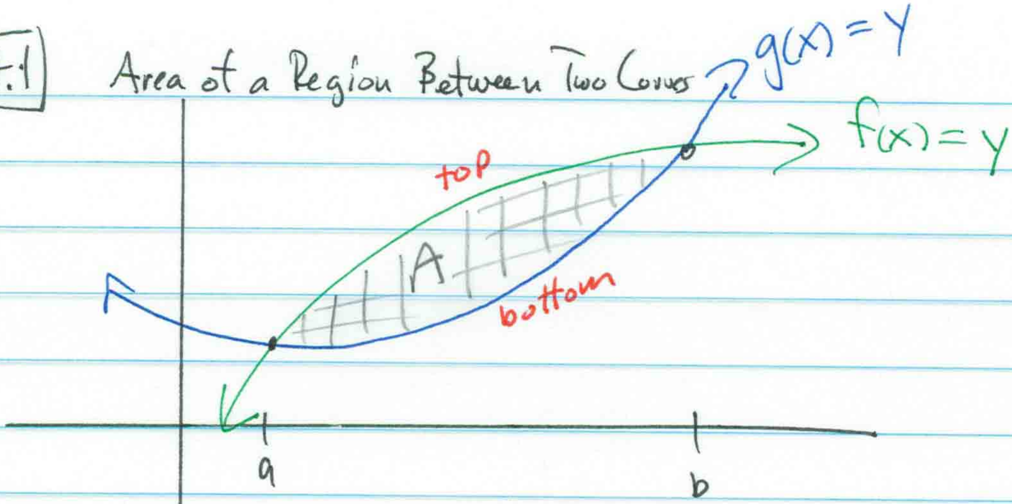
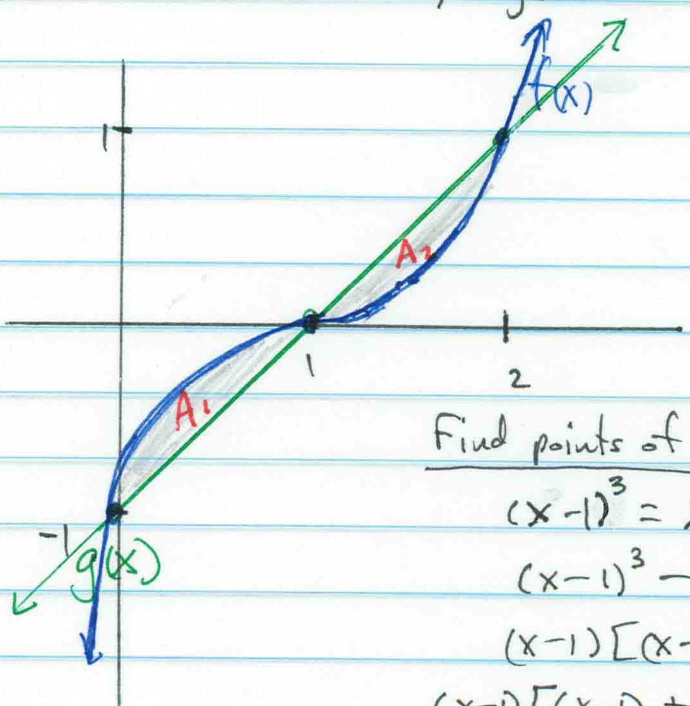
$\boxed{7.1}$ Area of a Region Between Two Curves



