12.2 Logarithmic Functions

The Definition of Logarithmic Functions

For $x > 0$ and $b > 0$, $b \neq 1$

$y = \log_b x$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the logarithmic function with base b.

Example 1: Write each equation in its equivalent exponential form.

a. $4 = \log_2 x$

b. $-1 = \log_3 x$

c. $\log_2 8 = y$

Example 2: Write each equation in its equivalent logarithmic form.

a. $2^6 = x$

b. $b^4 = 81$

c. $2^y = 128$

Example 3: Evaluate each of the following.

$a.\ \log_{10} 100$

$b.\ \log_{25} 5$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

$c.\ \log_5 \frac{1}{5}$

$d.\ \log_2 \frac{1}{16}$

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Basic Logarithmic Properties

Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

$\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x$ b raised to the logarithm with base b of a number equals that number.

Example 4: Evaluate each of the following.

a. $\log_8 8$

b. $\log_{1.5} 1.5$

c. $\log_8 1$

d. $\log_{1.7} 1$

e. $\log_8 8^5$

f. $\log_5 5^{2.3}$

g. $7^{\log_7 8.3}$

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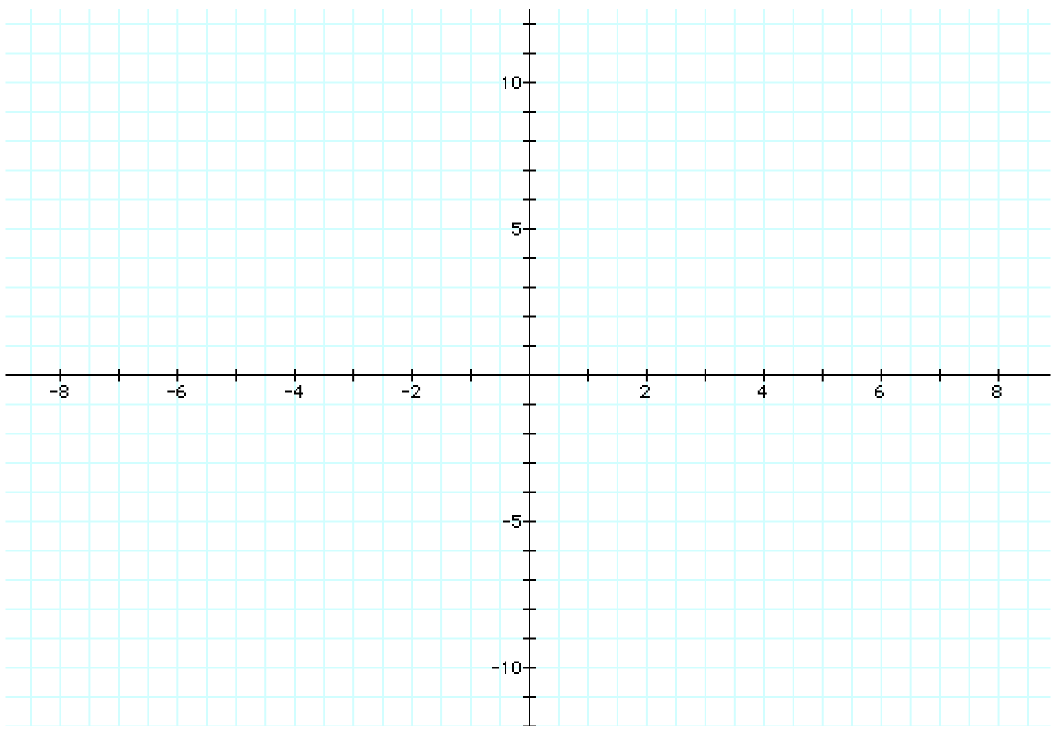
Graphs of Logarithmic Functions

The logarithmic function is the inverse of the exponential function with the same base. Thus the logarithmic function reverses the coordinates of the exponential function. The graph of the logarithmic function is the reflection of the exponential function about the line y=x.

Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_b x$.

