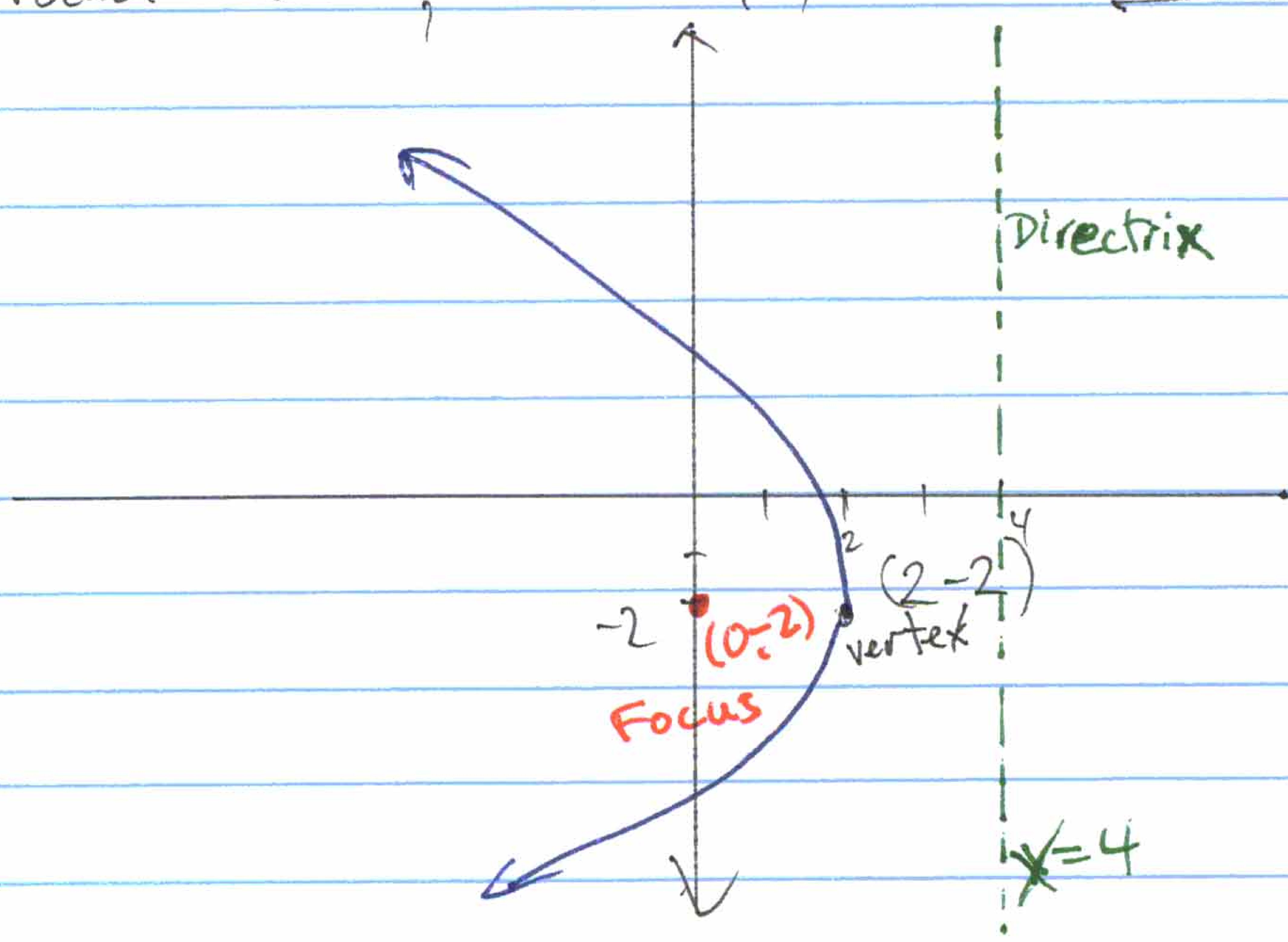
Focus: $(0,-2)$, Vertex: $(2,-2)$

Directrix:

