10.2 Rational Exponents

Definition of $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ represents a real number and $n \geq 2$ is an integer, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}.$$

If n is odd and

- a is positive, then $a^{\frac{1}{n}}$ is positive.
- a is negative, then $a^{\frac{1}{n}}$ is negative.
- a is zero, then $a^{\frac{1}{n}}$ is zero.

If n is even and

- a is positive, then $a^{\frac{1}{n}}$ is positive.
- a is negative, then $a^{\frac{1}{n}}$ is not a real number
- a is zero, then $a^{\frac{1}{n}}$ is also zero.

Example 1: Use radical notation to rewrite each expression. Simplify, if possible.

a. $36^{\frac{1}{2}} = \sqrt{36} = ?$

b. $(-8)^{\frac{1}{3}}$

c. $(9xy^2)^{\frac{1}{5}}$

d. $\left(x^2\right)^{\frac{1}{2}}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 2: Rewrite each expression using rational exponents.

a. $\sqrt[4]{5xy} = (5xy)^{\frac{1}{4}}$

b. $\sqrt[3]{3xy^2}$

c. $\sqrt[5]{4a^2b}$

d. $\sqrt{3xy}$

Definition of $a^{\frac{m}{n}}$

If $\sqrt[n]{a}$ represents a real number and $\frac{m}{n}$ is a positive rational number, $n \geq 2$, then

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \quad \text{and} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}.$$

Note that if n is even and a is negative, $\sqrt[n]{a}$ does not represent a real number and $a^{\frac{m}{n}}$ is not a real number.

Example 3: Use radical notation to rewrite each of the following and then simplify.

a. $16^{\frac{3}{2}} = \sqrt{16}^3 = 4^3 = ?$

b. $8^{\frac{2}{3}}$

c. $(-9)^{\frac{3}{2}}$

d. $-32^{\frac{3}{5}}$

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Example 4: Rewrite with rational exponents.

a. $\sqrt[4]{8^5}$

b. $\sqrt[3]{(3x)^2}$

c. $\left(\sqrt[6]{5xy}\right)^7$

d. $\sqrt[5]{25x^2}$

Definition of $a^{-\frac{m}{n}}$

If $a^{\frac{m}{n}}$ is a nonzero real number, then

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$$

Example 5: Rewrite each of the following with a positive exponent. Simplify, if possible. Assume all variables represent nonnegative quantities.

a. $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{\sqrt{49}} = ?$

b. $32^{-\frac{3}{5}}$

c. $\left(2ab\right)^{-\frac{2}{3}}$

d. $\left(-27\right)^{-\frac{2}{3}}$

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Properties of Rational Exponents

If m and n are rational exponents, and a and b are real numbers for which the following expressions are defined, then

1. $b^m \cdot b^n = b^{m+n}$
2. $\frac{b^m}{b^n} = b^{m-n}$
3. $\left(b^m\right)^n = b^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 6: Simplify the following expressions with rational exponents. Express all answers with positive exponents. Assume all variables represent nonnegative quantities.

a. $8^{\frac{2}{3}} \cdot 8^{\frac{4}{3}}$

b. $\frac{7^{\frac{5}{8}}}{7^{\frac{3}{8}}}$

c. $\left(3xy^2\right)^{\frac{5}{7}}$

d. $\frac{16x^{\frac{2}{3}}}{4x^{\frac{1}{2}}}$

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Simplifying Radical Expressions Using Rational Exponents

To simplify a radical expression by using rational exponents:

1. Rewrite each radical expression as an exponential expression with a rational exponent.
2. Simplify using properties of rational exponents.
3. Rewrite your answer in radical notation when rational exponents still appear.

Example 7: Use rational exponents to simplify. Assume all variables represent nonnegative quantities.

a. $\sqrt[3]{8x^2} = \left(8x^2\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(x^2\right)^{\frac{1}{3}} = ?$

b. $\sqrt{16xy^4}$

c. $\sqrt[6]{64x^3}$

d. $\sqrt{x} \bullet \sqrt[3]{x^2}$

e. $\sqrt{\sqrt[4]{x^3}}$

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Application of Rational Exponents

Example 8: The function $f(x) = 70x^{\frac{3}{4}}$ models the number of calories per day, f(x), that a person needs to maintain life in terms of that person's weight, x, in kilograms. (1 kilogram is approximately 2.2 pounds.) Use the model and a calculator to find how many calories per day are required to maintain life for a person who weighs 55 kilograms (about 121 pounds). Round your answer to the nearest calorie.

Example 9: Use your calculator to evaluate the following to three decimal places.

a. $(234)^{\frac{1}{4}}$

b. $(-655)^{\frac{2}{3}}$

c. $(45)^{\frac{3}{4}} + \sqrt[3]{47}$

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Answers Section 10.2

Example 1:

a. 6

b. –2

c. $\sqrt[5]{9xy^2}$

d. $|x|$

Example 2:

a. $(5xy)^{\frac{1}{4}}$

b. $(3xy^2)^{\frac{1}{3}}$

c. $(4a^2b)^{\frac{1}{5}}$

d. $(3xy)^{\frac{1}{2}}$

Example 3:

a. 64

b. 4

c. Not a real number

d. –8

Example 4:

a. $8^{\frac{5}{4}}$

b. $(3x)^{\frac{2}{3}}$

c. $(5xy)^{\frac{7}{6}}$

d. $(25x^2)^{\frac{1}{5}}$

Example 5:

a. $\frac{1}{7}$

b. $\frac{1}{8}$

c. $\frac{1}{(2ab)^{\frac{2}{3}}}$

d. $\frac{1}{9}$

Example 6:

a. 64

b. $7^{\frac{1}{4}}$

c. $3^{\frac{5}{7}}x^{\frac{5}{7}}y^{\frac{10}{7}}$

d. $4x^{\frac{1}{6}}$

Example 7:

a. $2x^{\frac{2}{3}}$

b. $4x^{\frac{1}{2}}y^2$

c. $2x^{\frac{1}{2}}$

d. $x^{\frac{7}{6}}$

e. $x^{\frac{3}{8}}$

Example 8:

a. x = 55 kg., f(55) ≅ 1414 calories

Example 9:

a. 3.911

b. 75.421

c. 20.983

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