10.1 Radical Expressions and Functions

Square Roots

If a is a nonnegative real number, the nonnegative number b such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the principal square root of a.

Example1: Simplify each of the following:

$a.\ \sqrt{100} = 10$

b. $\sqrt{\frac{9}{16}}$

c. $\sqrt{.04}$

d. $-\sqrt{100}$

e. $\sqrt{9+16}$

f. $\sqrt{36} + \sqrt{9+16}$

What happens when we try to evaluate the square root of a negative number? The square root of a negative number is not a real number.

Square Root Functions

Because each nonnegative real number, x, has precisely one principal square root, $\sqrt{x}$, there is a square root function defined by

$$f(x) = \sqrt{x}.$$

The domain of this function is $[0, \infty)$.

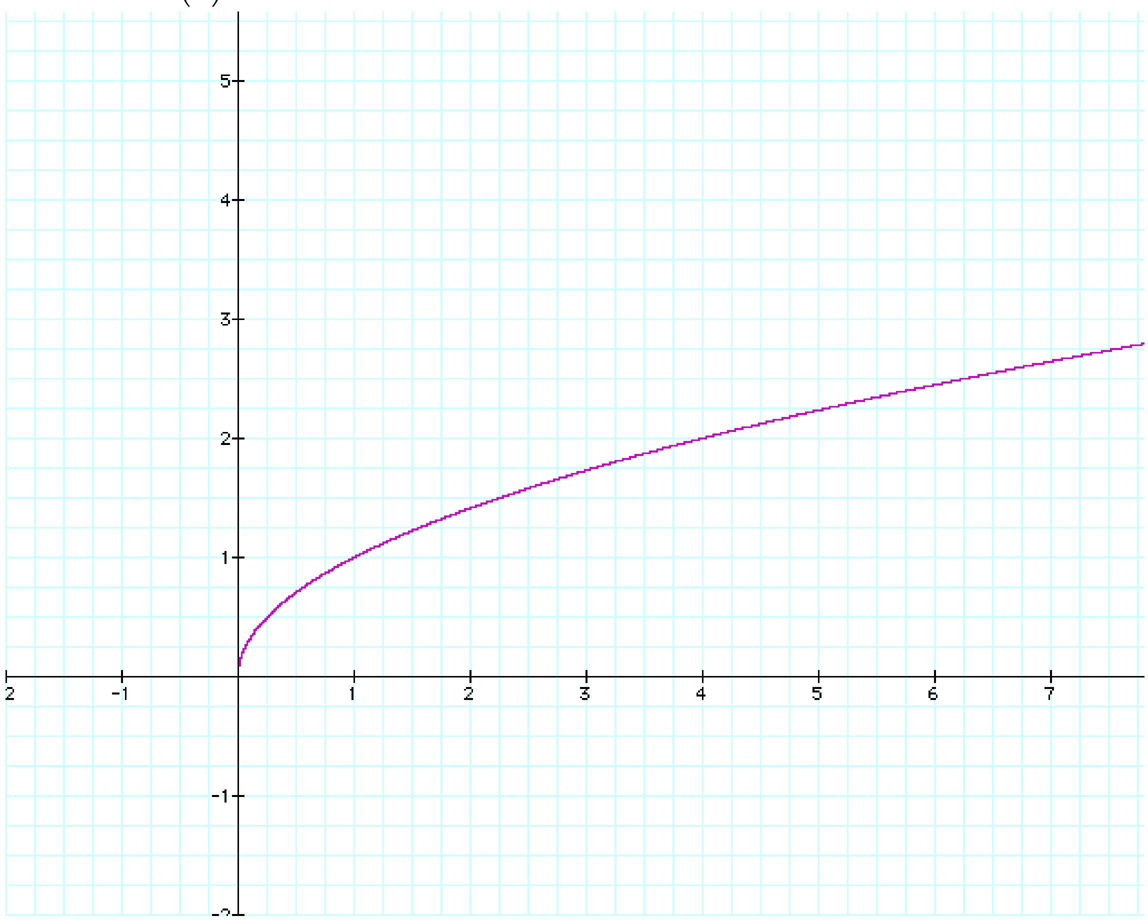
To graph $f(x) = \sqrt{x}$, construct a table of values by choosing nonnegative real numbers for x and calculating y. If you are not using a calculator, it is easiest to calculate y if you choose perfect squares for x. Plot these ordered pairs and then connect the points with a smooth curve. Remember that the domain is **nonnegative** real numbers.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Table of values for $f(x)=\sqrt{x}$