$x = (2) - (-2)$

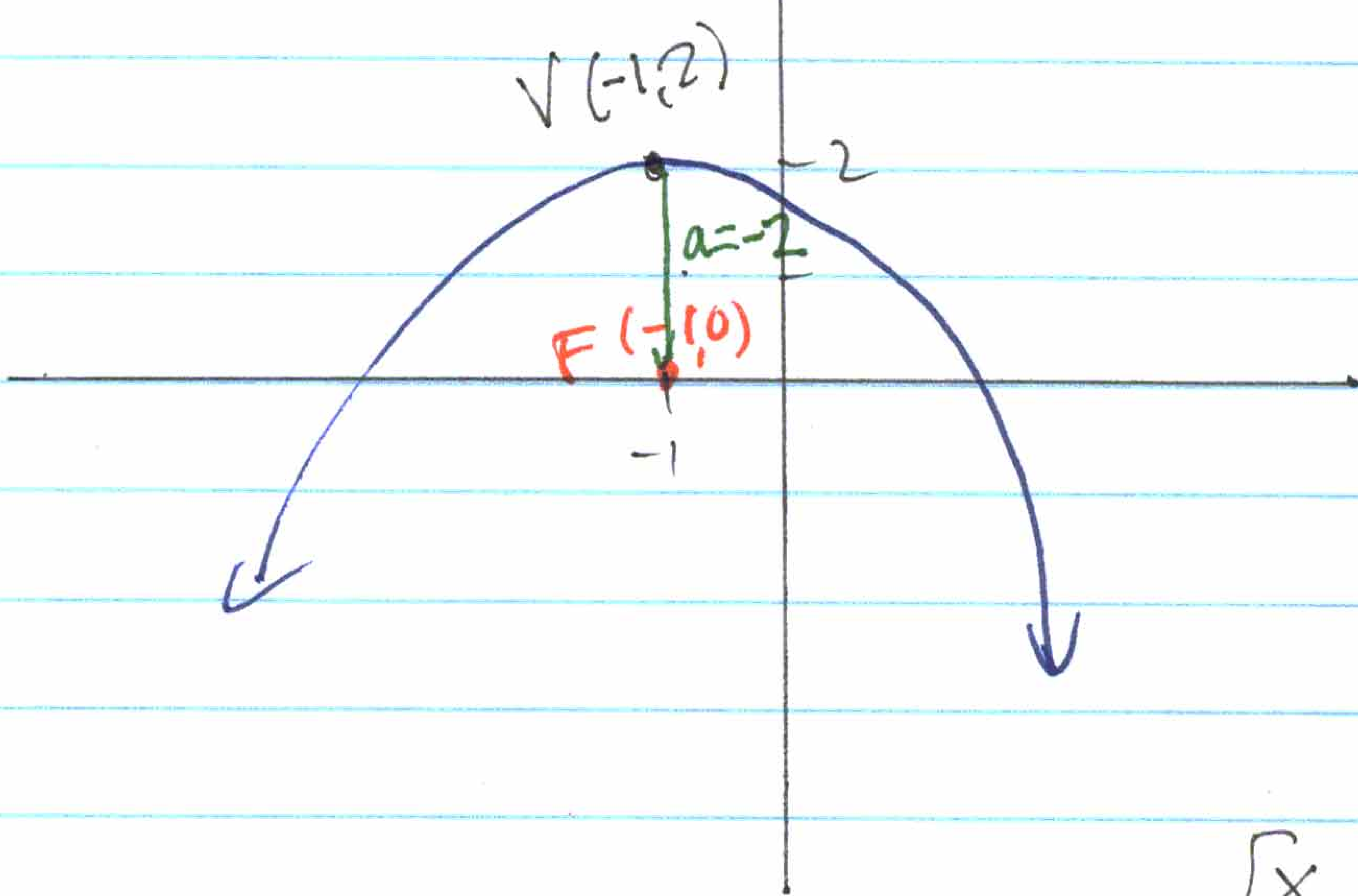
$x = 2 + 2$

$\underline{x = 4}$



$\boxed{10.1}$

#22 Find the eqvation of the parabola Vertex: $(-1,2)$ & Focus $(-1,0)$



Goal:

$(x-h)^2 = 4a(y-k)$

$a<0$ since the parabola opens down: $a=-2$

$h=-1, \quad k=2$

$[x-(-1)]^2 = 4(-2)[y-(2)]$

$$(x+1)^2 = -8(y-2)$$

#83: Find an equation of the tangent line to the parabola $y=ax^2$ at $x=x_0$. Prove that the x-intercept of this tangent line is $\left(\frac{x_0}{2}, 0\right)$.

