10.6 Radical Equations

Solving Radical Equations

In order to solve an equation that contains one or more nth roots,
1. If necessary, arrange the terms so that one radical is isolated on one side of the equation.
2. Raise both sides of the equation to the nth power to eliminate the nth root.
3. Solve the resulting equation. If this equation still contains nth roots, repeat steps I and 2 until all nth roots have been eliminated.
4. Test each proposed solution in the original equation to determine if it is a solution.

Example 1: Solve each radical equation. Note that each equation contains only one square root.

a. $\sqrt{5x-1}=8$

b. $\sqrt{4x-3}=5$

c. $\sqrt{5x-4}+2=6$

d. $\sqrt{2x+5}+11=6$

$\sqrt{2x+5}=-5$

$2x+5=(-5)^2$

$x=?$ Check this proposed solution to see if it works.

What is the solution, if any?

e. $x=\sqrt{6x+7}$

f. $x=\sqrt{3x+7}-3$

The following equations contain roots other than square roots. Remember that to eliminate an nth root, isolate the term containing the root, and then raise both sides to the nth power.

Example 2: Solve each equation.

a. $\sqrt[3]{x-1}=3$

$(\sqrt[3]{x-1})=(3^3)$ (Cube both sides)

$x-1=27$ (Solve this equation)

$x=28$

Answer : {28}

b. $\sqrt[3]{5x-1}=4$

c. $\sqrt[4]{7x+2}+15=17$ (Hint: Isolate the fourth root first)

d. $(x-3)^{\frac{1}{3}}=5$ (Hint: The $\frac{1}{3}$ power means cube root.)

Some radical equations contain more than one nth root. To solve these equations, radical terms must be isolated one-by-one and eliminated by raising both sides to the nth power.

Example 3: Solve each equation.

a. $\sqrt{x-8}=\sqrt{x}-2$ Isolate one radical.

$\left(\sqrt{x-8}\right)^2=\left(\sqrt{x}-2\right)^2$ Square both sides.

$x-8=\left(\sqrt{x}\right)^2-4\sqrt{x}+4$ Left side: $\left(\sqrt{a}\right)^2=a,$ Right side: FOIL

$x-8=x+4-4\sqrt{x}$ Simplify.

$-12=-4\sqrt{x}$ Isolate the remaining radical.

$3=\sqrt{x}$ Simplify.

$9=x$ Square both sides.

Check this proposed solution: {9}. Does it work?

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

b. $\sqrt{x-4}+\sqrt{x+4}=4$

c. $\sqrt{x-4}+\sqrt{x+1}=5$

Applications of Square Root Functions

Example 4: Out of a group of 50,000 births, the number of people, f(x) surviving to age x is modeled by the function

$$f(x) = 5000\sqrt{100 - x}.$$

a. How many people in the group are expected to survive to age 80?

b. At what age are 35,000 people in the group still surviving?

Answers Section 10.6

Example 1:
a. {13}
b. { 7}
c. { 4}
d. No solution, or $\varnothing$
e. { 7} (Note that the proposed solution x = −1 does not work)
f. { −2, −1}

Example 2:
a. { 28}
b. {13}
c. { 2}
d. { 128}

Example 3:
a. { 9}
b. { 5}
c. { 8}

Example 4:
a. f(x) = $5000\sqrt{20} \cong 22{,}360$ About 22,360 people are expected to survive to age 80.
b. At age 51 there are still 35,000 people in the group surviving.