x	$f(x)=\sqrt{x}$	(x,y)
0	$f(0)=\sqrt{0}=0$	(0,0)
1	$f(1)=\sqrt{1}=1$	(1,1)
4	$f(4)=?$	
9	$f(9)=?$	
16		

Graph of $f(x)=\sqrt{x}$.



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Evaluating Square Root Functions

To evaluate a square root function at a value of x, substitute that value into the function everywhere that x occurs.

Example 2: For each function, find the indicated function value.

a. $f(x) = \sqrt{2x+5}$, $f(2)$ Solution : $f(2) = \sqrt{2(2)+5} = ?$

b. $g(x) = \sqrt{x+13}$, $g(3)$

c. $g(x) = \sqrt{x+13}$, $g(-4)$

Finding the Domain of a Square Root Function

Because the square root of a negative number is not a real number, the square root function is defined only for values of the independent variable that produce a nonnegative radicand.

To find the domain of a square root function, set the radicand (quantity under the radical) greater than or equal to zero and solve the inequality.

Example 3: Find the domain of the given square root functions.

a. $f(x) = \sqrt{2x+6}$ Solution: $2x+6 \geq 0$

$$2x \geq -6$$
$$x \geq -3$$
$$[-3, \infty)$$

b. $g(x) = \sqrt{5x-12}$

c. $h(x) = \sqrt{3-2x}$

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Modeling With a Square Root Function

The graph of the square root function is increasing from left to right, but the rate of increase is decreasing. Thus, the square root function is often the appropriate function to use to model growth phenomena with decreasing rate of growth.

Example 4: The median height of boys in the United States from birth through age 60 months is given by the function

$$f(x) = 2.9\sqrt{x} + 20.1$$

where x is the age in months and f(x) is the median height. Use this function and a calculator to find the median height of 3-year old boys in the United States. Round your answer to the nearest tenth of an inch.

Simplifying Expressions of the Form $\sqrt{a^2}$

For any real number a, $\sqrt{a^2} = \lvert a \rvert$.
For any nonnegative real number a, $\sqrt{a^2} = \lvert a \rvert = a$

Example 5: Simplify each of the following:

a. $\sqrt{(-3)^2}$ Solution: $\sqrt{(-3)^2} = \lvert(-3)\rvert = 3$

b. $\sqrt{49x^4}$

c. $\sqrt{(x-9)^2}$

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Cube Roots and Cube Root Functions

The cube root of a real number a is written $\sqrt[3]{a}=b$ where $b^3 = a$. The cube root of a positive number is positive, and the cube root of a negative number is negative. The following table shows some cube roots of perfect cubes that you should memorize.

Table of Cube Roots of Some Perfect Cubes

$\sqrt[3]{1} = 1$
$\sqrt[3]{8} = 2$
$\sqrt[3]{-8} = -2$
$\sqrt[3]{27} = 3$
$\sqrt[3]{-27} = -3$
$\sqrt[3]{64} = 4$
$\sqrt[3]{-64} = -4$
$\sqrt[3]{125} = 5$
$\sqrt[3]{216} = 6$
$\sqrt[3]{1000} = 10$

Graph of the Cube Root Function

Because each real number has exactly one cube root, $f(x) = \sqrt[3]{x}$ is a function. It is called the cube root function. The domain of f is the set of all real numbers. To graph the function, construct a table of values by choosing perfect cubes for x and calculating f(x). Then plot each ordered pair and connect the ordered pairs with a smooth curve. A table of values and the graph follow.

