$\boxed{4.5}$ Integration by Substitution

#16 $\int t^3 \sqrt{t^4+5}\, dt$

Let $u = t^4+5$

$\frac{du}{dt} = 4t^3$

$\frac{du}{4t^3} = dt$

$= \int t^3 \cdot u^{1/2} \cdot \left(\frac{du}{4t^3}\right)$

$= \frac{1}{4} \int u^{1/2}\, du$

$= \frac{1}{4}\left[\frac{2}{3} \cdot u^{3/2}\right] + C$

$= \frac{1}{6} u^{3/2} + C$

$\boxed{= \frac{1}{6}(t^4+5)^{3/2} + C}$

check:

$\frac{d}{dt}\left[\frac{1}{6}(t^4+5)^{3/2}+C\right] = \frac{1}{6} \cdot \left[\frac{3}{2}(t^4+5)^{1/2}\right] \cdot \frac{d}{dt}(t^4+5)+0$

$= \frac{1}{4} \cdot (t^4+5)^{1/2} \cdot (4t^3)$

$= t^3(t^4+5)^{1/2}$, ✓

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#36 Solve: $\frac{dy}{dx} = \frac{10x^2}{\sqrt{1+x^3}}$

$$\int \left(\frac{dy}{dx}\right) dx = \int \left(\frac{10x^2}{\sqrt{1+x^3}}\right) dx$$

Let $u = 1+x^3$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3x^2} = dx$

$$\int dy = \int 10x^2 (1+x^3)^{-1/2} dx,$$

$$y = \int 10x^2 (u)^{-1/2} \cdot \left(\frac{du}{3x^2}\right)$$

$$y = \frac{10}{3} \int u^{-1/2} du$$

$$y = \frac{10}{3}\left[\frac{2}{1} \cdot u^{1/2}\right] + C$$

$$y = \frac{20}{3} u^{1/2} + C$$

$$\boxed{y = \frac{20}{3}(1+x^3)^{1/2} + C}$$

check: $\frac{d}{dx}(y) = \frac{d}{dx}\left[\frac{20}{3} \cdot (1+x^3)^{1/2} + c\right]$

$$\frac{dy}{dx} = \frac{20}{3}\left[\frac{1}{2} \cdot (1+x^3)^{-1/2} \cdot \frac{d}{dx}(1+x^3)\right] + 0$$

$$\frac{dy}{dx} = \frac{10}{3}(1+x^3)^{-1/2} \cdot (3x^2)$$

$$\frac{dy}{dx} = \frac{10x^2}{\sqrt{1+x^3}}, \checkmark$$

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#52. $\int \sqrt{\tan x}\ \sec^2 x\, dx$,

Let $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$\frac{du}{\sec^2 x} = dx$

$= \int u^{1/2} \cdot \sec^2 x \cdot \left(\frac{du}{\sec^2 x}\right)$

$= \int u^{1/2}\, du$

$= \frac{2}{3} \cdot u^{3/2} + c$

$$\boxed{= \frac{2}{3}(\tan x)^{3/2} + c}$$

check: $\frac{d}{dx}\left[\frac{2}{3}(\tan x)^{3/2} + c\right]$

$= \frac{2}{3}\left[\cdot \frac{3}{2} \cdot (\tan x)^{1/2} \cdot \frac{d}{dx}(\tan x)\right] + 0$

$= (\tan x)^{1/2} \cdot \sec^2 x$

$= \sqrt{\tan x}\ \sec^2 x$, ✓

"change of variables"

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#64 $\int x\sqrt{2x+1}\,dx$, | let $u=2x+1$, $\frac{du}{dx}=2$, $\frac{du}{2}=dx$ | $u-1=2x$, $\frac{u-1}{2}=x$ ★

$$= \int \left(\frac{u-1}{2}\right)(u)^{1/2}\left(\frac{du}{2}\right)$$

$$= \frac{1}{4}\int (u-1)\,u^{1/2}\,du$$

$$= \frac{1}{4}\int \left(u^{3/2}-u^{1/2}\right)du$$

$$= \frac{1}{4}\left\{\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + c$$

