11.2 The Quadratic Formula

Solving Quadratic Equations Using the Quadratic Formula.
By solving the general quadratic equation $ax^2 + bx + c = 0$ using the method of completing the square, one can derive the quadratic formula. The quadratic formula can be used to solve any quadratic equation.

The Quadratic Formula
The solutions of a quadratic equation in standard form $ax^2 + bx + c = 0$, with $a \neq 0$, are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1: Solve the given quadratic equations by using the quadratic formula.

$a.\ \ 2x^2 = 6x - 1$

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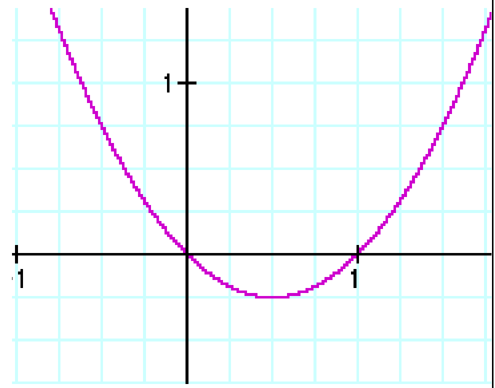
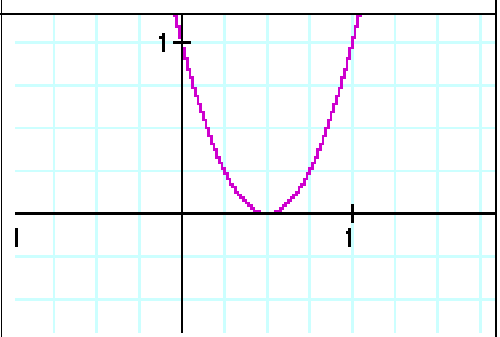
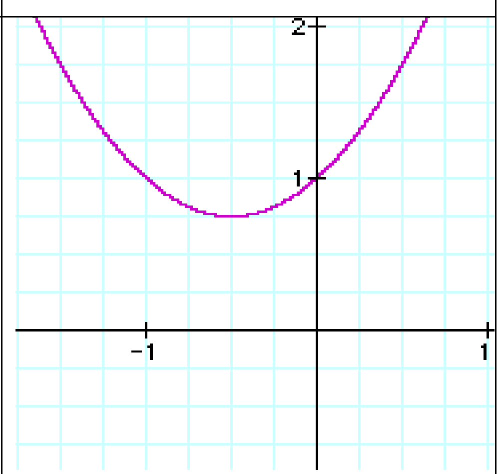
b. $3x^2 + 5 = -6x$

c. $3 + \dfrac{4}{x} = -\dfrac{2}{x^2}$

The Discriminant

The quantity b^2-4ac, which appears under the radical sign in the quadratic formula, is called the discriminant. The value of the discriminant for a given quadratic equation can be used to determine the kinds of solutions that the quadratic equation has.

The Discriminant and the Kinds of Solutions to $ax^2+bx+c=0$

Value of the Discriminant	**Kinds of Solutions**	**Graph of $y=ax^2+bx+c$**
$b^2-4ac>0$	Two unequal real solutions. Graph crosses the x-axis twice.	
$b^2-4ac=0$	One real solution (a repeated solution) that is a real number. Graph touches the x-axis.	
$b^2-4ac<0$	Two complex solutions that are not real and are complex conjugates of one another. Graph does not touch or cross the x-axis.	

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Example 2: For each equation, compute the discriminant. Then determine the number and types of solutions.

$a.\ \ x^2 + 6x + 9 = 0$

$b.\ \ 2x^2 - 7x - 4 = 0$

$c.\ \ 3x^2 - 2x + 4 = 0$

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Determining Which Method to Use To Solve a Quadratic Equation

Use the following chart as a guide to help you in finding the most efficient method to use to solve a given quadratic equation.

Method 1: $ax^2+bx+c=0$ and ax^2+bx+c can be factored easily	Factor and use the zero-product principle.	$\text{Ex}: 2x^2-3x+1=0$ $(2x-1)(x-1)=0$ $x = \frac{1}{2},\ x=1$
Method 2: $ax^2+c=0$ The quadratic equation has no x-term.	Solve for x^2 and use the square root property.	$\text{Ex}: 2x^2-18=0$ $2x^2=18$ $x^2=9$ $x=\pm 3$
Method 3: $u^2=d$ and u is a first degree polynomial	Use the square root property	$\text{Ex}: (2x-1)^2=9$ $2x-1=\pm 3$ $2x=1\pm 3$ $x=2,-1$
Method 4: $ax^2+bx+c=0$ and ax^2+bx+c cannot be factored or the factoring is too difficult	Use the quadratic formula.	$\text{Ex}: x^2+x+2=0$ $x=\frac{-1\pm\sqrt{(1)^2-4(1)(2)}}{2(1)}$ $x=\frac{-1\pm i\sqrt{7}}{2}$

Example 3: Match each equation with the proper technique given in the chart. Place the equation in the chart and solve it.

a. $(2x\text{-}3)^2=7$

b. $4x^2=-9$

c. $2x^2+3x=1$

d. $2x^2+3x=-1$

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Writing Quadratic Equations from Solutions

To find a quadratic equation that has a given solution set $\{a,b\}$, write the equation $(x-a)(x-b)=0$ and multiply and simplify.

Example 4: Find a quadratic equation that has the given solution set.

a. $\{-2,5\}$

b. $\left\{-\frac{1}{2},\frac{2}{5}\right\}$

c. $\{3i,-3i\}$

Applications of Quadratic Equations

Use your calculator to assist you in solving the following problem. Round your answer(s) to the nearest whole number.

Example 5: The number of fatal vehicle crashes per 100 million miles, f(x), for drivers of age x can be modeled by the quadratic function

$$f(x)=0.013x^2-1.19x+28.24$$

What age groups are expected to be involved in 3 fatal crashes per 100 million miles driven?

Example 6: Use your calculator to approximate the solutions of the following quadratic equations to the nearest tenth.

a. $2.1x^2 - 3.8x - 5.2 = 0$

b. $4.5x^2 - 10.2x + 1.3 = 0$

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Answers Section 11.2

Example 1:

a. $\left\{\frac{3+\sqrt{7}}{2},\frac{3-\sqrt{7}}{2}\right\}$

b. $\left\{\frac{-3+i\sqrt{6}}{3},\frac{-3-i\sqrt{6}}{3}\right\}$

c. $\left\{\frac{-2+i\sqrt{2}}{3},\frac{-2-i\sqrt{2}}{3}\right\}$

Example 2:

a. value of discriminant is 0, one real solution.

b. value of discriminant is 81, two real solutions.

c. value of discriminant is -44, two complex solutions that are not real and are complex conjugates of each other.

Example 3:

a. Method 3. $\left\{\frac{3+\sqrt{7}}{2},\frac{3-\sqrt{7}}{2}\right\}$

b. Method 2. $\left\{-\frac{3i}{2},\frac{3i}{2}\right\}$

c. Method 4. $\left\{\frac{-3+\sqrt{17}}{4},\frac{-3-\sqrt{17}}{4}\right\}$

d. Method 1. $\left\{-\frac{1}{2},-1\right\}$

Example 4:

a. $x^2-3x-10=0$

b. $10x^2+x-2=0$

c. $x^2+9=0$

Example 5: The age groups that can be expected to be involved in 3 fatal crashes per 100 million miles driven are ages 33 and 58.

Example 6:

a. 2.7 and –0.9

b. 0.1 and 2.1

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