10.7 Complex Numbers

The Imaginary Unit i

The imaginary unit i is defined as

$$i = \sqrt{-1} \text{ where } i^2 = -1.$$

If b is a positive number, then

$$\sqrt{-b} = \sqrt{b(-1)} = \sqrt{b}\sqrt{-1} = i\sqrt{b}$$

Example 1: Write each square root of a negative number as a multiple of i.

a. $\sqrt{-5} = \sqrt{-1 * 5} = \sqrt{-1} * \sqrt{5} = i\sqrt{5}$

b. $\sqrt{-25}$

c. $\sqrt{-16}$

Complex Numbers and Imaginary Numbers

The set of all numbers in the form

a + bi

with real numbers a and b, and i, the imaginary unit, is called the set of complex numbers. The real number a is called the real part, and the real number b is called the imaginary part, of the complex number a + bi. Complex numbers can be further described as either:

- Imaginary, if a=0
- Real, if b = 0,
- Complex but not real, if neither a nor b is zero.

Example 2:

a. Consider the complex number 8 + 5i. What is the imaginary part of the number? The real part? How can you further describe this number?

b. Consider the complex number 8. What is the imaginary part of this complex number? How can you further describe this number?

c. Consider the complex number 5i. What is the imaginary part of this number? The real part? How can you further describe this number?

Note: Portions of this document are excerpted from the textbook *Algebra: A Combined Approach* by Elayn Martin-Gay.

Adding and Subtracting Complex Numbers

To add or subtract complex numbers:

1. (a + bi) + (c + di) = (a + c) + (b + d)i
   In words, add complex numbers by adding the real parts, adding the imaginary parts, and expressing the result as a complex number.
2. (a + bi) – (c + di) = (a – c) + (b – d)i
   In words, subtract complex numbers by subtracting the real parts, subtracting the imaginary parts, and expressing the result as a complex number.

Example 3: Simplify the following.

a. $(2-3i)+(5+7i)$

b. $(10+8i)-(2-6i)$

Multiplying Complex Numbers

To multiply complex numbers, use the distributive law and the FOIL method.

Example 4: Simplify the following.

a. $(2-3i)(5+7i)$

b. $7i(3\text{-}i)$

The product rule for radicals only applies to real numbers. If a radical does not represent a real number, you must write the radical as a multiple of i before you use the product rule.

Example 5: Simplify the following.

a. $\sqrt{-4}\sqrt{-9}=2i*3i=?$

b. $\sqrt{-5}\sqrt{-6}$

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Conjugates and Division of Complex Numbers

The conjugate of the complex number a + bi is the complex number a – bi. When a complex number is multiplied by its conjugate, the result is a real number.

Example 6: Multiply each complex number by its conjugate.

a. 7i

b. $3+7i$

c. $6+5i$

To divide two complex numbers, write in fraction form and then multiply the numerator and the denominator by the conjugate of the denominator.

Example 7: Simplify.

$a.\ \frac{3-i}{7i}$

$b.\ \frac{3-2i}{6+5i}=\frac{3-2i}{6+5i}*\frac{6-5i}{6-5i}=?$

$c.\ \frac{3-i}{7-i}$

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Powers of i

The powers of i cycle through four values: i, –1, –i, and 1.

Example 8: Simplify.

$a.\ i^1$

$b.\ i^2$

$c.\ i^3$

$d.\ i^4$

$e.\ i^5$

$f.\ i^6$

$g.\ i^7$

$h.\ i^8$

Simplifying Powers of i

To simplify a power of i
1. Express the given power of i in terms of i^2.
2. Replace i^2 by –1 and simplify. Use the fact that –1 to an even power is 1 and –1 to an odd power is –1.

Example 9: Simplify.

$a.\ i^{12} = \left(i^2\right)^6 = \left(-1\right)^6 = 1$

$b.\ i^{31} = \left(i^2\right)^{15} \bullet i = \left(-1\right)^{15} \bullet i = -1 \bullet i = -i$

$c.\ i^{52}$

$d.\ i^{79}$

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Answers Section 10.7

Example 1:
a. $i\sqrt{5}$
b. 5i
c. 4i

Example 2:
a. 8 is the real part and 5 is the imaginary part. The number is a complex number that is a "complex number that is not real".

b. 8 is the real part and 0 is the imaginary part. The number is a complex number that is "real".

c. 0 is the real part and 5 is the imaginary part. The number is a complex number that is "imaginary".

Example 3:
a. 7 + 4i
b. 8 + 14i

Example 4:
a. 31 – I
b. 7 + 21i

Example 5:
a. –6
b. $-\sqrt{30}$

Example 6:
a. 49
b. 58
c. 61

Example 7:
a. $\frac{-21i-7}{49}$
b. $\frac{8-27i}{61}$
c. $\frac{22-4i}{50}$

Example 8:
a. i
b. –1
c. –i
d. 1
e. i
f. –I
g. –i
h. 1

Example 9:
a. 1
b. –i
c. 1
d. –i

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