$y' = 2ax$

at $x=x_0$, the slope of the tangent line is $m_{Tan} = 2a(x_0)$

$m_{Tan} = 2ax_0$

The equation of the tangent at $x=x_0$ is

$y - y_1 = m_{Tan}(x-x_1)$ with $(x_1, y_1) = (x_0, ax_0^2)$

$y - (ax_0^2) = 2ax_0[x-(x_0)]$

$y - ax_0^2 = 2ax_0(x-x_0)$

$ax_0^2 + y - ax_0^2 = 2ax_0x - 2ax_0^2 + ax_0^2$

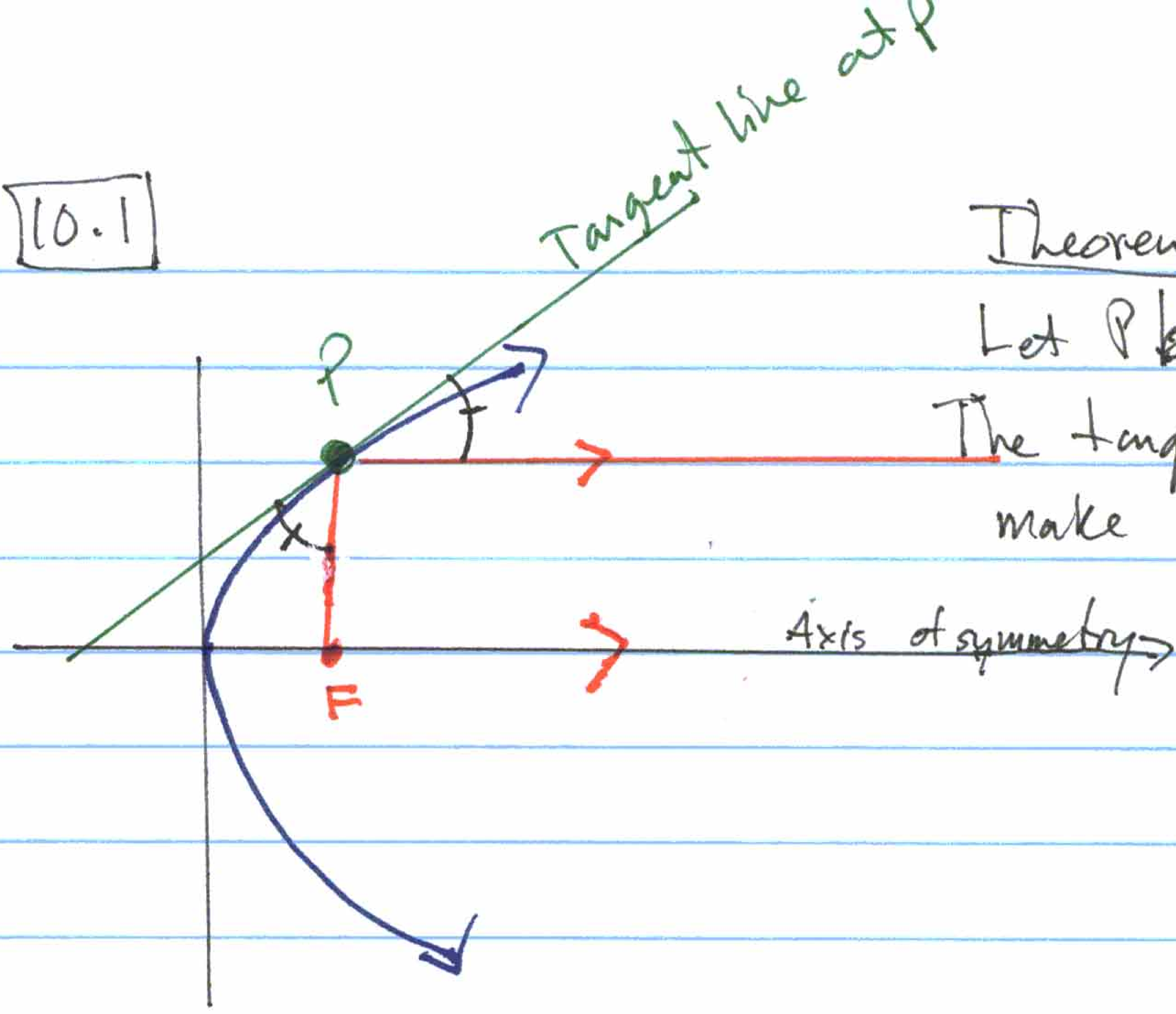
$y = 2ax_0x - ax_0^2$

x-intercept: $y=0$, $\quad 0 = 2ax_0x - ax_0^2$

$ax_0^2 = 2ax_0x$

$$\boxed{\frac{x_0}{2} = x}$$

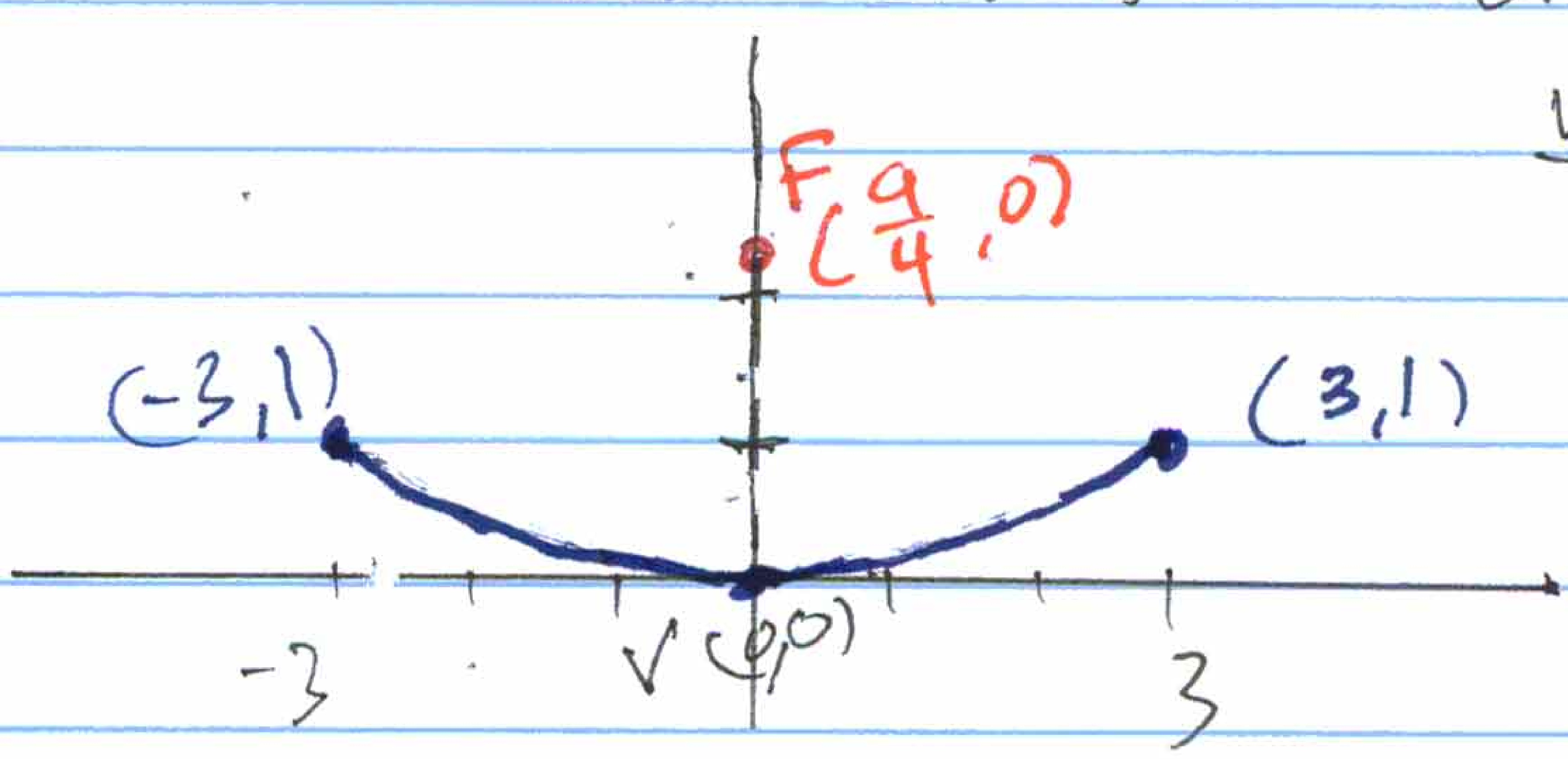
10.1



Theorem 10.2: Reflective Property of a Parabola
Let P be a point on a parabola. The tangent line to the parabola at P make equal angles with the lines PF & the line through P that is parallel to the axis of symmetry.

#81 Solar Collector: A solar collector for heating water is constructed with a sheet of stainless steel that is formed into the shape of a parabola. The water will flow through a pipe that is located at the focus of the parabola. At what distance from the vertex is the pipe?

→ The Pipe is in the focus position because of the Reflective Property of a Parabola. The Sun's rays are parallel to the axis of symmetry and at every point they are reflected and "collected" at the focus.



use: $x^2 = 4ay$

$(3)^2 = 4a(1)$

$9 = 4a$

$\boxed{\frac{9}{4} = a}$

The distance is $\frac{9}{4}$ meters, or $2\frac{1}{4}$ meters.

complete the square for x & y

10.1

#34 $16x^2+25y^2-64x+150y+279=0$

Goal:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

center: (h,k)

$a^2-c^2=b^2$

Foci: $(h\pm c, k)$

Vertices: $(h\pm a, k)$, $(h, k\pm b)$

$$(16x^2-64x)+(25y^2+150y)=-279$$

$$16(x^2-4x)+25(y^2+6y)=-279$$

$$16(x^2-4x+4)+25(y^2+6y+9)=-279+64+225$$

$\left[\frac{1}{2}(-4)\right]^2=(-2)^2=4$

$\left[\frac{1}{2}(6)\right]^2=(3)^2=9$

$16\cdot4=64$

$25\cdot9=225$

$$16(x-2)^2+25(y+3)^2=10$$

$$\frac{16}{10}(x-2)^2+\frac{25}{10}(y+3)^2=\frac{10}{10}$$

$$\frac{(x-2)^2}{\frac{10}{16}}+\frac{(y+3)^2}{\frac{10}{25}}=1$$

$$\boxed{\frac{(x-2)^2}{\frac{5}{8}}+\frac{(y+3)^2}{\frac{2}{5}}=1}$$

$h=2$, $k=-3$

center $=(2,-3)$

$a^2=\frac{5}{8}$, $a=\frac{\sqrt{5}}{\sqrt{8}}$ $a=\frac{\sqrt{5}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{a=\frac{\sqrt{10}}{4}}$

$b^2=\frac{2}{5}$, $b=\frac{\sqrt{2}}{\sqrt{5}}$, $b=\frac{\sqrt{2}\cdot\sqrt{5}}{\sqrt{5}\sqrt{5}}$

$\boxed{b=\frac{\sqrt{10}}{5}}$

$c^2=a^2-b^2$

$c^2=\frac{5}{8}-\frac{2}{5}$

$c^2=\frac{25}{40}-\frac{16}{40}=\frac{9}{40}$

$c=\frac{\sqrt{9}}{\sqrt{40}}=\frac{3}{2\sqrt{10}}=\frac{3\cdot\sqrt{10}}{2\sqrt{10}\sqrt{10}}$

