$\boxed{5.6}$ Inverse Trigonometric Functions : Differentiation

#42 $f(t) = \arcsin(t^2)$, find $f'(t)$

$$f'(t) = \frac{d}{dt}\left[\arcsin(t^2)\right]$$

$$= \frac{1}{\sqrt{1-(t^2)^2}} \cdot \frac{d}{dt}\left[t^2\right]$$

$$= \frac{1}{\sqrt{1-t^4}} \cdot (2t)$$

$$\boxed{f'(t) = \frac{2t}{\sqrt{1-t^4}}}$$

OR

$$f'(t) = \frac{d}{dt}\left[\arcsin(t^2)\right]$$

let $u = t^2$

$\frac{du}{dt} = 2t$

$$= \frac{d}{dt}\left[\arcsin(u)\right]$$

$$= \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dt}$$

$$= \frac{1}{\sqrt{1-(t^2)^2}} \cdot (2t)$$

$$\boxed{f'(t) = \frac{2t}{\sqrt{1-t^4}}}$$

$\boxed{5.6}$

 # 48 $h(x) = x^2 \arctan x$, find $h'(x)$

$$\frac{d}{dx}\left[h(x)\right] = \frac{d}{dx}\left[x^2 \arctan x\right]$$

$$h'(x) = (\arctan x)\frac{d}{dx}(x^2) + (x^2)\frac{d}{dx}\left[\arctan x\right]$$

$$h'(x) = (\arctan x)(2x) + (x^2)\left(\frac{1}{1+x^2}\right)$$

$$\boxed{h'(x) = 2x \arctan x + \frac{x^2}{1+x^2}}$$

or

$$\boxed{h'(x) = 2x \tan^{-1}(x) + \frac{x^2}{1+x^2}}$$

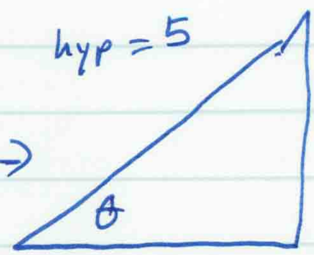
 # 17 (a) Evaluate: $\sin(\arctan \frac{3}{4})$

Let $\theta = \arctan \frac{3}{4}$

$\tan(\theta) = \tan(\arctan 3/4)$

$\tan(\theta) = \frac{3}{4}$

hyp = 5



opp = 3

adj = 4

$\tan(\theta) = \frac{opp}{adj}$

$(4)^2 + (3)^2 = hyp^2$

$16 + 9 = hyp^2$

$25 = hyp^2$

$5 = hyp$

So, $\sin(\arctan 3/4) = \sin(\theta)$

$= \frac{opp}{hyp}$

$= \frac{3}{5}$

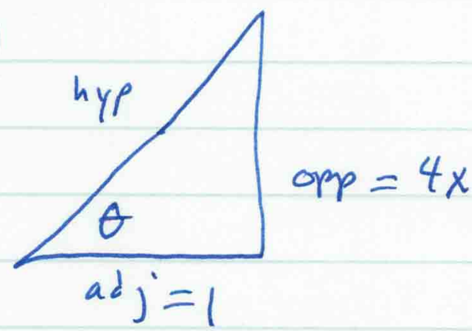
$\boxed{5.6}$

#22 write an algebraic form for
$\sec(\arctan 4x)$

Let $\theta = \arctan 4x$

$\tan(\theta) = \tan(\arctan 4x)$

$\tan(\theta) = 4x$



$\tan(\theta) = \dfrac{opp}{adj} = \dfrac{4x}{1}$

$(1)^2 + (4x)^2 = hyp^2$

$1 + 16x^2 = hyp^2$

$\sqrt{1+16x^2} = hyp$

So, $\sec(\arctan 4x)$

$= \sec(\theta)$

$= \dfrac{hyp}{adj}$

$= \dfrac{\sqrt{1+16x^2}}{1}$

$\boxed{= \sqrt{1+16x^2}}$