$\boxed{5.4}$ Logarithmic, exponential, & other Transcendental Functions

#38 $y = x^2 e^{-x}$, find $\frac{dy}{dx}$

$$\frac{d}{dx}[y] = \frac{d}{dx}[x^2 e^{-x}]$$

$$\frac{dy}{dx} = (x^2)\frac{d}{dx}(e^{-x}) + (e^{-x})\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2(e^{-x})\frac{d}{dx}(-x) + (e^{-x})(2x)$$

$$\frac{dy}{dx} = x^2(e^{-x})(-1) + (e^{-x})(2x)$$

$$\frac{dy}{dx} = xe^{-x}(-x + 2)$$

$$\boxed{\frac{dy}{dx} = xe^{-x}(2-x)}$$

#58 $e^{xy} + x^2 - y^2 = 10$, find $\frac{dy}{dx}$

$$\frac{d}{dx}\left(e^{xy} + x^2 - y^2\right) = \frac{d}{dx}(10)$$

$$\frac{d}{dx}(e^{xy}) + \frac{d}{dx}(x^2) - \frac{d}{dx}(y^2) = 0$$

$$e^{xy} \cdot \frac{d}{dx}(xy) + 2x - 2y \cdot \frac{dy}{dx} = 0$$

$$e^{xy} \cdot \left[(x)\frac{d}{dx}(y) + (y)\frac{d}{dx}(x)\right] + 2x - 2y\frac{dy}{dx} = 0$$

$$e^{xy}\left[x\frac{dy}{dx} + y\right] + 2x - 2y\frac{dy}{dx} = 0$$

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$$e^{xy}x\frac{dy}{dx}+e^{xy}y+2x-2y\frac{dy}{dx}=0$$

$$e^{xy}y+2x=2y\frac{dy}{dx}-e^{xy}x\frac{dy}{dx}$$

$$e^{xy}y+2x=(2y-e^{xy})\frac{dy}{dx}$$

$$\boxed{\frac{e^{xy}y+2x}{2y-e^{xy}}=\frac{dy}{dx}}$$

#88 $\int \frac{e^{1/x^2}}{x^3}dx$,

let $u=\frac{1}{x^2}$

$u=x^{-2}$

$\frac{du}{dx}=-2x^{-3}$

$\frac{du}{dx}=\frac{-2}{x^3}$

$-\frac{x^3}{2}du=dx$

$$=\int \frac{e^u}{x^3}\cdot\left(-\frac{x^3}{2}du\right)$$

$$=-\frac{1}{2}\int e^u du$$

$$=-\frac{1}{2}(e^u)+C$$

$$\boxed{=-\frac{1}{2}e^{1/x^2}+C}$$

check: $\frac{d}{dx}\left[-\frac{1}{2}e^{1/x^2}+c\right]$

$=-\frac{1}{2}\left[(e^{1/x^2})\frac{d}{dx}\left(\frac{1}{x^2}\right)\right]+0$

$=-\frac{1}{2}e^{1/x^2}\cdot\left(\frac{-2}{x^3}\right)$

$=\frac{e^{1/x^2}}{x^3}$ ✓

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#90 $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$

let $u = 1+e^{2x}$

$\frac{du}{dx} = e^{2x} \cdot 2$

$\frac{du}{2e^{2x}} = dx$

$$= \int \frac{e^{2x}}{(u)} \cdot \left(\frac{du}{2e^{2x}}\right)$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2} \cdot \ln|u| + C$$

$$= \frac{1}{2}\ln|1+e^{2x}| + C$$

$$\boxed{= \frac{1}{2}\ln(1+e^{2x}) + C}$$

check; $\frac{d}{dx}\left[\frac{1}{2}\ln(1+e^{2x}) + C\right]$

$$= \frac{1}{2} \cdot \left(\frac{1}{1+e^{2x}}\right) \cdot \frac{d}{dx}(1+e^{2x}) + C$$

$$= \frac{1}{2}\left(\frac{1}{1+e^{2x}}\right)(e^{2x}) \cdot \frac{d}{dx}(2x)$$

$$= \frac{1}{2}\left(\frac{1}{1+e^{2x}}\right)(e^{2x})(2)$$

$$= \frac{e^{2x}}{1+e^{2x}} \quad \checkmark$$