$\boxed{c=\frac{3\sqrt{10}}{20}}$

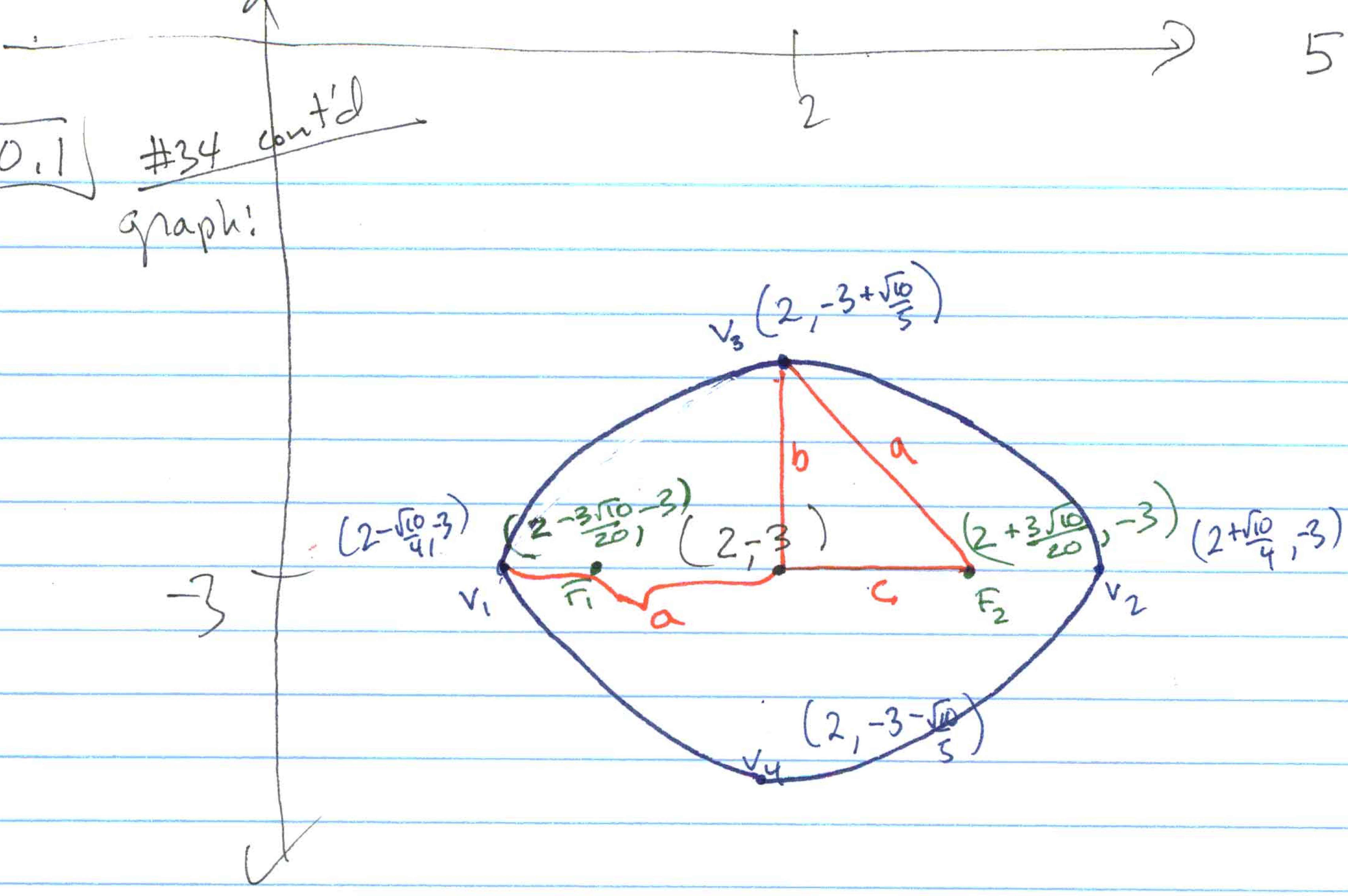
Foci: $\left(2\pm\frac{3\sqrt{10}}{20}, -3\right)$

Vertices: $\left(2\pm\frac{\sqrt{10}}{4}, -3\right)$ &

$\left(2, -3\pm\frac{\sqrt{10}}{5}\right)$

$\boxed{10.1}$ #34 cont'd

graph:



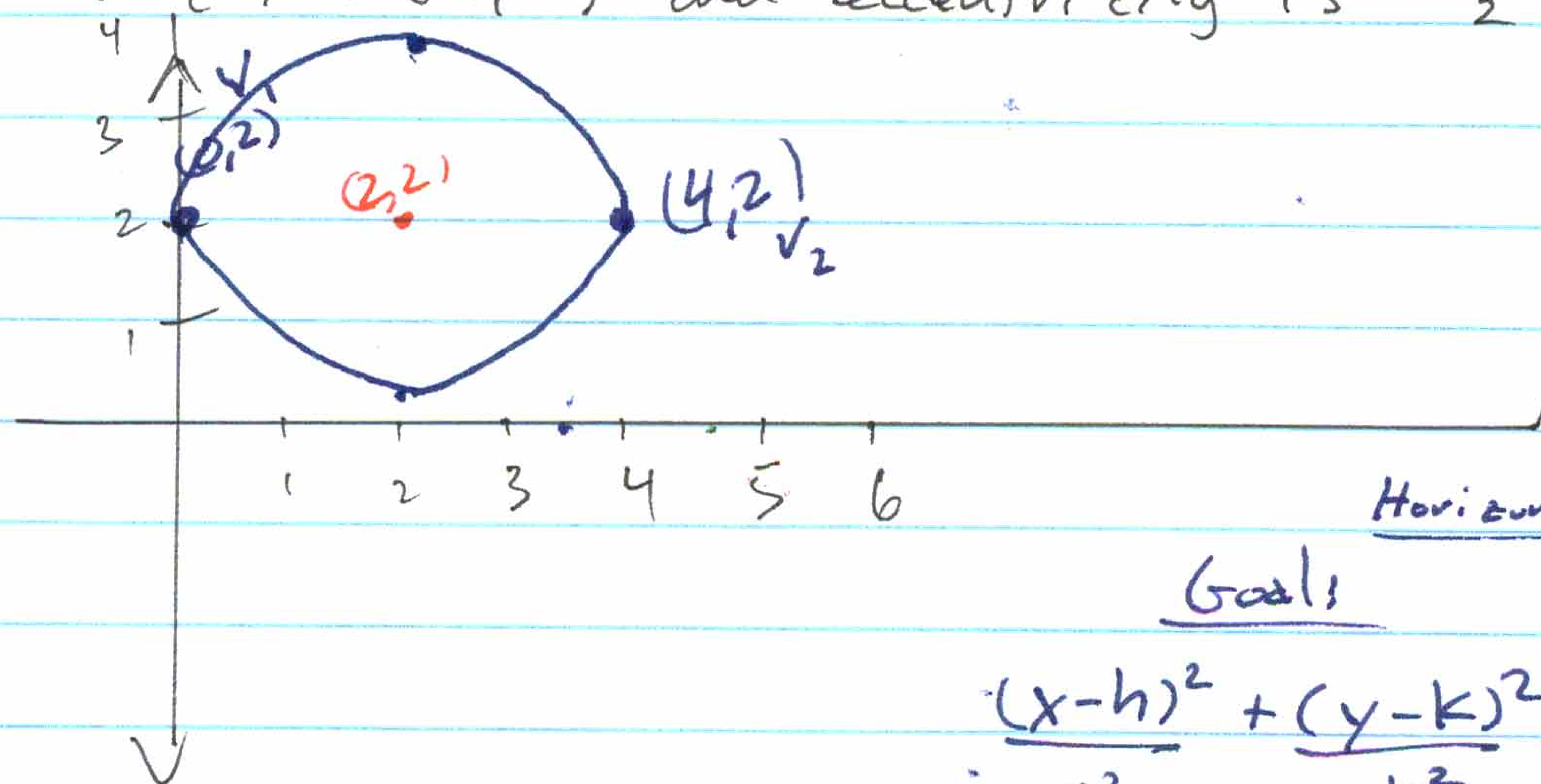
Theorem 10.4 · Reflective Property of an Ellipse
Let P be a point on an ellipse. The tangent line to the ellipse at point P makes equal angles with the lines through P and the foci.

