8.1 Basic Integration Rules

#24

$$\int \frac{2x}{x-4}\,dx$$

$$= \int \left(2 + \frac{8}{x-4}\right) dx$$

$$= \int 2\,dx + \int \frac{8}{x-4}\,dx$$

$$= 2(x) + 8\int \frac{1}{x-4}\,dx,$$

$$= 2x + 8\int \frac{1}{u}\,du$$

$$= 2x + 8 \cdot \ln|u| + c$$

$$\boxed{= 2x + 8\ln|x-4| + C}$$

Long Division:

$$\begin{array}{r} 2 \quad R\ 8 \\ x-4\,\overline{)\,2x + 0} \\ -(2x - 8) \\ \hline 8 \end{array}$$

$$\frac{2x}{x-4} = 2 + \frac{8}{x-4}$$

let $u = x-4$

$\frac{du}{dx} = 1$

$du = dx$

#12 $\int \sec(3x)\tan(3x)\,dx$

$$= \int \sec(u)\tan(u)\left(\frac{du}{3}\right)$$

$$= \frac{1}{3} \cdot \int \sec(u)\tan(u)\,du$$

$$= \frac{1}{3}[\sec u] + C = \boxed{\frac{1}{3}\sec(3x) + C}$$

let $u = 3x$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$\boxed{8.1}$

#34 $\displaystyle\int \csc^2(x)\cdot e^{\cot(x)}\,dx$, let $u=\cot(x)$

$\dfrac{du}{dx}=-\csc^2(x)$

$\dfrac{du}{-\csc^2(x)}=dx$

$$=\int \csc^2(x)\cdot e^u\cdot\left(\frac{du}{-\csc^2(x)}\right)$$

$$=-\int e^u\,du$$

$$=-\left[e^u\right]+C$$

$$\boxed{=-e^{\cot(x)}+C}$$

#36

$$\int \frac{5}{3e^x-2}\,dx$$

let $u=e^x$

$\ln(u)=\ln(e^x)$

$\ln(u)=x$

$\dfrac{d}{dx}\left(\ln(u)\right)=\dfrac{d}{dx}(x)$

$\dfrac{1}{u}\cdot\dfrac{du}{dx}=1$

$\dfrac{du}{u}=dx$

or

$\dfrac{du}{dx}=e^x$

$\dfrac{du}{dx}=u$

$\dfrac{du}{u}=dx$

$$=\int \frac{5}{3u-2}\cdot\left(\frac{du}{u}\right)$$

$$=5\int \frac{1}{u(3u-2)}\,du$$

$$=\frac{5}{2}\int \frac{2}{u(3u-2)}\,du$$

$$=\frac{5}{2}\int \frac{3u-3u+2}{u(3u-2)}\,du=\frac{5}{2}\int \frac{3u-(3u-2)}{u(3u-2)}\,du$$

$$=\frac{5}{2}\int \frac{3u}{u(3u-2)}\,du-\frac{5}{2}\int \frac{3u-2}{u(3u-2)}\,du$$

8.1

#36 cont'd

let $z = 3u - 2$

$\frac{dz}{du} = 3$

$\frac{dz}{3} = du$

$$= \frac{5}{2}\int \frac{3}{3u-2}\,du - \frac{5}{2}\int \frac{1}{u}\,du$$

$$= \frac{5}{2}\int \frac{3}{z}\left(\frac{dz}{3}\right) - \frac{5}{2}\cdot \ln|u| + c$$

$$= \frac{5}{2}\int \frac{1}{z}\,dz - \frac{5}{2}\ln|e^x| + c$$

$$= \frac{5}{2}\ln|z| - \frac{5}{2}\ln|e^x| + c$$

$$= \frac{5}{2}\ln|3u-2| - \frac{5}{2}\ln|e^x| + c$$

$$= \frac{5}{2}\ln|3e^x-2| - \frac{5}{2}\ln|e^x| + c$$

$$= \frac{5}{2}\ln\left|\frac{3e^x-2}{e^x}\right| + c$$

$$= \frac{5}{2}\ln\left|\frac{3e^x}{e^x} - \frac{2}{e^x}\right| + c$$

$$\boxed{= \frac{5}{2}\ln|3 - 2e^{-x}| + c}$$

$\boxed{8.1}$ ★ Not the result in the Solution Manual ???

