10.5 Multiplying with More Than One Term and Rationalizing Denominators

Multiplying Radicals with More Than One Term

To multiply radicals with more than one term, use the distributive law. If the two expressions are both binomials, you may use the FOIL method.

Example 1: Simplify.

$a.\ \sqrt{7}\left(x+\sqrt{10}\right)=\sqrt{7}*x+\sqrt{7}*\sqrt{10}=?$

$b.\ \sqrt[3]{x}\left(\sqrt[3]{x}+\sqrt[3]{14}\right)$

$c.\ \left(5+\sqrt{7}\right)\left(3-4\sqrt{7}\right)=5*3-5*4\sqrt{7}+\sqrt{7}*3-\sqrt{7}*4\sqrt{7}=?$

$d.\ \left(5\sqrt{2}+\sqrt{7}\right)\left(3\sqrt{2}-4\sqrt{7}\right)$

$e.\ \left(5+\sqrt{7}\right)^2$

$f.\ \left(5+\sqrt{7}\right)\left(3-4\sqrt{7}\right)$

$g.\ \left(5-\sqrt{7}\right)\left(5+\sqrt{7}\right)$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Rationalizing Denominators Containing One Term

To rationalize the denominator of a radical expression, you must rewrite the expression as an equivalent expression that does not contain any radicals in the denominator. When the denominator contains a single radical with an nth root, multiply the numerator and the denominator by a radical of index n that produces a perfect nth power in the denominator's radicand.

Example 2: Rationalize each denominator.

$$a.\quad \frac{\sqrt{10}}{\sqrt{3}} = \frac{\sqrt{10}}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = ?$$

$$b.\quad \frac{\sqrt{3}}{\sqrt{x}}$$

$$c.\quad \sqrt{\frac{12}{5\text{x}}}$$

$$d.\quad \frac{\sqrt[3]{14}}{\sqrt[3]{3}}$$

$$e.\quad \frac{\sqrt[3]{6}}{\sqrt[3]{7x^2}} = \frac{\sqrt[3]{6}}{\sqrt[3]{7x^2}} * \frac{\sqrt[3]{7^2 x}}{\sqrt[3]{7^2 x}} = \frac{?}{\sqrt[3]{7^3 x^3}} = ?$$

$$f.\quad \frac{\sqrt[3]{3x}}{\sqrt[3]{y^2}}$$

$$g.\quad \frac{\sqrt[4]{5\text{xy}}}{\sqrt[4]{2\text{z}^2}}$$

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Rationalizing Denominators Containing Two Terms

Radical expressions that involve the sum and the difference of the same two terms are called conjugates.

Example 3: Find the conjugate of each expression.

a. $\sqrt{2}+3$

b. $5-3\sqrt{5}$

c. $\sqrt[3]{x}-3$

To rationalize a denominator that contains two terms, multiply both numerator and denominator by the conjugate of the denominator.

Example 4: Rationalize each denominator.

a. $\dfrac{15}{\sqrt{6}+1}$

b. $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{7}-1}$

c. $\dfrac{\sqrt{x}+2}{\sqrt{x}-1}$

d. $\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{6}+\sqrt{2}}$

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Answers Section 10.5

Example 1:

a. $\sqrt{7}x + \sqrt{70}$

b. $\sqrt[3]{x^2} + \sqrt[3]{14x}$

c. $-13 - 17\sqrt{7}$

d. $2 - 17\sqrt{14}$

e. $32 + 10\sqrt{7}$

f. $-13 - 17\sqrt{7}$

g. 18

Example 2:

a. $\dfrac{\sqrt{30}}{3}$

b. $\dfrac{\sqrt{3x}}{x}$

c. $\dfrac{2\sqrt{15x}}{5x}$

d. $\dfrac{\sqrt[3]{126}}{3}$

e. $\dfrac{\sqrt[3]{294x}}{7x}$

f. $\dfrac{\sqrt[3]{3xy}}{y}$

g. $\dfrac{\sqrt[4]{40xyz^2}}{2z}$

Example 3:

a. $\sqrt{2} - 3$

b. $5 + 3\sqrt{5}$

c. $\sqrt[3]{x} + 3$

Example 4:

a. $3\left(\sqrt{6} - 1\right)$

b. $\dfrac{\sqrt{35} + \sqrt{21} + \sqrt{5} + \sqrt{3}}{6}$

c. $\dfrac{x + 3\sqrt{x} + 2}{x - 1}$

d. $\dfrac{\sqrt{18} - \sqrt{42} - \sqrt{6} + \sqrt{14}}{4}$

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