12.1 Exponential Functions

Definition of the Exponential Function

The exponential function f with base b is defined by

$$f(x) = b^x \text{ or } y = b^x$$

where b is a positive constant other than 1 and x is any real number.

Example 1: The exponential function $f(x) = 13.49(0.967)^x - 1$ describes the number of 0-rings expected to fail, f(x), when the temperature is x°F. On the morning the Challenger was launched, the temperature was 31°F, colder than any previous experience. Find the number of 0-rings expected to fail at this temperature.

Graphing Exponential Functions

Characteristics of Exponential Functions of the Form $f(x) = b^x$

1. The domain of $f(x) = b^x$ consists of all real numbers. The range consists of all positive real numbers.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point (0,1) because $f(0) = b^0$ = 1. The y-intercept is (0,1).
3. The graph of $f(x) = b^x$ may be either strictly increasing or strictly decreasing:
   - If b > 1, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
   - If b < 1, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
4. The graph of $f(x) = b^x$ approaches, but does not cross, the x-axis. The x-axis, or y = 0, is a horizontal asymptote.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

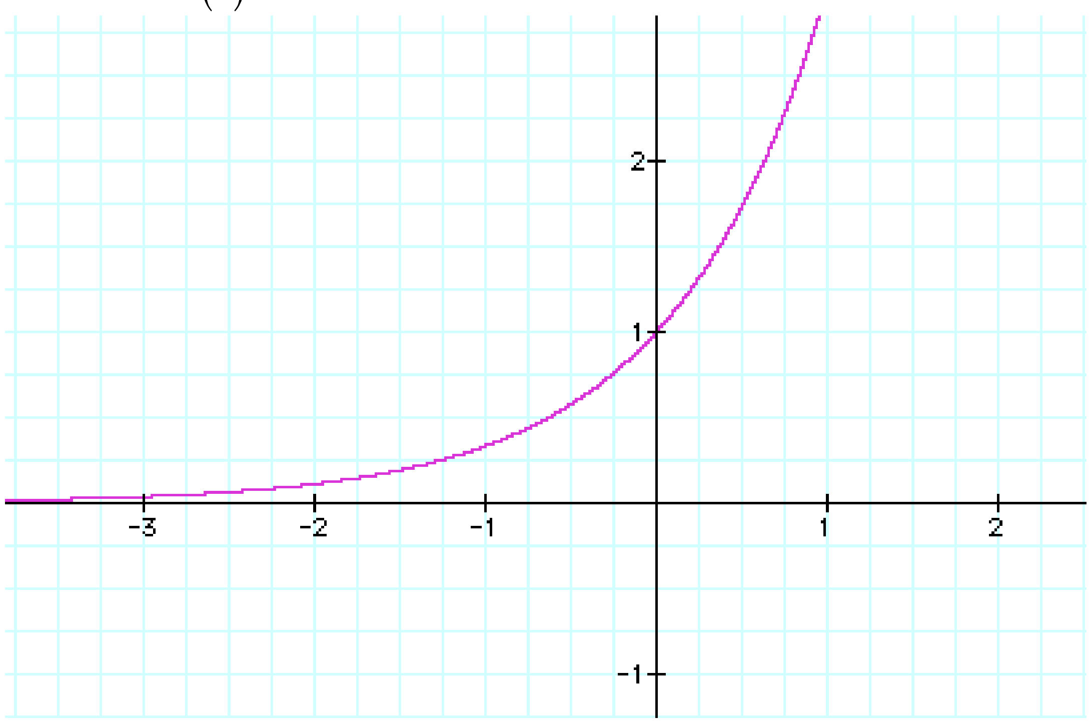
Example 2: Graph $f(x) = 3^x$

1. The domain is all real numbers. The range is all positive real numbers.
2. The y-intercept is (0,1).
3. Since b > 1, the graph is increasing.
4. y = 0 is a horizontal asymptote

Table of Coordinates

x	f(x)	(x, f(x))
–2	$f(-2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$	$\left(-2, \frac{1}{9}\right)$
–1	$f(-1) = 3^{-1} = ?$	
0	$f(0) = 3^0 = ?$	
1	$f(1) = ?$	
2		

