5.7 Inverse Trigonometric Functions: Integration

#10:

$$\int \frac{t}{t^4+16}\,dt$$

$$= \int \frac{t}{(t^2)^2+(4)^2}\,dt$$

$$= \int \frac{t}{(u)^2+(4)^2}\cdot\left(\frac{du}{2t}\right)$$

$$= \frac{1}{2}\int \frac{1}{u^2+(4)^2}\,du$$

$$= \frac{1}{2}\left[\frac{1}{a}\arctan\left(\frac{u}{a}\right)\right] + C$$

$$= \frac{1}{2}\left[\frac{1}{4}\arctan\left(\frac{u}{4}\right)\right] + C$$

$$\boxed{= \frac{1}{8}\arctan\left(\frac{t^2}{4}\right) + C}$$

, let $u = t^2$

$\frac{du}{dt} = 2t$

$\frac{du}{2t} = dt$

$a = 4$

check!

$$\frac{d}{dt}\left[\frac{1}{8}\arctan\left(\frac{t^2}{4}\right)+C\right]$$

$$= \frac{1}{8}\left[\frac{d}{dt}[\arctan(w)\right] + 0$$

$$= \frac{1}{8}\left[\frac{1}{1+w^2}\right]\cdot\frac{dw}{dt}$$

$$= \frac{1}{8}\left[\frac{1}{1+\left(\frac{t^2}{4}\right)^2}\right]\left(\frac{t}{2}\right)$$

$$= \frac{1}{16}\cdot\left[\frac{t}{1+\frac{t^4}{16}}\right]$$

$$= \frac{t}{16+t^4}$$ ✓

, let $w = \frac{t^2}{4}$

$\frac{dw}{dt} = \frac{1}{4}\cdot(2t)$

$\frac{dw}{dt} = \frac{t}{2}$

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#12 $\int \frac{1}{x\sqrt{x^4-4}}\,dx$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$= \int \frac{1}{x\sqrt{(x^2)^2-(2)^2}}\,dx$

$= \int \frac{1}{x\sqrt{(u)^2-(2)^2}} \cdot \left(\frac{du}{2x}\right)$

$= \frac{1}{2}\int \frac{1}{x^2\sqrt{u^2-(2)^2}}\,du$

$a = 2$

$= \frac{1}{2}\int \frac{1}{u\sqrt{u^2-(2)^2}}\,du$

$= \frac{1}{2}\left[\frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}\right] + C$

$= \frac{1}{2}\left[\frac{1}{2}\operatorname{arcsec}\frac{|u|}{2}\right] + C$

$= \frac{1}{4}\operatorname{arcsec}\frac{|x^2|}{2} + C$

$= \frac{1}{4}\operatorname{arcsec}\left(\frac{x^2}{2}\right) + C$ ← check ??

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36 $\int \frac{2}{\sqrt{-x^2+4x}}\,dx$

"complete the square"

$-x^2+4x = -(x^2-4x)$

$= -(x^2-4x+\boxed{4})+\boxed{4}$

$\left[\frac{1}{2}\cdot(-4)\right]^2 = (-2)^2 = 4$

$= -(x^2-4x+4)+4$

$= 4-(x^2-4x+4)$

$= 4-(x-2)^2$ ✓

$= \int \frac{2}{\sqrt{4-(x-2)^2}}\,dx$

Let $u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$= 2\int \frac{1}{(2)^2-(u)^2}\cdot du$

$a = 2$

$= 2\left[\arcsin\frac{u}{a}\right]+C$

$= 2\left[\arcsin\left(\frac{x-2}{2}\right)\right]+C$

$\boxed{= 2\arcsin\left(\frac{x-2}{2}\right)+C}$