11.1 The Square Root Property and Completing the Square

Review of Quadratic Equations and Functions

Following is a summary of what you have already studied about quadratic equations and quadratic functions.

1. A quadratic equation in x can be written in the standard form

   $$ax^2 + bx + c = 0, \quad a \neq 0$$

2. Some quadratic equations can be solved by factoring.
3. The polynomial function of the form

   $$f(x) = ax^2 + bx + c, \; a \neq 0$$

   is a quadratic function. Graphs of quadratic functions are called parabolas. The shape of the graph is cuplike.
4. The real solutions of $ax^2 + bx + c = 0$ correspond to the x-intercepts for the graph of the quadratic function $f(x) = ax^2 + bx + c$.

Example 1: Consider the quadratic function given by

$$f(x) = 2x^2 - 9x + 4.$$

Find the x-intercepts of the graph.

The Square Root Property

If u is an algebraic expression and d is a nonzero real number, then $u^2 = d$ has exactly two solutions:

$$\text{If } u^2 = d, \text{ then } u = \sqrt{d} \text{ or } u = -\sqrt{d}.$$

Equivalently,

$$\text{If } u^2 = d, \text{ then } u = \pm\sqrt{d}.$$

This property can be used to solve quadratic equations that are written in the form $u^2 = d$.

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Example 2: Solve the following quadratic equations by using the square root property.

$a.\ 5x^2 = 125$

$x^2 = 25$ Divide both sides by 5 to isolate x^2.

$x = +\sqrt{25}$, or $x = -\sqrt{25}$ Apply the square root property.

$x = 5$, or $x = -5$ Simplify.

Answer: The solution set is $\{-5, 5\}$.

$b.\ 7x^2 = 64$

$c.\ 2x^2 - 11 = 0$

$d.\ 3x^2 + 18 = 0$

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$e.\ (x-3)^2=36$

$f.\ (2x+7)^2=10$

$g.\ (4x-3)^2=-9$

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Completing the Square

How do you solve a quadratic if the quadratic can't be factored, is not given in the form $u^2 = d$, and can't be rewritten in the $u^2 = d$ form by transposing terms in the equation? Interestingly enough, all quadratics can be rewritten in the $u^2 = d$ form by using a technique called "completing the square".

Finding the Term Needed to Complete the Square If $x^2 + bx$ is a binomial, then by adding $\left(\frac{b}{2}\right)^2$, which is the square of half of the coefficient of x, a perfect square trinomial will result. That is, $$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example 3: Find the term needed to complete the square.

a. $x^2 + 2x$

b. $x^2 + 5x$

c. $x^2 - 7x$

Example 4: Determine if each of the following is a perfect square trinomial. Factor each perfect square trinomial.

a. $x^2 + 6x + 9$

b. $x^2 + 5x + \frac{25}{4}$

c. $x^2 + \frac{1}{2}x + \frac{1}{16}$

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Solving Quadratic Equations by Completing the Square

To solve a quadratic equation by completing the square:

1. Rewrite the equation in the form $x^2 + bx = c$.
2. Add to both sides the term needed to complete the square.
3. Factor the perfect square trinomial, and solve the resulting equation by using the square root property.

Example 5: Solve by completing the square.

a. $x^2 + 6x - 7 = 0$

$x^2 + 6x \quad = 7$ Add 7 to both sides.

$x^2 + 6x + 9 = 7 + 9$ Add $\left(\frac{b}{2}\right)^2$ *to* both sides.

$(x+3)^2 = 16$ Factor the left side.

Now, use the square root property to complete the solution.

b. $x^2 + 8x + 5 = 0$

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$c.\ 2x^2 + 8x + 5 = 0$ (Hint: You must divide by 2 before you complete the square.)

Compound Interest Applied Problems

Suppose that an amount of money, P, is invested at interest rate r, compounded annually. In t years, the amount, A, or balance, in the account is given by the formula

$$A = P(1+r)^t$$

Example 6: You invested $3000 in an account whose interest is compounded annually. After 2 years, the amount, or balance, in the account is $4320. Find the annual interest rate.

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Applied Problems Using the Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the legs have length a and b, and the hypotenuse has length c, then

$$a^2 + b^2 = c^2$$

Example 7: A 50-foot supporting wire is to be attached to an antenna. The wire is anchored 20 feet from the base of the antenna. How high up the antenna is the wire attached? Express your answer in simplified radical form, and then find a decimal approximation to the nearest tenth of a foot.

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Using the Distance Formula

The distance, d, between the points (x_1, y_1) and (x_2, y_2), is given by the Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 8:
a. Find the distance between the points (6,–1) and (9,3).

b. Find the exact distance between the given points, and then use your calculator to approximate the distance to two decimal places.
(7,4) and (–1,–5)

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Answers Section 11.1

Example 1: $(4,0)$, $\left(\frac{1}{2},0\right)$

Example 2:

a. $\{-5,5\}$

b. $\left\{-\frac{8\sqrt{7}}{7},\frac{8\sqrt{7}}{7}\right\}$

c. $\left\{-\frac{\sqrt{22}}{2},\frac{\sqrt{22}}{2}\right\}$

d. $\{-i\sqrt{6},i\sqrt{6}\}$

e. $\{9,-3\}$

f. $\left\{\frac{\sqrt{10}-7}{2},\frac{-\sqrt{10}-7}{2}\right\}$

g. $\left\{\frac{3+3i}{4},\frac{3-3i}{4}\right\}$

Example 3:

a. 1

b. $\frac{25}{4}$

c. $\frac{49}{4}$

Example 4:

a. $(x+3)^2$

b. $\left(x+\frac{5}{2}\right)^2$

c. $\left(x+\frac{1}{4}\right)^2$

Example 5:

a. $\{-7,1\}$

b. $\{-4-\sqrt{11},-4+\sqrt{11}\}$

c. $\left\{\frac{-4-\sqrt{6}}{2},\frac{-4+\sqrt{6}}{2}\right\}$

Example 6: The annual interest rate is 20%.

Example 7: The wire is attached 45.8 feet up the antenna.

Example 8:

a. 5

b. $\sqrt{145}\approx 12.04$

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