$f(x) = 3^x$



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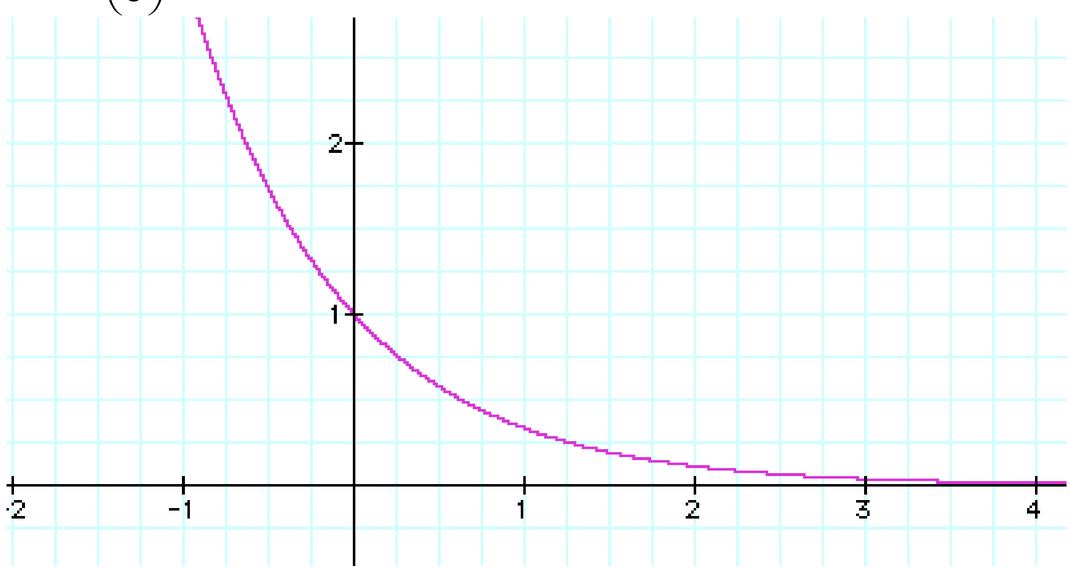
Example 3: Graph $f(x)=\left(\frac{1}{3}\right)^x$

1. The domain is all real numbers. The range is all positive real numbers.
2. The y-intercept is (0,1).
3. Since b < 1, the graph is decreasing.
4. y = 0 is a horizontal asymptote

Table of Coordinates

x	f(x)	(x, f(x))
–2	$f(-2)=\left(\frac{1}{3}\right)^{-2}=\frac{1}{3^{-2}}=9$	$(-2,9)$
–1	$f(-1)=\left(\frac{1}{3}\right)^{-1}=\frac{1}{3^{-1}}=?$	
0	$f(0)=\left(\frac{1}{3}\right)^{0}=?$	
1	$f(1)=?$	
2		

$f(x)=\left(\frac{1}{3}\right)^x$



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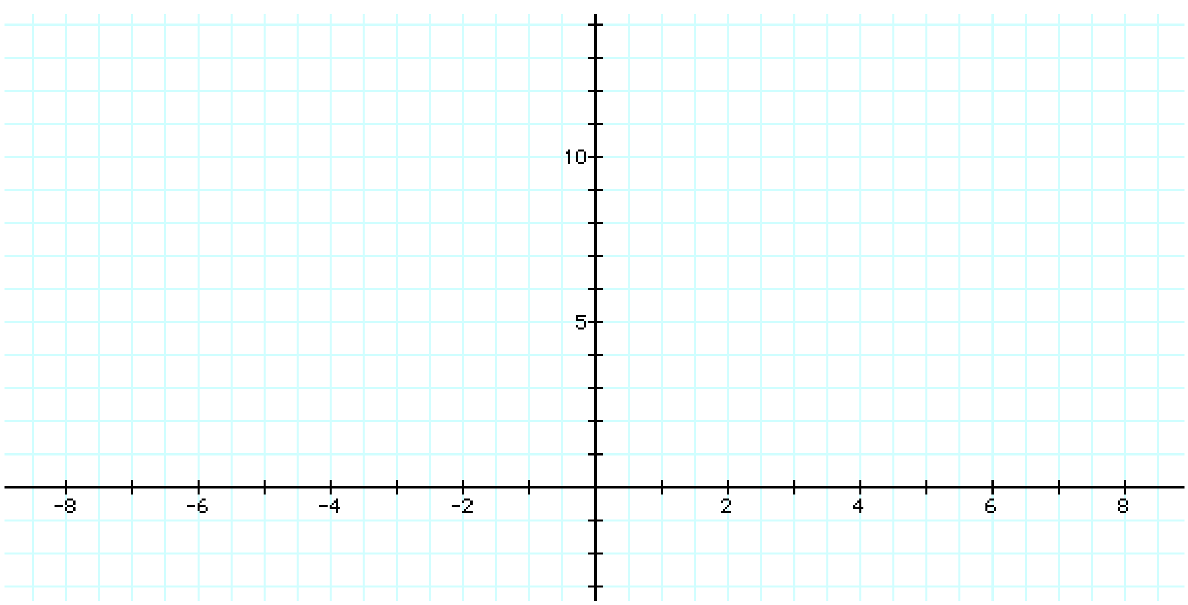
Example 4: Sketch the graph of $f(x) = 2^x$ and $g(x) = 2^{x+1}$ on the same coordinate axes. What is the relationship between the two graphs?



