# 11.4 Equations in Quadratic Form

## Quadratic Form

An equation that is quadratic in form is an equation that can be expressed as a quadratic equation using an appropriate substitution.
In symbols:

- equation in quadratic form $ax^{2n} + bx^{n} + c = 0$
- substitution $t = x^{n}$
- resulting quadratic equation: $at^{2} + bt + c = 0$

Example 1: Choose an appropriate substitution and write the given equations as a quadratic equation in t.

a. $x^{4} - 10x^{2} + 9 = 0$

b. $x^{\frac{1}{2}} - 10x^{\frac{1}{4}} + 9 = 0$

c. $2x - \sqrt{x} - 10 = 0$

d. $(x+3)^{2} + 7(x+3) - 18 = 0$

e. $x^{-2} - x^{-1} - 6 = 0$

## Solving Equations That Are Quadratic in Form

To solve equations that are quadratic in form:
1. Choose an appropriate substitution and rewrite the original equation as a quadratic equation in t.
2. Solve the quadratic equation in t.
3. Use the original substitution and the t-solutions to find the x-solutions.
4. Check your solutions. If at any time during the solution process you raised both sides of an equation to an even power, a check is required, since raising both sides to an even power may introduce extraneous solutions.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

Example 2: Solve the given equations.

$a.\ \ x^4 - 10x^2 + 9 = 0$

*b.* $x^6 - 10x^3 + 9 = 0$

c. $x^{\frac{1}{2}} - 10x^{\frac{1}{4}} + 9 = 0$

$d.\ 2x - \sqrt{x} - 10 = 0$

e. $(x+3)^2 + 7(x+3) - 18 = 0$

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$f.\ x^{-2} - x^{-1} - 6 = 0$

**Finding x-intercepts of a Quadratic-in-Form Function**

To find x-intercepts of a function, substitute 0 for f(x) and solve the resulting equation.

Example 3: Find the x-intercepts of the given functions.

$a.\ f(x) = x^4 - 13x^2 + 36$

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$b.\ f(x) = x^{\frac{2}{3}} - 9x^{\frac{1}{3}} + 8$

## Answers Section 11.4

Example 1:

a. Let t=$x^2$. $t^2 - 10t + 9 = 0$

b. Let t=$x^{\frac{1}{4}}$. $t^2 - 10t + 9 = 0$

c. Let t=$\sqrt{x}$. $2t^2 - t - 10 = 0$

d. Let t=$(x+3)$. $t^2 + 7t - 18 = 0$

e. Let t=$x^{-1}$. $t^2 - t - 6 = 0$

Example 2:

a. $\{-3,-1,1,3\}$

b. $\{1,\sqrt[3]{9}\}$

c. $\{1,6561\}$

d. $\left\{\frac{25}{4}\right\}$

e. $\{-1,-12\}$

f. $\left\{-\frac{1}{2},\frac{1}{3}\right\}$

Example 3:

a. $x$-intercepts are $(\pm 2,0)$ and $(\pm 3,0)$

b. x-intercepts are (1,0) and (512,0)

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