Table of values for $f(x) = \sqrt[3]{x}$

x	$f(x) = \sqrt[3]{x}$	(x,y)
–8	$f(-8) = \sqrt[3]{-8} = -2$	$(-8,-2)$
–1	$f(-1) = \sqrt[3]{-1} = -1$	$(-1,-1)$
0	$f(0) = \sqrt[3]{0} = ?$	
1	$f(1) = ?$	
8		

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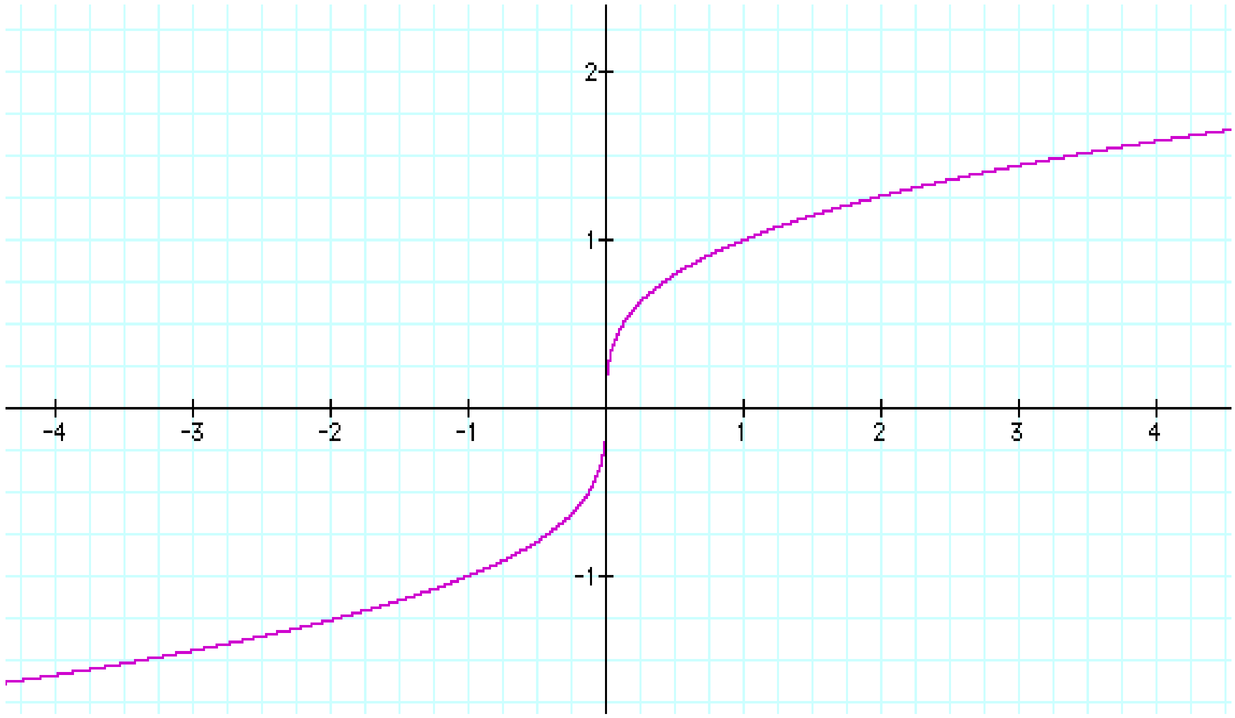

Note that the cube root function is always increasing as the graph is traced from left to right.

Evaluating Cube Root Functions
To evaluate a cube root function at a value of x, substitute that value into the function everywhere that x occurs.