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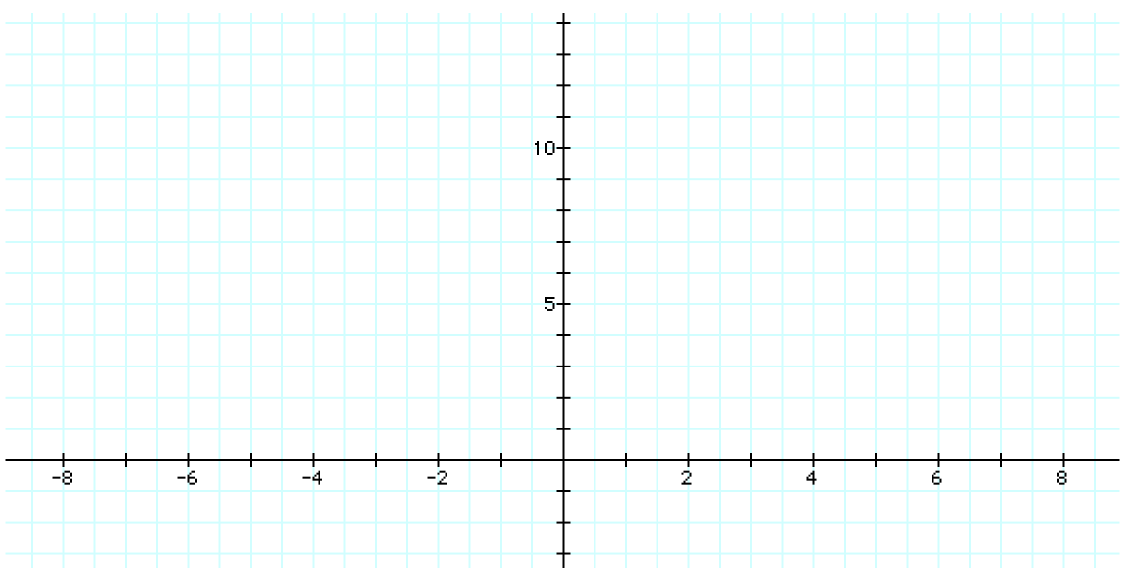
Example 5: Sketch the graph of $f(x) = 2^x$ and $g(x) = 2^x - 3$ on the same coordinate axes. What is the relationship between the two graphs?



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In general, the graph of $g(x) = b^x + d$ is simply the graph of $f(x) = b^x$ shifted d units up, if d > 0, or d units down, if d < 0. The horizontal asymptote of $g(x) = b^x + d$ is y = d.
The graph of $g(x) = b^{x+c}$ is simply the graph of $f(x) = b^x$ shifted c units left, if c > 0, or c units right, if c < 0. The horizontal asymptote of $g(x) = b^{x+c}$ is y = 0.

Example 6: For each of the following pairs of functions, tell how the graph of the second function can be obtained from the graph of the first function. Give the equation of the horizontal asymptote.

$a.\ f(x) = 4^x,\ g(x) = 4^{x-3}$

$b.\ f(x) = 4^x,\ g(x) = 4^{x+3}$

$c.\ f(x) = 4^x,\ g(x) = 4^x + 2$

$d.\ f(x) = 4^x,\ g(x) = 4^x - 3$

$e.\ f(x) = 4^x,\ g(x) = 4^{x+1} - 2$

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The Natural Base e

An irrational number, symbolized by the letter e, appears as a base in many applied exponential functions. This irrational number is approximately equal to 2.72. The number e is called the natural base, and the function $f(x) = e^x$ is called the natural exponential function.

Example 7: The function $f(x) = 6e^{0.013x}$ describes world population, f(x), in billions, x years after 2000 subject to a growth rate of 1.3% annually. Use the function to find the world population in 2050.

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Compound Interest

Formulas for Compound Interest

After t years, the balance, A, in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compoundings per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$

Example 8: Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% if the money is

a. compounded semiannually

b. compounded monthly

c. compounded continuously.