Definition of Eccentricity of an Ellipse: $e = \frac{c}{a}$.

The eccentricity of an ellipse measures the "ovalness" of an ellipse.
we know $0 < c < a$, & for every ellipse $0 < e < 1$.
For an ellipse that is nearly circular, the foci are close to the center and the ratio $\frac{c}{a}$ is small. For an elongated ellipse, the foci are close to the vertices and the ratio is close to 1.

10.1

#40 Find the equation of the ellipse if vertices are $(0,2)$ & $(4,2)$ and eccentricity is $\frac{1}{2}$.



Horizontal Major Axis

Goal:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Center: $(2,2)$

$h=2$, $k=2$

$a=2$

$e=\frac{1}{2}$

$e=\frac{c}{a}$

$\frac{1}{2}=\frac{c}{2}$

$c=1$

$c^2=a^2-b^2$

$(1)^2=(2)^2-b^2$

$1=4-b^2$

$-3=-b^2$

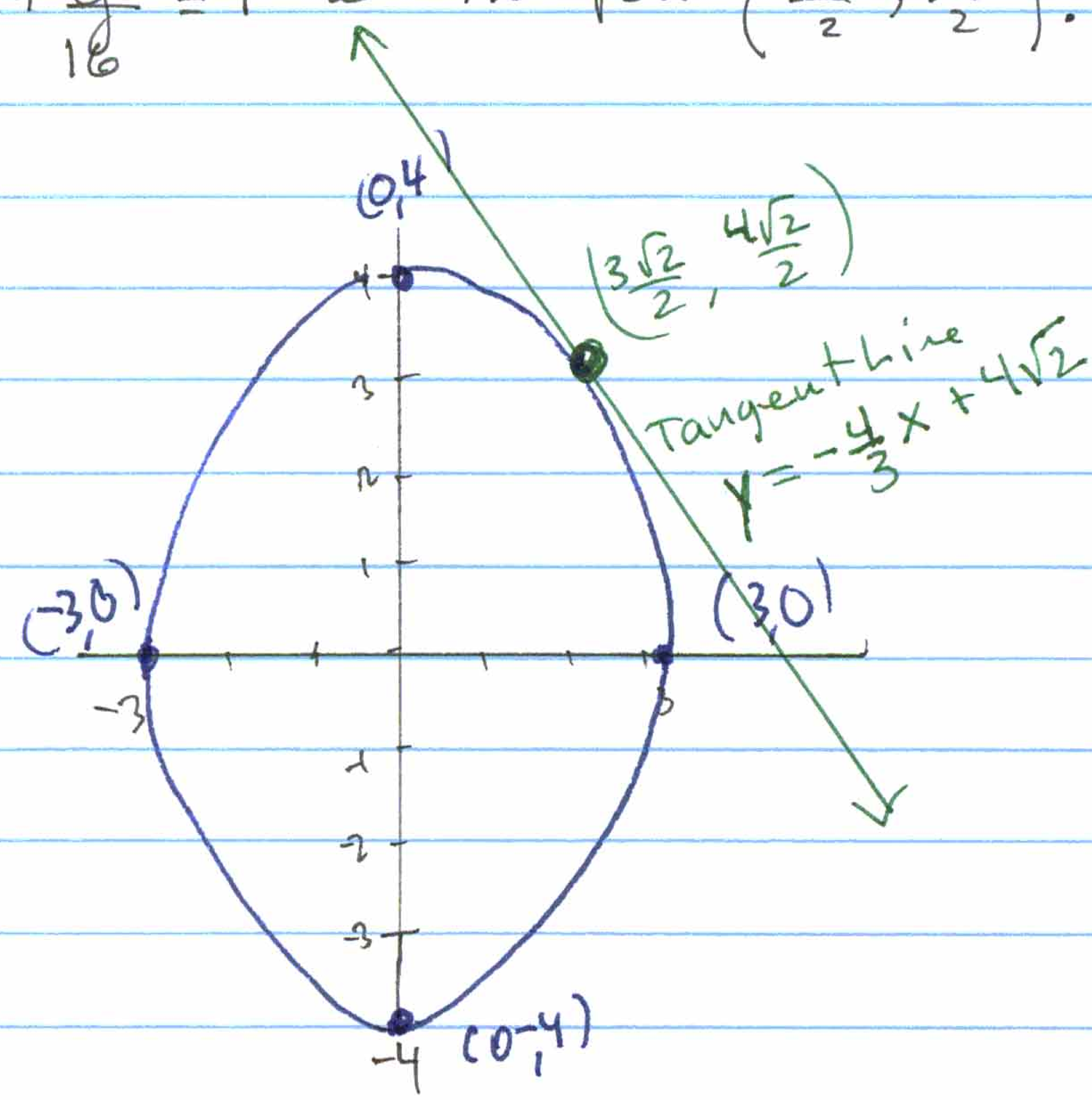
$3=b^2$

$b=\sqrt{3}$

$$\boxed{\frac{(x-2)^2}{4}+\frac{(y-2)^2}{3}=1}$$

10.1

Example: Find the equation of the tangent line of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ at the point $\left(\frac{3\sqrt{2}}{2}, \frac{4\sqrt{2}}{2}\right)$.



