# 10.3 Multiplying and Simplifying Radical Expressions

## The Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$

The product of two nth roots is the nth root of the product.

Note that in order to multiply two radicals, the radicals must have the same index.

Example 1: Multiply. Assume all variables represent nonnegative quantities.

a. $\sqrt{5} \cdot \sqrt{6} = \sqrt{30}$

b. $\sqrt[3]{x} \cdot \sqrt[3]{3y}$

c. $\sqrt[5]{5x^2} \cdot \sqrt[5]{2x^2}$

d. $\sqrt{2x+1} \cdot \sqrt{3x+7}$

## Simplifying Radical Expressions by Factoring

A radical expression whose index is n is said to be simplified when its radicand has no factors that are perfect nth powers.

To simplify a radical expression of index n:

1. Write the radicand in factored form, where all possible perfect nth powers appear as one of the factors.
2. Use the product rule to take the nth root of each factor.
3. Find the nth root of the perfect nth power.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 2: Simplify. Assume all variables represent nonnegative quantities.

*a.* $\sqrt{18} = \sqrt{3^2 \bullet 2} = \sqrt{3^2} \bullet \sqrt{?} = ?$

*b.* $\sqrt{72}$

*c.* $\sqrt[3]{108}$

*d.* $\sqrt[4]{162}$

*e.* $\sqrt{50x^3}$

*f.* $\sqrt{27xy^5}$

*g.* $\sqrt[3]{2x^5y^4}$

**Using Absolute Values When Simplifying Even Roots:** For the remainder of the chapter, assume that no radicands involve negative quantities raised to even powers. Thus, absolute value bars are not necessary when taking even roots. One exception is the process of simplifying a **radical function**. In this case, absolute value bars must be used when taking even roots of even powers.

Example 3: Simplify.

$$f(x) = \sqrt{2x^2 + 12x + 18}$$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

**Perfect nth Powers**

When simplifying radicals of index n that involve variables raised to various powers, note that the perfect nth powers have exponents that are divisible by n.

Example 4: Simplify.

a. $\sqrt{3x^5y^3} = \sqrt{3x^4xy^2y} = \sqrt{3}\sqrt{x^4}\sqrt{x} \bullet \sqrt{y^2}\sqrt{y} = ?$

b. $\sqrt{8x^4y^6}$

c. $\sqrt[3]{3x^4y^3}$

d. $\sqrt[3]{12a^5b^4}$

e. $\sqrt[4]{24a^9b^5}$

f. $\sqrt[5]{64x^{10}y^7}$

g. $\sqrt[6]{2x^{10}y^8}$

h. $\sqrt[7]{128x^{10}y^5}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

## Multiplying and Simplifying Radicals

To multiply radicals that have the same index, n:

- Use the product rule for nth roots to multiply the radicals, and
- Simplify the result by factoring and taking the nth root of the factors that are perfect nth powers.

Example 5: Multiply and simplify.

a. $\sqrt{12} \bullet \sqrt{6}$

b. $\sqrt{2x} \bullet \sqrt{8x}$

c. $3\sqrt{15} \bullet 5\sqrt{6}$

d. $\sqrt{5xy} \bullet \sqrt{10xy^2}$

e. $\sqrt[3]{4x^2} \bullet \sqrt[3]{4x}$

*f*. $\sqrt[5]{8x^4y^3} \bullet \sqrt[5]{8xy^4}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

## Answers Section 10.3

Example 1:

a. $\sqrt{30}$

b. $\sqrt[3]{3xy}$

c. $\sqrt[5]{10x^4}$

d. $\sqrt{6x^2+17x+7}$

Example 2:

a. $3\sqrt{2}$

b. $6\sqrt{2}$

c. $3\sqrt[3]{4}$

d. $3\sqrt[4]{2}$

e. $5x\sqrt{2x}$

f. $3y^2\sqrt{3xy}$

g. $xy\sqrt[3]{2x^2y}$

Example 3: f(x) = $\sqrt{2}|x+3|$

Example 4:

a. $x^2y\sqrt{3xy}$

b. $2x^2y^3\sqrt{2}$

c. $xy\sqrt[3]{3x}$

d. $ab\sqrt[3]{12a^2b}$

e. $a^2b\sqrt[4]{24ab}$

f. $2x^2y\sqrt[5]{2y^2}$

g. $xy\sqrt[6]{2x^4y^2}$

h. $2x\sqrt[7]{x^3y^5}$

Example 5:

a. $6\sqrt{2}$

b. $4x$

c. $45\sqrt{10}$

d. $5xy\sqrt{2y}$

e. $2x\sqrt[3]{2}$

f. $2xy\sqrt[5]{2y^2}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.