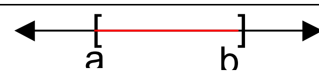
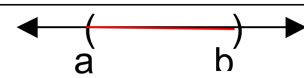
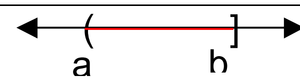
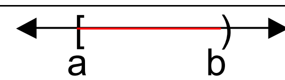
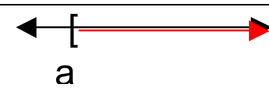
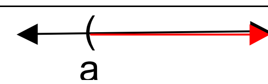
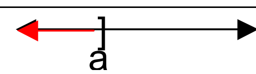
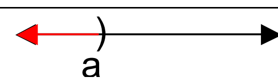
11.5 Polynomial and Rational Inequalities

Interval Notation-Review

Intervals can be expressed in interval notation, set-builder notation or graphically on the number line. The following chart shows the different notations. You may use interval notation, inequality notation or set-builder notation to depict intervals.

Let a and b represent two real numbers with a < b.

Type of Interval	Interval Notation	Set-Builder Notation	Graph on the Number Line
Closed Interval	[a,b]	$\{x \mid a \leq x \leq b\}$	
Open Interval	(a,b)	$\{x \mid a < x < b\}$	
Half-Open Interval	(a,b]	$\{x \mid a < x \leq b\}$	
Half-Open Interval	[a,b)	$\{x \mid a \leq x < b\}$	
Interval That Is Not Bounded on the Right	[a,∞)	$\{x \mid a \leq x < \infty\}$ or $\{x \mid x \geq a\}$	
Interval That Is Not Bounded on the Right	(a,∞)	$\{x \mid a < x < \infty\}$ or $\{x \mid x > a\}$	
Interval That Is Not Bounded on the Right	(-∞,a]	$\{x \mid -\infty < x \leq a\}$ or $\{x \mid x \leq a\}$	
Interval That Is Not Bounded on the Right	(-∞,a)	$\{x \mid -\infty < x < a\}$ or $\{x / x < a\}$	
Interval That Is Not Bounded on the Right	(-∞,∞)	$\{x \mid -\infty < x < \infty\}$ or {x|x is a real no.}	

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 1: Write each inequality in interval notation.

a. $x \geq -3$

b. $5 < x < \infty$

c. $x < 7$

d. $-4 \leq x < \infty$

Example 2: Write each interval in set-builder notation.

a. $[-4,\infty)$

b. $(-\infty,5)$

c. $(-7, -2]$

d. $(-1,4)$

Example 3: Graph each interval on the number line.

a. $[-4,\infty)$

b. $(-\infty,5)$

c. $(-3, -2]$

e. $[-2,2]$

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Polynomial Inequalities

Definition of a Polynomial Inequality

A polynomial inequality is any inequality that can be put in one of the forms

$$f(x) > 0 \qquad f(x) \geq 0$$

$$f(x) < 0 \qquad f(x) \leq 0$$

where f(x) is a polynomial. Recall that a polynomial is a single term or the sum or difference of terms all of which have variables in numerators only and which have only whole number exponents.

Solving Polynomial Inequalities

Solutions to a polynomial inequality

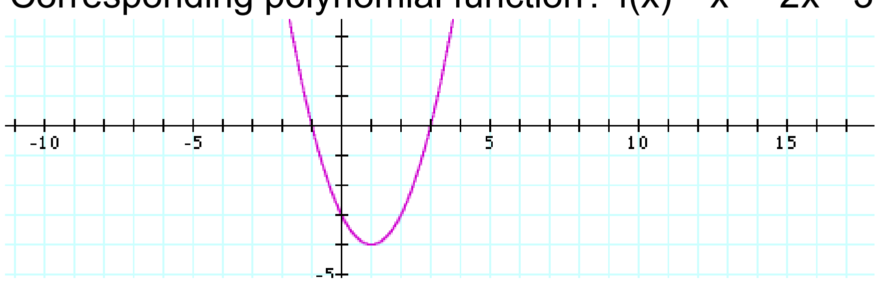
- $f(x) > 0$ consists of the x-values for which the graph of f(x) lies above the x-axis.
- $f(x) \geq 0$ consists of the x-values for which the graph of f(x) lies above the x-axis or is touching or crossing the x-axis.
- $f(x) < 0$ consists of the x-values for which the graph lies below the x-axis.
- $f(x) \leq 0$ consists of the x-values for which the graph lies below the x-axis or is touching or crossing the x-axis.

Thus the x-values at which the graph moves from below-to-above or above-to-below the x-axis are crucial values. These x-values are the solutions to the equation $f(x) = 0$. They are **boundary points** for the inequality.