$$\frac{d}{dx}\left[\frac{x^2}{9}+\frac{y^2}{16}\right]=\frac{d}{dx}[1]$$

$$\frac{d}{dx}\left[\frac{x^2}{9}\right]+\frac{d}{dx}\left[\frac{y^2}{16}\right]=0$$

$$\frac{1}{9}\cdot 2x+\frac{1}{16}\cdot 2y\frac{dy}{dx}=0$$

$$\frac{2}{9}x+\frac{1}{8}y\frac{dy}{dx}=0$$

$$\frac{1}{8}y\frac{dy}{dx}=\frac{-2x}{9}$$

$$\frac{8}{y}\cdot\frac{1}{8}y\frac{dy}{dx}=\frac{-2x}{9}\cdot\frac{8}{y}$$

$$\boxed{\frac{dy}{dx}=\frac{-16x}{9y}}$$

m_{Tan} at $x=\frac{3\sqrt{2}}{2}$ & $y=\frac{4\sqrt{2}}{2}$

$$m_{Tan}=\frac{-16\left(\frac{3\sqrt{2}}{\cancel{2}}\right)}{9\left(\frac{4\sqrt{2}}{\cancel{2}}\right)}$$

$$m_{TAN}=\frac{-4}{3}$$

$y-y_1=m_{Tan}(x-x_1)$ with $(x_1,y_1)=\left(\frac{3\sqrt{2}}{2}, \frac{4\sqrt{2}}{2}\right)$

$$y-\left(\frac{4\sqrt{2}}{2}\right)=\frac{-4}{3}\left[x-\left(\frac{3\sqrt{2}}{2}\right)\right]$$

$$y-\frac{4\sqrt{2}}{2}=\frac{-4}{3}x+2\sqrt{2}$$

$$2\sqrt{2}+y-2\sqrt{2}=-\frac{4}{3}x+2\sqrt{2}+2\sqrt{2}$$

$$\boxed{y=-\frac{4}{3}x+4\sqrt{2}}$$

Goal: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, $a^2+b^2=c^2$

Center: (h,k)

Foci: $(h, k\pm c)$

Vertices: $(h, k\pm a)$

Asymptotes: $y-k = \pm\frac{a}{b}(x-h)$

$\boxed{10.1}$

#56 Find the center, foci, and vertices of the hyperbola. Graph the hyperbola and its asymptotes.

$\leftarrow$ complete the square for x & y

$3y^2 - x^2 + 6x - 12y = 0$

$3y^2 - 12y \quad - x^2 + 6x = 0$

$3(y^2-4y) - 1\cdot(x^2-6x) = 0$

$3(y^2-4y+4) - 1(x^2-6x+9) = 0 + 12 - 9$

$[\frac{1}{2}(-4)]^2 = (-2)^2 = 4$ | $[\frac{1}{2}(-6)]^2 = (-3)^2 = 9$

$3\cdot 4 = 12$, $-1\cdot 9 = -9$

$3(y-2)^2 - 1(x-3)^2 = 3$

$\frac{3}{3}(y-2)^2 - \frac{1}{3}(x-3)^2 = \frac{3}{3}$

$\boxed{\frac{(y-2)^2}{1} - \frac{(x-3)^2}{3} = 1}$ $\longrightarrow$

$h=3, k=2 \quad a^2=1, b^2=3$

Center: $(3,2)$

$a=1$, $b=\sqrt{3}$

$c^2 = a^2+b^2$

$c^2 = 1+3$

$c^2 = 4$

$c = 2$

Foci: $(3,4)$ & $(3,0)$

Vertices: $(3,3)$ & $(3,1)$

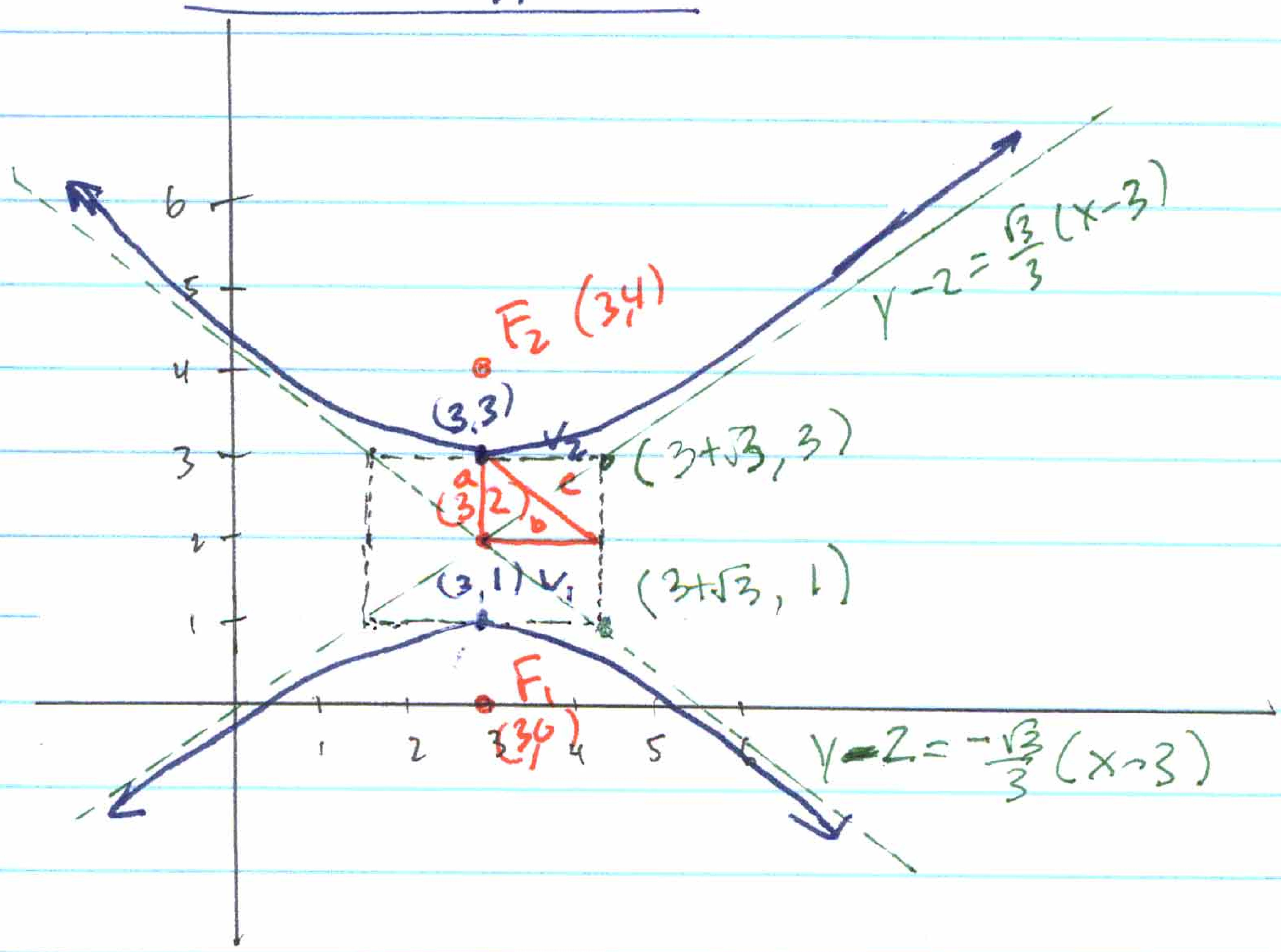
Asymptotes:

$y-2 = \pm\frac{1}{\sqrt{3}}(x-3)$

$y-2 = \pm\frac{\sqrt{3}}{3}(x-3)$

$\uparrow$

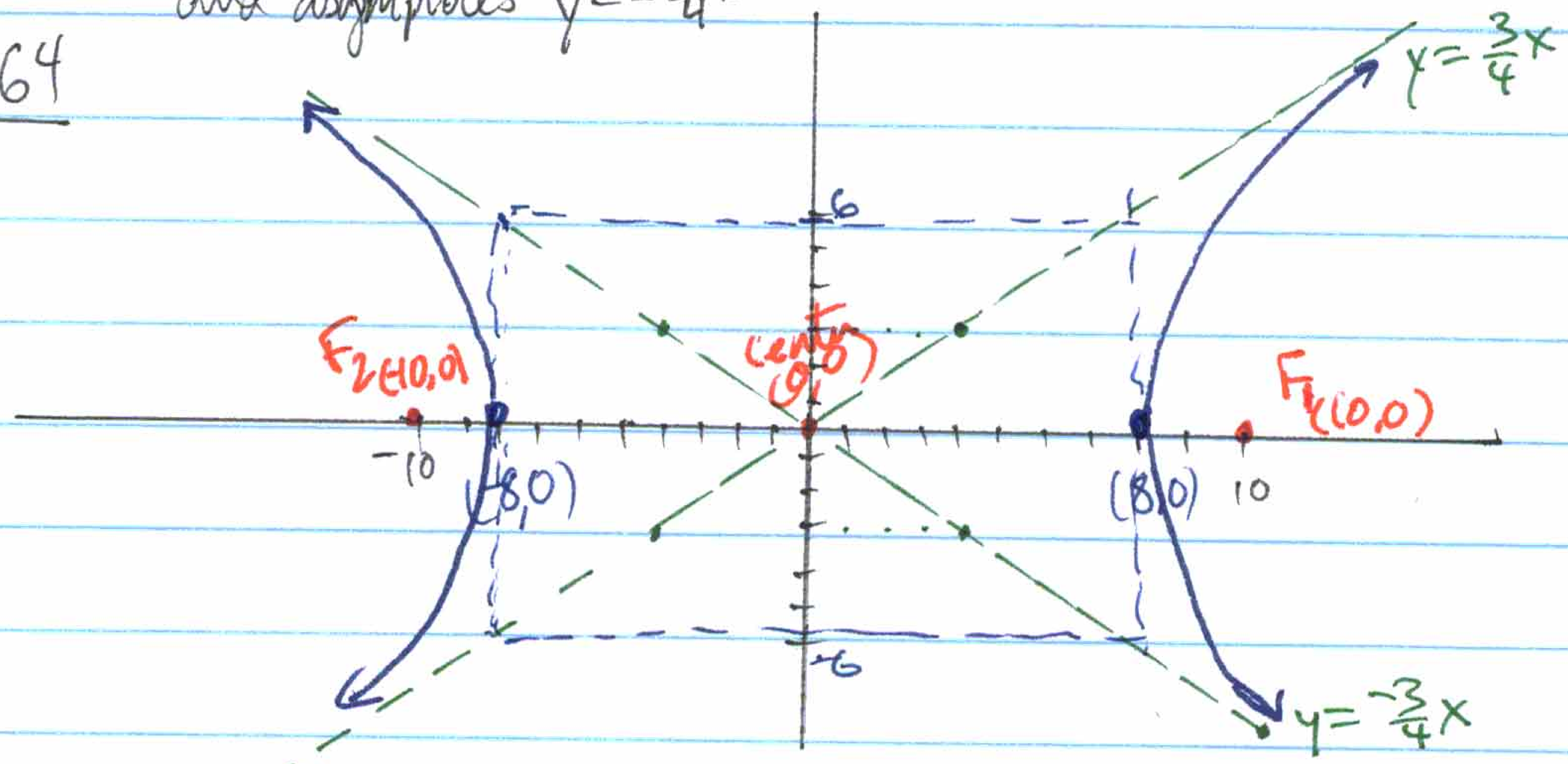
Vertical Hyperbola:



10.1

#64

Find the equation of the hyperbola with Focus at (10,0) and asymptotes $y=\pm\frac{3}{4}x$.



Goal: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \rightarrow \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Asymptotes: $y-k=\pm\frac{b}{a}(x-h) \longrightarrow y=\pm\frac{b}{a}x \longrightarrow \pm\frac{3}{4}x=\pm\frac{b}{a}x$

