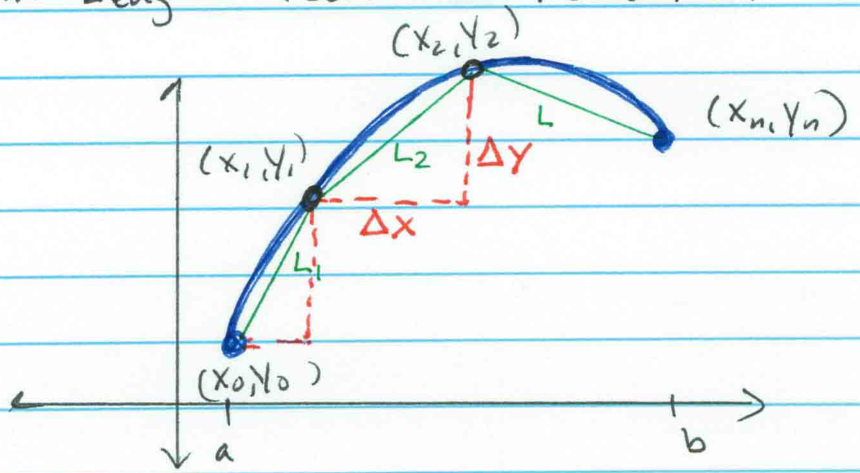
7.4 Arc Length & Surfaces of Revolution



$$L_1 = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \quad , \quad L_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Arc Length} \approx \sum_{i=1}^{n} \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$$S \approx \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$$S \approx \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2 (\Delta x_i)^2}$$

$$S \approx \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot (\Delta x_i)$$

$$S = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot (\Delta x_i)$$

$$\boxed{S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx}$$ Arc Length

$\boxed{7.4}$

#8 $y = \frac{3}{2}x^{2/3} + 4$, on $[1, 27]$

$$y' = \frac{d}{dx}\left[\frac{3}{2}x^{2/3} + 4\right]$$

$$y' = \frac{3}{2}\left[\frac{2}{3}x^{-1/3}\right] + 0$$

$$y' = x^{-1/3}$$

$$S = \int_1^{27} \sqrt{1 + \left(\frac{1}{x^{1/3}}\right)^2}\, dx$$

$$S = \int_1^{27} \sqrt{1 + \frac{1}{x^{2/3}}}\, dx$$

$$S = \int_1^{27} \sqrt{\frac{x^{2/3}}{x^{2/3}} + \frac{1}{x^{2/3}}}\, dx$$

$$S = \int_1^{27} \frac{\sqrt{x^{2/3} + 1}}{\sqrt{x^{2/3}}}\, dx$$

$$S = \int_1^{27} \frac{\sqrt{x^{2/3} + 1}}{x^{1/3}}\, dx$$

$$S = \int_{u=2}^{u=10} \frac{\sqrt{u}}{x^{1/3}} \cdot \left(\frac{3}{2}x^{1/3}\, du\right)$$

$$S = \frac{3}{2}\int_{u=2}^{u=10} u^{1/2}\, du$$

Let $u = x^{2/3} + 1$

$$\frac{du}{dx} = \frac{2}{3}x^{-1/3}$$

$$\frac{du}{dx} = \frac{2}{3x^{1/3}}$$

$$\frac{3}{2}x^{1/3}\, du = dx$$

$x = 1$, $u = (1)^{2/3} + 1$

$u = 1 + 1$

$\underline{u = 2}$

$x = 27$, $u = (27)^{2/3} + 1$

$u = (3)^2 + 1$

$u = 9 + 1$

$\underline{u = 10}$

$\boxed{7.4}$

#8 cont'd

$$S = \frac{3}{2}\left[\frac{2}{3} \cdot u^{3/2}\right]_2^{10}$$

$$S = \left[u^{3/2}\right]_2^{10}$$

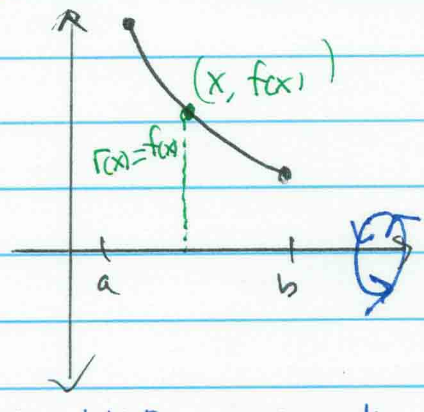
$$\underline{\underline{S = (10)^{3/2} - (2)^{3/2}}}$$

$$S \approx 28.79$$

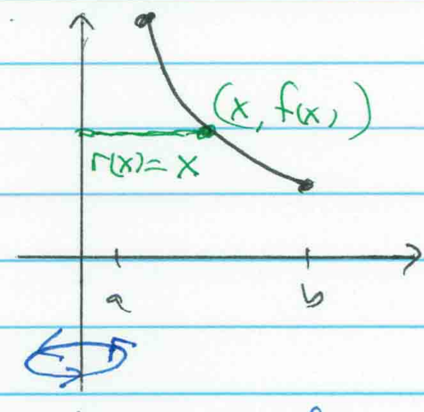
<u>Area of the Surface of Revolution</u> : $y = f(x)$ is <u>cts</u> on $[a,b]$