$$= \frac{1}{10}u^{5/2} - \frac{1}{6}u^{3/2} + c$$

$$\boxed{= \frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C}$$, or

$$= \frac{6}{60}u^{5/2} - \frac{10}{60}u^{3/2} + c$$

$$= \frac{2}{60}u^{3/2}\left[3u^{2/2} - 5\right] + C$$

$$= \frac{1}{30}u^{3/2}\left[3u-5\right] + C$$

$$= \frac{1}{30}(2x+1)^{3/2}\left[3(2x+1)-5\right] + C$$

$$= \frac{1}{30}(2x+1)^{3/2}\left[6x-2\right] + C$$

$$\boxed{= \frac{1}{15}(2x+1)^{3/2}(3x-1) + C}$$

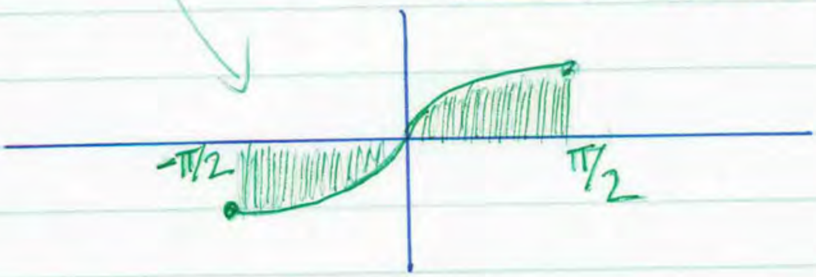
Even & ODD Functions

$$\int_{-\pi/2}^{\pi/2} \frac{\sin(x)}{\sqrt{1+\cos(x)}}\,dx \quad , \quad \begin{array}{|l} \text{let } u = 1+\cos(x) \\ \frac{du}{dx} = -\sin(x) \\ \frac{du}{-\sin(x)} = dx \end{array}$$

not needed

$= 0$, since $y = \frac{\sin(x)}{\sqrt{1+\cos(x)}}$ is an <u>odd</u> function

Not Area



<u>#103</u>** $\displaystyle\int_{-\pi/4}^{\pi/4} \sin^2(x)\cos(x)\,dx$, $\begin{array}{|l} \text{let } u = \sin(x) \\ \frac{du}{dx} = \cos(x) \\ \frac{du}{\cos x} = dx \end{array}$ $\begin{array}{|l} x=0 \\ u=\sin(0) \\ u=0 \\ \hline x=\pi/4 \\ u=\sin(\pi/4) \\ u=\frac{\sqrt{2}}{2} \end{array}$

$= 2\cdot\displaystyle\int_0^{\pi/4} \sin^2(x)\cos(x)\,dx$, since $y=\sin^2(x)\cos(x)$ is an <u>even</u> function!

$$= 2\int_{u=0}^{u=\frac{\sqrt{2}}{2}} (u)^2\cdot\cos(x)\cdot\left(\frac{du}{\cos(x)}\right)$$

$$= 2\int_0^{\frac{\sqrt{2}}{2}} u^2\,du$$

$$= 2\left[\frac{1}{3}\cdot u^3\right]_0^{\sqrt{2}/2}$$

$$= \frac{2}{3}\left[u^3\right]_0^{\sqrt{2}/2}$$

$$= \frac{2}{3}\left[\left(\frac{\sqrt{2}}{2}\right)^3 - (0)^3\right]$$

$$= \frac{2}{3}\left[\frac{2\sqrt{2}}{8}\right]$$

$$\boxed{= \frac{\sqrt{2}}{6}}$$

1-26-09

Notes of Math 155

$\boxed{4.5}$

#47 $\int \frac{1}{\theta^2}\cos\left(\frac{1}{\theta}\right)d\theta$

Let $u = \frac{1}{\theta}$

$u = \theta^{-1}$

$\frac{du}{d\theta} = -1 \cdot \theta^{-2}$

$\frac{du}{-\theta^{-2}} = d\theta$

$-\theta^2 du = d\theta$

$$= \int \frac{1}{\theta^2}\cos(u)(-\theta^2 du)$$

$$= -\int \cos(u)\,du$$

$$= -\sin(u) + c$$

$$\boxed{= -\sin\left(\tfrac{1}{\theta}\right) + C}$$

#67 $\int \frac{x^2-1}{\sqrt{2x-1}}dx$

Let $u = 2x-1$,

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

solve for x:

$u+1 = 2x$

$\frac{u+1}{2} = x$ ★

$$= \int \frac{x^2-1}{\sqrt{u}}\left(\frac{du}{2}\right)$$

$$= \frac{1}{2}\int \left(\left[\frac{u+1}{2}\right]^2 - 1\right)u^{-1/2}du$$

$$= \frac{1}{2}\int \left(\frac{u^2}{4} + \frac{u}{2} + \frac{1}{4} - \frac{4}{4}\right)u^{-1/2}du$$

$$= \frac{1}{2}\int \left(\frac{u^2}{4} + \frac{u}{2} - \frac{3}{4}\right)u^{-1/2}du$$

$$= \frac{1}{2}\int \left(\frac{u^{3/2}}{4} + \frac{u^{1/2}}{2} - \frac{3}{4}u^{-1/2}\right)du$$

$$= \frac{1}{2}\left[\frac{1}{4}\cdot\frac{2}{5}u^{5/2} + \frac{1}{2}\cdot\frac{2}{3}u^{3/2} - \frac{3}{4}\cdot\frac{2}{1}u^{1/2}\right] + C$$

#67 cont.

$$= \frac{1}{2}\left[\frac{1}{10}u^{5/2} + \frac{1}{3}u^{3/2} - \frac{3}{2}u^{1/2}\right] + C$$

$$= \frac{1}{20}u^{5/2} + \frac{1}{6}u^{3/2} - \frac{3}{4}u^{1/2} + C$$

$$\boxed{= \frac{1}{20}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} - \frac{3}{4}(2x-1)^{1/2} + C}$$