Vertices: $(h\pm a,k)$

$a^2+b^2=c^2 \longrightarrow a^2+b^2=10^2$

$a^2+b^2=100$

$\frac{3}{4}=\frac{b}{a}$

$a\cdot\frac{3}{4}=b$

$\boxed{\frac{3a}{4}=b}$

$a^2+\left(\frac{3a}{4}\right)^2=100$

$a^2+\frac{9a^2}{16}=100$

$\frac{16a^2}{16}+\frac{9a^2}{16}=100$

$\frac{16}{25}\cdot\frac{25a^2}{16}=100\cdot\frac{16}{25}$

$a^2=64$

$\boxed{a=8} \longrightarrow b=\frac{3(8)}{4}$

$\boxed{b=6}$

$$\boxed{\frac{x^2}{64}-\frac{y^2}{36}=1}$$

$\boxed{10.1}$ Find the equations for the tangent line at $x=4$

#66 $\quad \dfrac{y^2}{4} - \dfrac{x^2}{2} = 1$

Center $= (h,k) = (0,0)$

$a^2 = 4$, $\quad b^2 = 2$

$\boxed{a=2}$ $\quad \boxed{b=\sqrt{2}}$ $\quad$ Foci: $(0, \pm\sqrt{6})$

$a^2 + b^2 = c^2$ $\quad$ Vertices: $(0, \pm 2)$

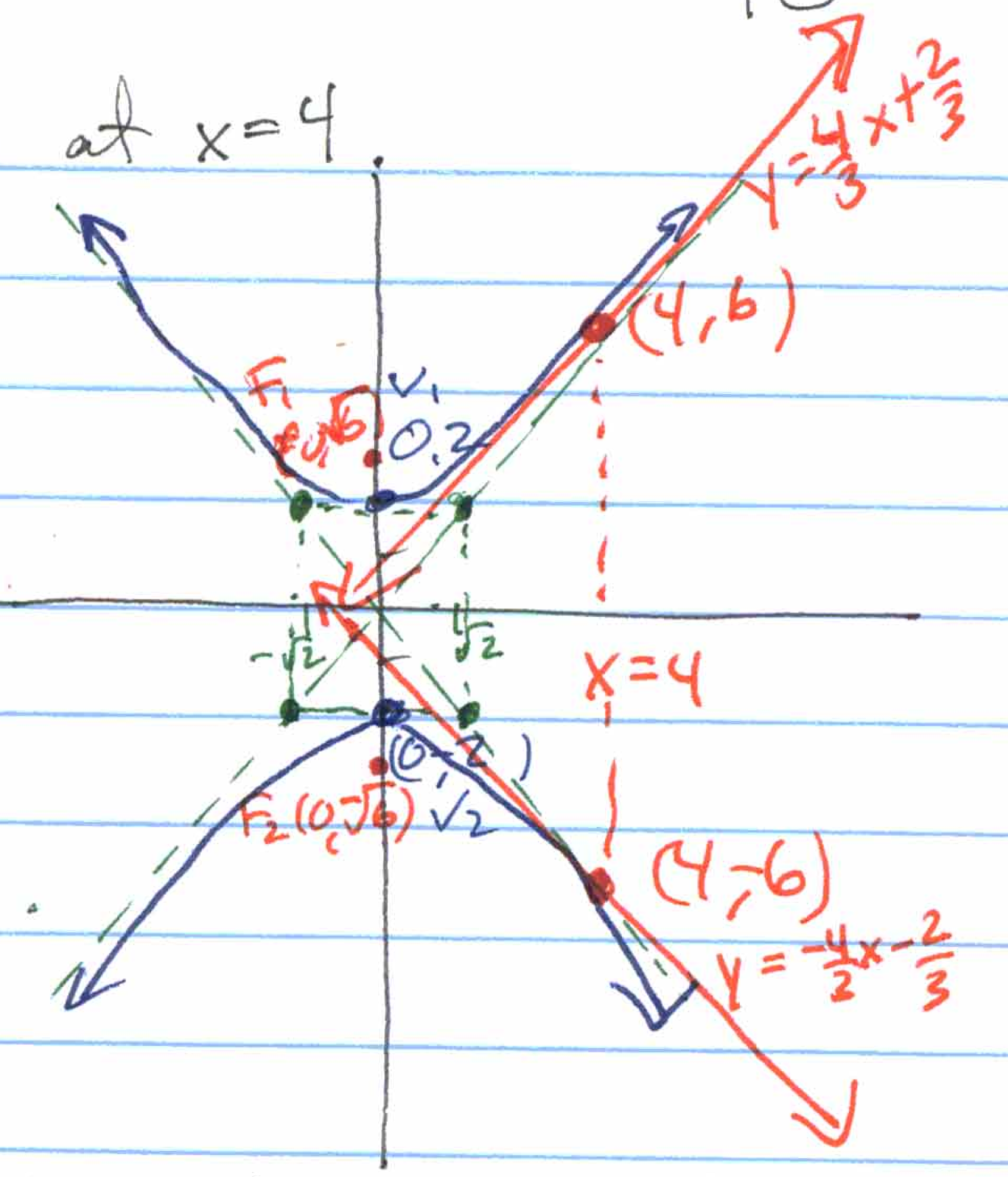
$4 + 2 = c^2$

$6 = c$

$\boxed{\sqrt{6} = c}$

Asymptotes: $\quad y = \pm \dfrac{2}{\sqrt{2}} x$

$y = \pm \dfrac{\sqrt{2}}{1} x$



at $x=4$, Find y.

$\dfrac{y^2}{4} - \dfrac{(4)^2}{2} = 1$

$\dfrac{y^2}{4} - \dfrac{16}{2} = 1$

$8 + \dfrac{y^2}{4} - 8 = 1 + 8$

$4 \cdot \dfrac{y^2}{4} = 9 \cdot 4$

$y^2 = 36$

$\boxed{y = \pm 6}$

$\dfrac{d}{dx}\left[\dfrac{y^2}{4} - \dfrac{x^2}{2}\right] = \dfrac{d}{dx}[1]$

$\dfrac{1}{4}\dfrac{d}{dx}[y^2] - \dfrac{1}{2}\dfrac{d}{dx}[x^2] = 0$

$\dfrac{1}{4}(2y)\dfrac{dy}{dx} - \dfrac{1}{2}(2x) = 0$

$\dfrac{y}{2}\dfrac{dy}{dx} - x = 0$

$\dfrac{2}{y} \cdot \dfrac{y}{2}\dfrac{dy}{dx} = \dfrac{x}{1} \cdot \dfrac{2}{y}$

$\dfrac{dy}{dx} = \dfrac{2x}{y}$

$M_{TAN_1}\Big|_{x=4} = \dfrac{2(4)}{(6)} = \dfrac{4}{3}$ $\quad$ & $\quad$ $M_{TAN_2}\Big|_{x=4} = \dfrac{2(4)}{(-6)} = \dfrac{-4}{3}$

$y - y_1 = m_{tan}(x - x_1)$

$y - (6) = \frac{4}{3}[x - (4)]$

$y - 6 = \frac{4}{3}x - \frac{16}{3}$

$y = \frac{4}{3}x - \frac{16}{3} + \frac{18}{3}$

$\boxed{y = \frac{4}{3}x + \frac{2}{3}}$

$y + 6 = -\frac{4}{3}(x - 4)$

$y + 6 = -\frac{4}{3}x + \frac{16}{3}$

$y = -\frac{4}{3}x + \frac{16}{3} - \frac{18}{3}$

$\boxed{y = -\frac{4}{3}x - \frac{2}{3}}$