S = area of the surface of revolution formed by revolving the graph of f about a <u>horizontal</u> or <u>vertical</u> axis

$r(x)$ = distance between graph of f & axis of revolution.



<u>Horizontal</u> Axis of Revolution



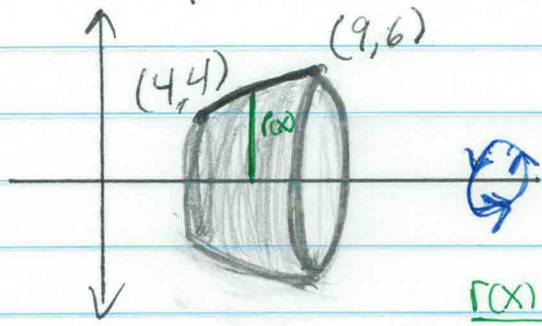
<u>Vertical</u> Axis of Revolution

$$S = 2\pi \int_a^b r(x) \cdot \sqrt{1 + [f'(x)]^2}\, dx$$

$\boxed{7.4}$

#40

$y = 2\sqrt{x}$, revolving about x-axis



$r(x) = 2\sqrt{x}$

$f(x) = 2x^{1/2}$

$f'(x) = 2 \cdot \frac{1}{2} x^{-1/2}$

$f'(x) = x^{-1/2}$

$f'(x) = \frac{1}{x^{1/2}}$

$$\text{Surface Area} = 2\pi \int_4^9 2\sqrt{x} \cdot \sqrt{1 + \left(\frac{1}{x^{1/2}}\right)^2}\, dx$$

$$= 2\pi \int_4^9 2\sqrt{x} \cdot \sqrt{1 + \frac{1}{x}}\, dx$$

$$= 4\pi \int_4^9 \sqrt{x+1}\, dx$$

$$= 4\pi \int_{u=5}^{u=10} \sqrt{u}\, (du)$$

Let $u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

$x = 4, \quad u = (4) + 1$

$u = 5$

$x = 9, \quad u = (9) + 1$

$u = 10$

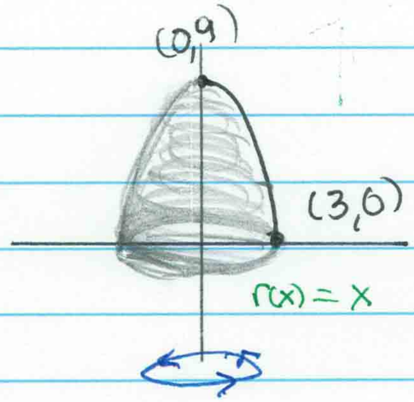
$$= 4\pi \left[\frac{2}{3} u^{3/2}\right]_5^{10}$$

$$S = \frac{8}{3}\pi \left(10^{3/2} - 5^{3/2}\right)$$

$$S \approx 171.258$$

$\boxed{7.4}$

#44 $y = 9 - x^2$, revolving about y-axis



$y' = -2x$

$$\text{Surface Area} = 2\pi \int_0^3 x \cdot \sqrt{1 + (-2x)^2}\, dx$$

$$= 2\pi \int_0^3 x\sqrt{1 + 4x^2}\, dx$$

$$= 2\pi \int_{u=1}^{u=37} x\sqrt{u}\left(\frac{du}{8x}\right)$$

$$= \frac{\pi}{4}\int_1^{37} u^{1/2}\, du$$

$$= \frac{\pi}{4}\left[\frac{2}{3}u^{3/2}\right]_1^{37}$$

$$= \frac{\pi}{6}\left[(37)^{3/2} + (1)^{3/2}\right]$$

$$= \frac{\pi}{6}\left[37^{3/2} + 1\right]$$

$$\approx 117.319$$

Let $u = 1 + 4x^2$

$\frac{du}{dx} = 8x$

$\frac{du}{8x} = dx$

$x = 0$, $u = 1 + 4(0)^2$

$\underline{u = 1}$

$x = 3$, $u = 1 + 4(3)^2$

$u = 1 + 4(9)$

$u = 1 + 36$

$\underline{u = 37}$