#41

$$\int \frac{1}{\cos\theta - 1}\,d\theta = \int \frac{1}{-1\cdot(1-\cos(\theta))}\,d\theta$$

$$= -\int \frac{1}{1-\cos(\theta)}\,d\theta$$

$$= -\int \frac{1}{2\sin^2(\frac{\theta}{2})}\,d\theta$$

$$= -\int \frac{1}{2\sin^2(u)}\left(2\,du\right)$$

$$= -\int \csc^2(u)\,du$$

$$= -\left[-\cot(u)\right] + C$$

$$\boxed{= \cot\left(\frac{\theta}{2}\right) + C}$$

★ $\boxed{\frac{1+\cos\theta}{\sin\theta} + C}$

Trig. Identity

$\sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2}$

$2\sin^2(\alpha) = 1-\cos(2\alpha)$

$2\alpha = \theta$, $\alpha = \frac{\theta}{2}$

Let $u = \frac{\theta}{2}$

$\frac{du}{d\theta} = \frac{1}{2}$

$2\,du = d\theta$

$\boxed{8.1}$

#42

$$\int \frac{2}{3(\sec x - 1)}dx = \frac{2}{3}\int \frac{1}{(\sec x - 1)}\left(\frac{\sec x + 1}{\sec x + 1}\right)dx$$

$$= \frac{2}{3}\int \frac{\sec(x)+1}{\sec^2(x)-1}dx$$

$$= \frac{2}{3}\int \frac{\sec(x)+1}{\tan^2(x)}dx$$

$$= \frac{2}{3}\int \frac{\sec(x)}{\tan^2(x)}dx + \frac{2}{3}\int \frac{1}{\tan^2(x)}dx$$

$$= \frac{2}{3}\int \left(\frac{1}{\cos(x)}\right)\left(\frac{\cos^2(x)}{\sin^2(x)}\right)dx + \frac{2}{3}\int \cot^2(x)\,dx$$

$$= \frac{2}{3}\int \frac{\cos(x)}{\sin^2(x)}dx + \frac{2}{3}\int (\csc^2(x)-1)dx$$

$$= \frac{2}{3}\int \frac{\cos(x)}{u^2}\left(\frac{du}{\cos(x)}\right) + \frac{2}{3}\int \csc^2(x)dx - \frac{2}{3}\int 1dx$$

$$= \frac{2}{3}\int u^{-2}du + \frac{2}{3}[-\cot(x)] - \frac{2}{3}(x) + C$$

$$= \frac{2}{3}\left[\frac{u^{-1}}{-1}\right] - \frac{2}{3}\cot(x) - \frac{2}{3}x + C$$

$$= -\frac{2}{3}\left(\frac{1}{\sin(x)}\right) - \frac{2}{3}\cot(x) - \frac{2}{3}x + C$$

$$\boxed{= -\frac{2}{3}\left[\csc(x) + \cot(x) + x\right] + C}$$

$\sec^2(x) - 1 = \tan^2(x)$

$\cot^2(x) = \csc^2(x) - 1$

let $u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$\frac{du}{\cos(x)} = dx$

8.1

#48

$$\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx$$

$$= \int \frac{1}{(x-1)\sqrt{[2(x-1)]^2-1}}\,dx$$

$$= \int \frac{1}{\left(\frac{u}{2}\right)\sqrt{u^2-1}}\left(\frac{du}{2}\right)$$

$$= \int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$= \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

$$= \frac{1}{1}\operatorname{arcsec}\frac{|2(x-1)|}{1} + C$$

$$\boxed{= \operatorname{arcsec}|2(x-1)| + C}$$

$4x^2-8x+3$
$=4(x^2-2x)+3$
$=4(x^2-2x+1)+3-4$
$\left[\frac{1}{2}(-2)\right]^2=(-1)^2=1$
$=4(x-1)^2-1$
$=[2(x-1)]^2-1$

let $u=2(x-1)$
$u=2x-2$
$\frac{du}{dx}=2$
$\frac{du}{2}=dx$

$u=2(x-1)$
$\frac{u}{2}=x-1$

$a^2=1$, $a=1$

#64

$$\int_1^e \frac{1-\ln(x)}{x}\,dx = \int_{u=1}^{u=0} \frac{u}{x}(-x\,du)$$

$$= -\int_1^0 u\,du = \int_0^1 u\,du$$

$$= \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}-0 = \boxed{\frac{1}{2}}$$

let $u=1-\ln(x)$
$\frac{du}{dx}=-\frac{1}{x}$
$-x\,du=dx$

$x=e$, $u=1-\ln(e)$
$u=1-1$
$u=0$

$x=1$, $u=1-\ln(1)$
$u=1-0$
$u=1$