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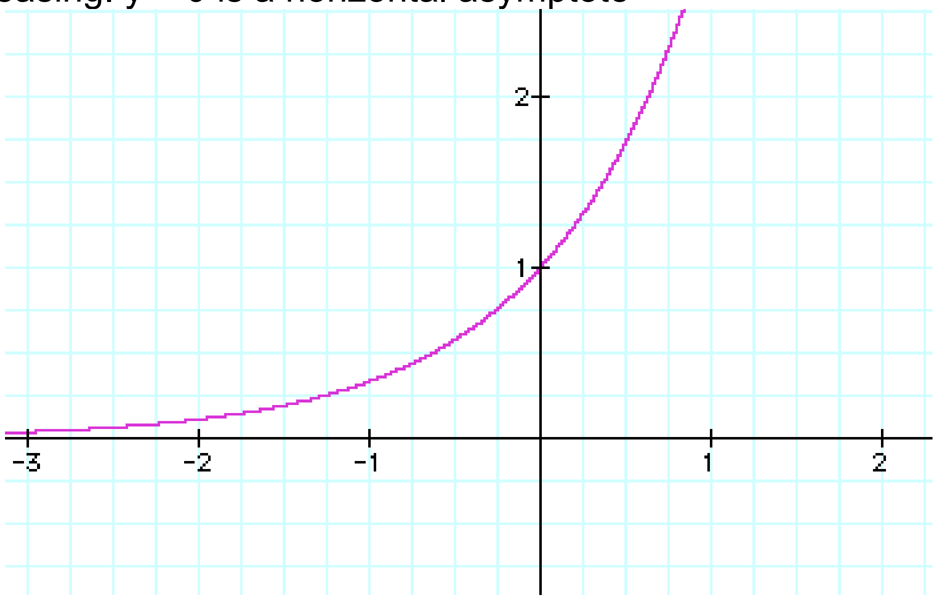
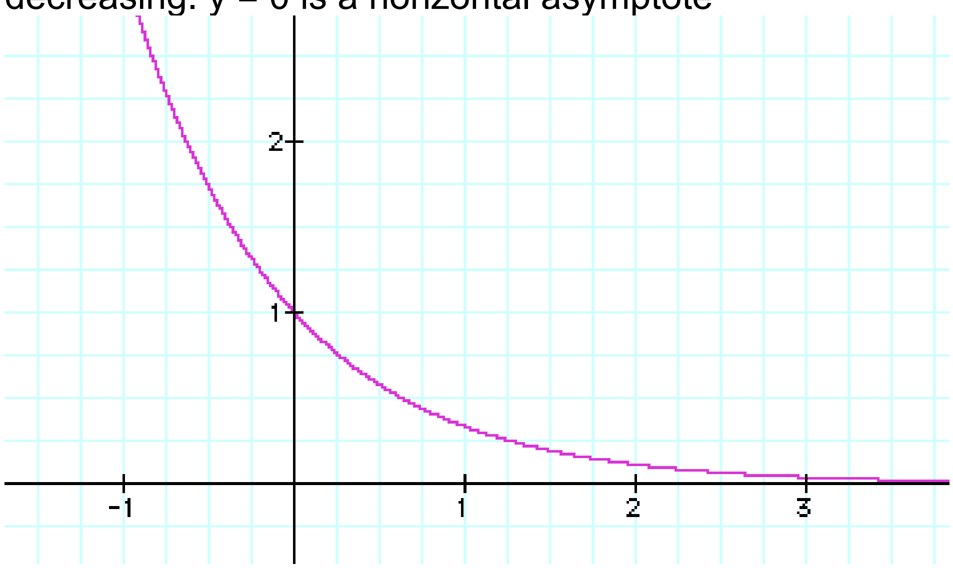
Answers Section 12.1

Example 1: f(31) ≅ 3.77. We would expect about 4 O-rings to fail.

Example 2: The domain is all real numbers. The range is all positive real numbers. The y-intercept is (0,1). Since b > 1, the graph is increasing. y = 0 is a horizontal asymptote



Example 3: The domain is all real numbers.The range is all positive real numbers.The y-intercept is (0,1). Since b < 1, the graph is decreasing. y = 0 is a horizontal asymptote