Example 4: Solve the given inequality by using the graph of the corresponding polynomial function.

Inequality: $x^2 - 2x - 3 \geq 0$

Corresponding polynomial function: $f(x) = x^2 - 2x - 3$



Solution: ?

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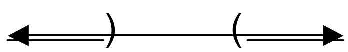
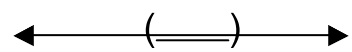
Procedure for Solving Polynomial Inequalities Algebraically

1. Express the inequality in the standard form $f(x) > 0$ or $f(x) < 0$.
2. Solve the equation $f(x) = 0$. The real solutions are the boundary points.
3. Locate these boundary points on a number line, thereby dividing the number line into test intervals. If the inequality symbol is “<” or “>”, exclude all boundary points from the test intervals.
4. Choose one representative number within each test interval. If substituting that value into the original inequality produces a true statement, then all real numbers in the test interval belong to the solution set. If substituting that value into the original inequality produces a false statement, then no real number in the test interval belongs to the solution set.
5. Write the solution set, selecting the interval(s) that produced a true statement. The graph of the solution set on a number line usually appears as


or


Example 5: Solve the given inequality.

a. $x^2 - 2x - 3 \geq 0$

Boundary points:

Graph boundary points on a number line:

Identify intervals and complete chart:

Intervals	Representative Number	Substitute into Inequality	Conclusion
$(-\infty,-1)$	-2	$(-2)^2 - 2(-2) - 3 \geq 0$ $5 \geq 0$	True. Thus $(-\infty,-1]$ belongs to sol’n set

Write the solution in interval notation.

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b. $(x+1)(x-2)(x+4)<0$

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Solving Rational Inequalities

A rational inequality is an inequality that can be put in one of the forms:

$$\frac{P(x)}{Q(x)} \leq 0 \quad \frac{P(x)}{Q(x)} \geq 0$$

$$\frac{P(x)}{Q(x)} < 0 \quad \frac{P(x)}{Q(x)} > 0$$

Procedure for Solving Rational Inequalities:

1. Write the inequality so that one side is zero and the other side is a single quotient.
2. Find the boundary points by setting the numerator and the denominator equal to zero.
3. Locate the boundary points on a number line.
4. Use the boundary points to establish test intervals. If the inequality symbol is "<" or ">", exclude all boundary points from the test intervals. Also, exclude any boundary points that make the denominator equal to zero.
5. Take one representative number within each test interval and substitute that number into the original inequality to determine if the inequality is true or false at that representative number.
5. The solution set consists of the intervals that produced a true statement.

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Example 6: Solve the given inequality. Write your answers in interval notation.

a. $\frac{x+5}{x+2} < 0$

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b. $\frac{x}{x+2} \geq 2$

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Applications

Quadratic and rational inequalities can be used to solve applied problems.

Example 7: A model rocket is launched from the top of a cliff 80 feet above sea level. The function

$$s(t) = -16t^2 + 64t + 80$$

models the rocket's height above the water, s(t), in feet, t seconds after it was launched. During which time period will the rocket's height exceed that of the cliff?

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Answers Section 11.5

Example1:

a. $[-3, \infty)$

b. $(5, \infty)$

c. $(-\infty, 7)$

d. $[-4, \infty)$

Example 2:

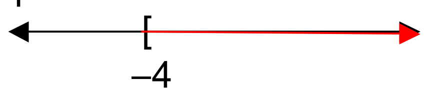
a. $\{x|x \geq -4\}$ or $\{x| -4 \leq x < \infty\}$

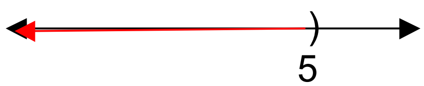
b. $\{x|x < 5\}$ or $\{x| -\infty \leq x < 5\}$

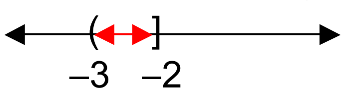
c. $\{x| -7 < x \leq -2\}$

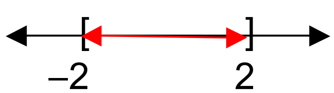
d. $\{x| -1 < x < 4\}$

Example 3:

a. 

b. 

c. 

d. 

Example 4: $(-\infty, -1] \cup [3, \infty)$

Example 5:

a. $(-\infty, -1] \cup [3, \infty)$

b. $(-\infty, -4) \cup (-1, 2)$

Example 6:

a. $(-5, -2)$

b. $[-4, -2)$

Example 7: (0,4) The rocket is above the cliff between 0 and 4 seconds.

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