Example 6: Find the indicated function values.

a. $f(x)=\sqrt[3]{x+3}$, $f(24)$ Solution: $f(24)=\sqrt[3]{24+3}=?$

b. $g(x)=\sqrt[3]{3x-4}$, $g(4)$

c. $h(x)=\sqrt[3]{1-3x}$, $h(3)$

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Simplifying Cube Roots

For any real number a, $\sqrt[3]{a^3} = a$.

Example 7: Simplify each of the following.

a. $\sqrt[3]{8x^3}$

b. $\sqrt[3]{-27x^3}$

c. $\sqrt[3]{1000x^3}$

Even and Odd *n*th Roots

Radical expressions can have roots other than square roots and cube roots.

The radical expression $\sqrt[n]{a}$ means the *n*th root of a. The number n is called the index, and a is called the radicand.

In general, $\sqrt[n]{a} = b$ if $b^n = a$.

Example 8: Simplify each of the following.

a. $\sqrt[4]{16}$ Solution: $\sqrt[4]{16} = 2$

b. $\sqrt[5]{243}$

c. $\sqrt[6]{64}$

d. $\sqrt[5]{-32}$

If the index is an odd number, then the root is said to be an odd root. An odd root of a positive number is a positive number, and an odd root of a negative number is a negative number.
If the index is an even number, then the root is said to be an even root. Since we choose the principal root when the index is even, the even root of a positive number is a positive number. The even root of a negative number is a not a real number.

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Example 9: Find the indicated root, or state that the expression is not a real number.

a. $\sqrt[4]{-16}$

c. $\sqrt[6]{-64}$

b. $\sqrt[5]{-243}$

d. $\sqrt[7]{-128}$

Simplifying Expressions of the Form $\sqrt[n]{a^n}$

For any real number a,

If n is even, $\sqrt[n]{a^n} = |a|$

If n is odd, $\sqrt[n]{a^n} = a$

Example 10: Simplify each of the following:

a. $\sqrt[4]{x^4}$

b. $\sqrt[5]{(x-5)^5}$

c. $\sqrt[3]{(2x+1)^3}$

d. $\sqrt{(3x-1)^2}$

Using a Calculator to Find Roots

The square root key on the TI-83 calculator is located on the left side above the x^2 key. Other roots are found in the MATH menu. Press the "MATH" key, and scroll down to $\sqrt[3]{\ }$ for cube roots or $\sqrt[x]{\ }$ for roots higher than 3. To use the $\sqrt[x]{\ }$ function, enter the index first, select $\sqrt[x]{\ }$, enter the radicand, and then press enter for the desired root.

Example 11: Use your calculator to find the following. Round to three decimal places.

a. $\sqrt{151}$

b. $\sqrt[3]{58}$

c. $\sqrt[5]{546}$

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Answers Section 10.1

Example 1:
- a. 10
- b. $\frac{3}{4}$
- c. 0.2
- d. –10
- e. 5
- f. 11

Example 2:
- a. f(2) = 3
- b. f(3) = 4
- c. g(–4) = 3

Example 3:
- a. $[-3, \infty)$
- b. $\left[\frac{12}{5}, \infty\right)$
- c. $\left(-\infty, \frac{3}{2}\right]$

Example 4: x = 36, f(36) = 37.5
The median height of a 3 yr. old boy in the U.S. is 37.5 inches.

Example 5:
- a. 3
- b. $7x^2$
- c. $|x-9|$

Example 6:
- a. f(24) = 3
- b. g(4) = 2
- c. h(3) =–2

Example 7:
- a. 2x
- b. –3x
- c. 10x

Example 8:
- a. 2
- b. 3
- c. 2
- d. –2

Example 9:
- a. Not a real number
- b. –3
- c. Not a real number
- d. –2

Example 10:
- a. $|x|$
- b. x – 5
- c. 2x + 1
- d. $|3x-1|$

Example 11:
- a. 12.288
- b. 3.871
- c. 3.527

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