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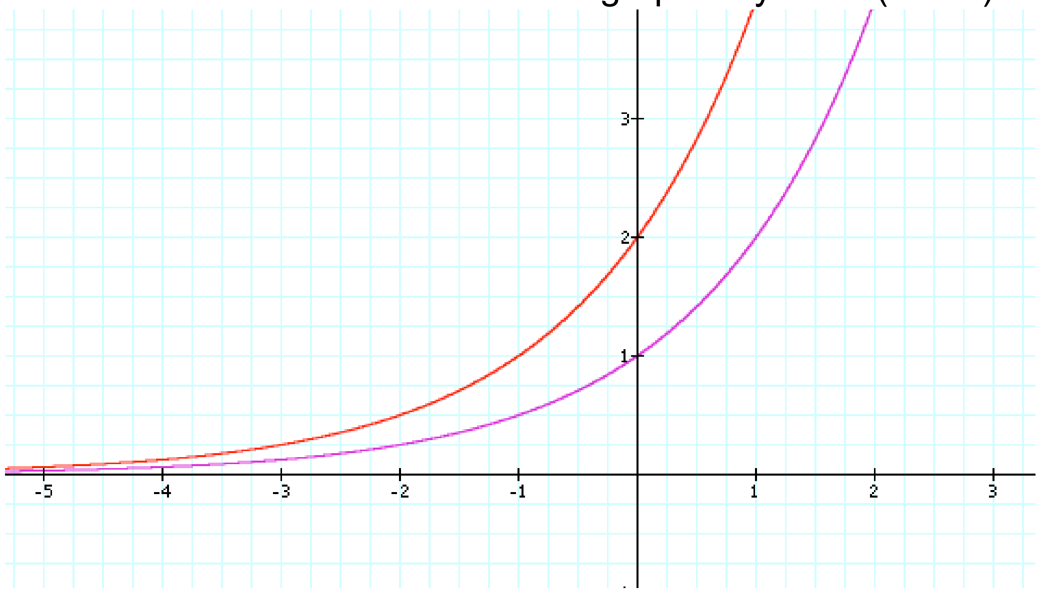
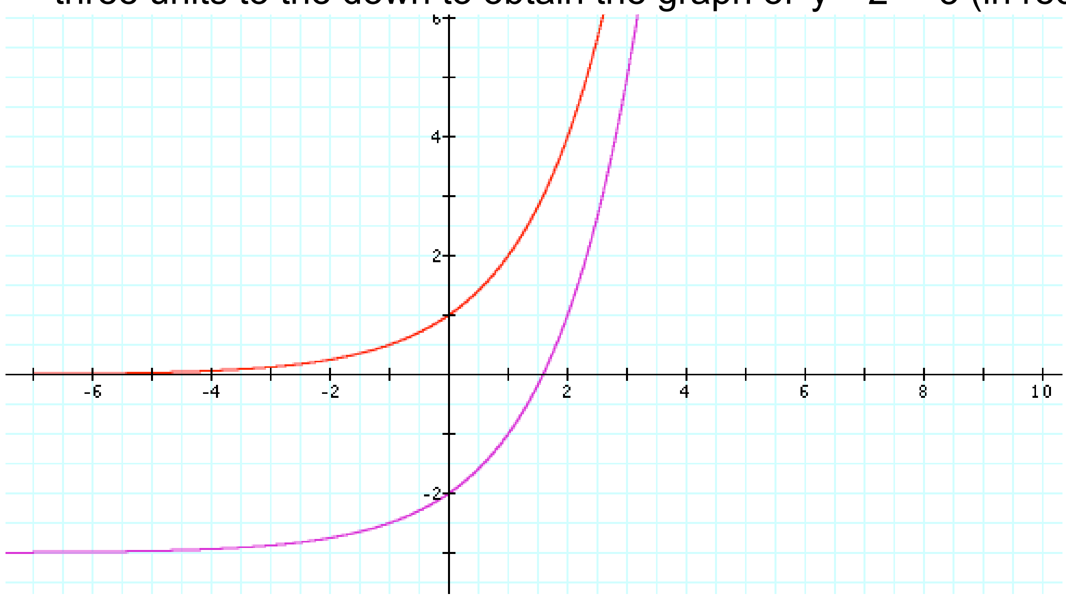
Example 4: Note that the graph of $y = 2^x$ (in purple) has been shifted one unit to the left to obtain the graph of $y = 2^{x+1}$ (in red)



Example 5: Note that the graph of $y = 2^x$ (in purple) has been shifted three units to the down to obtain the graph of $y = 2^x - 3$ (in red)



Example 6:

a. c = –3, shift 3 units right. Equation of horizontal asymptote: y = 0.
b. c = 3, shift 3 units left. Equation of horizontal asymptote: y = 0.
c. d = 2, shift 2 units up. Equation of horizontal asymptote: y = 2.
d. d = –3, shift 3 units down. Equation of horizontal asymptote: y = –3.

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e. $c = 1$ and $d = -2$, shift 1 unit left and 2 units down. Equation of horizontal asymptote: $y = -2$.

Example 7: $f(50) \cong 11.49$. The world population in 2050 is projected to be about 11.5 billion.

Example 8:

a. $9,479.19
b. $9,560.92
c. $9,577.70