$$A = \int_a^b [f(x) - g(x)]\,dx$$

$$A = \int_a^b [\text{top} - \text{bottom}]\,dx$$

#6 $f(x) = (x-1)^3$, $g(x) = x - 1$



$A = A_1 + A_2$

$$A_1 = \int_0^1 [f(x) - g(x)]\,dx$$

$$A_2 = \int_1^2 [g(x) - f(x)]\,dx$$

Find points of intersection: $f(x) = g(x)$

$$(x-1)^3 = x - 1$$

$$(x-1)^3 - (x-1) = 0$$

$$(x-1)[(x-1)^2 - 1] = 0$$

$$(x-1)[(x-1)+1][(x-1)-1] = 0$$

$$(x-1)(\ x\ )(x-2) = 0$$

Either $x - 1 = 0$, $x = 0$, or $x - 2 = 0$

$\boxed{x = 1}$ $\boxed{x = 0}$ $\boxed{x = 2}$

$\boxed{7.1}$

#6 cont'd

$$A = A_1 + A_2$$

$$= \int_0^1 [f(x) - g(x)]\,dx + \int_1^2 [g(x) - f(x)]\,dx$$

$$= \int_0^1 [(x-1)^3 - (x-1)]\,dx + \int_1^2 [(x-1) - (x-1)^3]\,dx$$

★ $(x-1)^3 - (x-1)$

$= (x-1)[(x-1)^2 - 1]$

$= (x-1)[(x-1)+1][(x-1)-1]$

$= (x-1)(x)(x-2)$ ✓

$= (x^2 - x)(x-2)$

$= x^3 - 2x^2 - x^2 + 2x$

$= \underline{x^3 - 3x^2 + 2x}$

$(x-1) - (x-1)^3$

$= x - 1 - [x^3 - 3x^2 + 3x - 1]$

$= x - 1 - x^3 + 3x^2 - 3x + 1$

$= \underline{-x^3 + 3x^2 - 2x}$

★★ $(x-1)^3 = (x-1)(x-1)(x-1)$

$= (x^2 - 2x + 1)(x-1)$

$= x^3 - 2x^2 + x - x^2 + 2x - 1$

$= \underline{x^3 - 3x^2 + 3x - 1}$

$$= \int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (-x^3 + 3x^2 - 2x)\,dx$$

$$= \left[\frac{x^4}{4} - 3\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2}\right]_0^1 + \left[-\frac{x^4}{4} + 3\cdot\frac{x^3}{3} - 2\cdot\frac{x^2}{2}\right]_1^2$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[-\frac{x^4}{4} + x^3 - x^2\right]_1^2$$

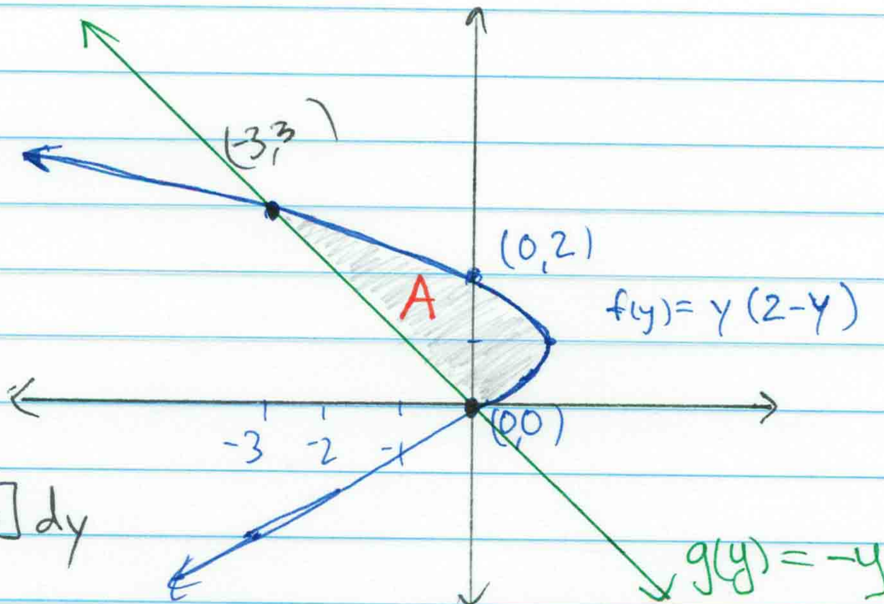
$$= \left(\frac{1}{4} - 1 + 1\right) - (0) + (-4 + 8 - 4) - \left(-\frac{1}{4} + 1 - 1\right)$$

$\boxed{7.1}$

$= \frac{1}{4} + \frac{1}{4}$

$A = \frac{1}{2}$

#28



$$A = \int_0^3 [f(y) - g(y)]\, dy$$

Find points of intersection:

$f(y) = g(y)$

$y(2-y) = -y$

$2y - y^2 = -y$

$0 = y^2 - 3y$

$0 = y(y-3)$

Either $y = 0$, or $y - 3 = 0$

$\boxed{y=0}$ $\boxed{y=3}$

$$A = \int_0^3 [y(2-y) - (-y)]\, dy$$

$$A = \int_0^3 [2y - y^2 + y]\, dy$$

$$A = \int_0^3 [3y - y^2]\, dy$$

$$A = \left[3 \cdot \frac{y^2}{2} - \frac{y^3}{3}\right]_0^3$$

$\boxed{7.1}$

#28 cont'd

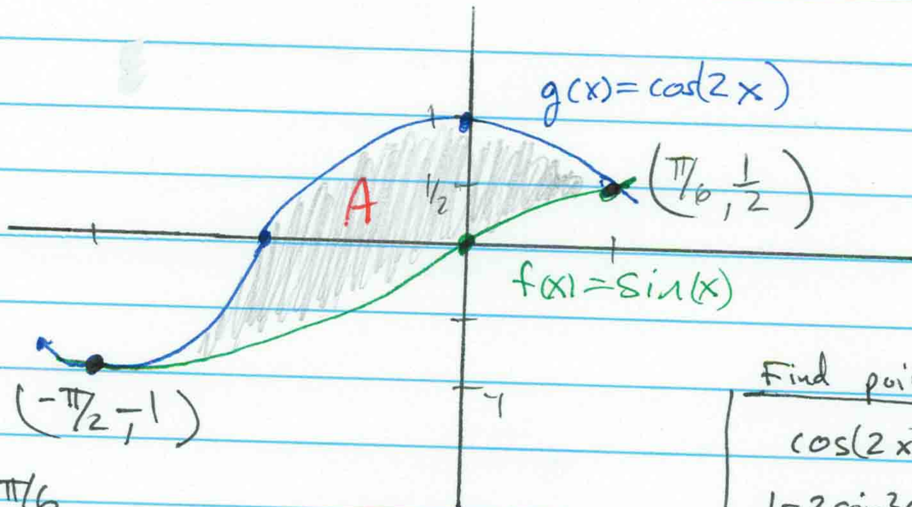
$$A = \left[\frac{3}{2}(3)^2 - \frac{(3)^3}{3}\right] - \left[\frac{3}{2}(0)^2 - \frac{(0)^3}{3}\right]$$

$$A = \left[\frac{3}{2}\cdot 9 - 9\right] - 0$$

$$A = \frac{27}{2} - \frac{18}{2}$$

$$A = \frac{9}{2}$$

#44



$$A = \int_{-\pi/2}^{\pi/6} [\cos(2x) - \sin(x)]\,dx$$

$$A = \int_{-\pi/2}^{\pi/6} \cos(2x)\,dx - \int_{-\pi/2}^{\pi/6} \sin(x)\,dx$$

Find points of intersection:

$\cos(2x) = \sin(x)$

$1 - 2\sin^2(x) = \sin(x)$

$0 = 2\sin^2(x) + \sin(x) - 1$

$0 = (2\sin(x) - 1)(\sin(x) + 1)$

Either

$2\sin(x) - 1 = 0$, or $\sin(x) + 1 = 0$

$2\sin(x) = 1$	$\sin(x) = -1$
$\sin(x) = \frac{1}{2}$	$x = -\pi/2$
$x = \pi/6$	

on $-\pi/2 \le x \le \pi/6$

$\boxed{7.1}$

#44 cont'd

$$A = \int_{u=-\pi}^{u=\pi/3} \cos(u) \cdot \left(\frac{du}{2}\right) - \int_{x=-\pi/2}^{x=\pi/6} \sin(x)\,dx$$

let $u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$x = \pi/6$, $u = 2(\pi/6)$, $u = \pi/3$

$x = -\pi/2$, $u = 2(-\pi/2)$, $u = -\pi$

$$A = \frac{1}{2}\Big[\sin(u)\Big]_{-\pi}^{\pi/3} - \Big[-\cos(x)\Big]_{x=-\pi/2}^{x=\pi/6}$$

$$A = \frac{1}{2}\left[\sin(\pi/3) - \sin(-\pi)\right] + \left[\cos(\pi/6) - \cos(-\pi/2)\right]$$

$$A = \frac{1}{2}\left[\frac{\sqrt{3}}{2} - 0\right] + \left[\frac{\sqrt{3}}{2} - 0\right]$$

$$A = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} \cdot \frac{2}{2}$$

$$A = \frac{3\sqrt{3}}{4}$$