1. The x-intercept is 1. There is no y-intercept.
2. The y-axis is a vertical asymptote.
3. If b > 1, the function is increasing. If 0 < b < 1, the function is decreasing.
4. The graph is smooth and continuous. It has no sharp corners or gaps.

Example 5: Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ on the same coordinate system.



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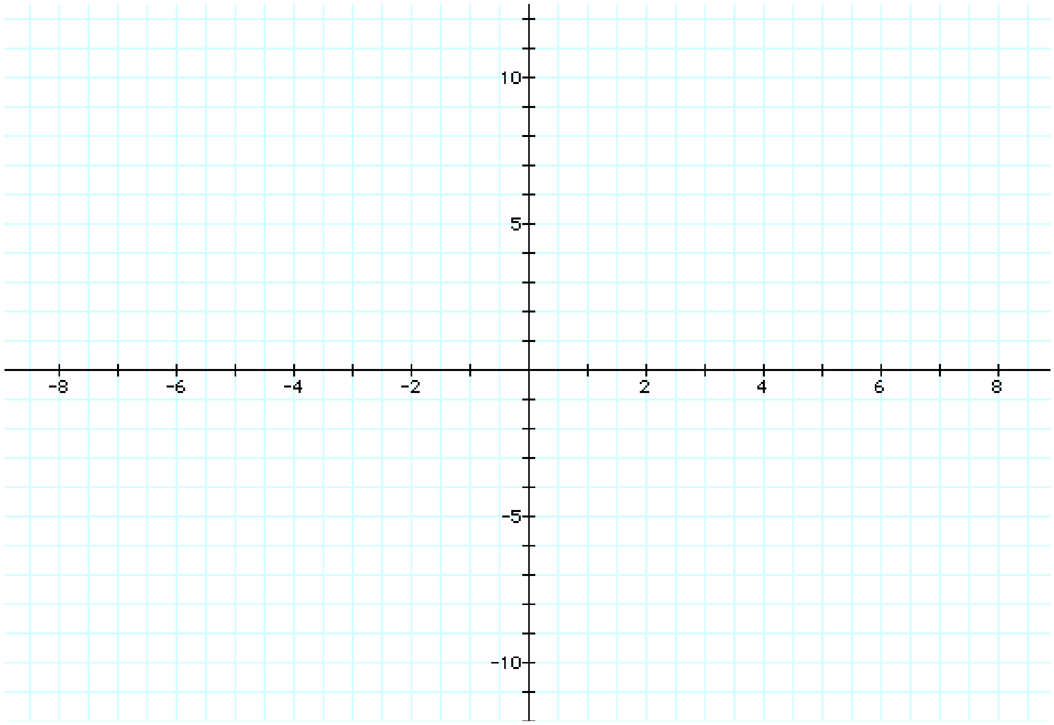
Example 6: Graph $f(x) = \log_{\frac{1}{2}} x$



The Domain of a Logarithmic Function

In the expression $y=\log_b x$, x is the number produced when y is used as an exponent with base b, b > 0. Since b is always positive, x must also be positive. Thus the domain of the logarithmic function is x > 0, or all positive real numbers.

In general:

domain of $f(x) = \log_b(x + c)$ consists of all x for which x + c > 0.

Example 7: Find the domain of the logarithmic function.

$f(x) = \log_5(x - 7)$

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Common Logarithms

The logarithmic function with base 10 is called the common logarithmic function.

The function $f(x) = \log_{10} x$ is usually expressed simply as $f(x) = \log x$.

Most calculators have a "log" key that can be used to perform calculations with base-10 logarithms.

Logarithmic functions may be used to model some growth functions that start with rapid growth and then level off.

Example 8: The percentage of adult height attained by a girl who is x years old can be modeled by

$$f(x) = 62 + 35\log(x-4)$$

where x represents the girl's age (from 5 to 15) and f(x) represents the percentage of adult height. What percentage of adult height has a 10-year old girl attained?

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Natural Logarithms

The logarithmic function with base e is called the natural logarithmic function.

The function $f(x) = \log_e x$ is usually expressed simply as $f(x) = \ln x$.

Most calculators have an "ln" key that can be used to perform calculations with base-e logarithms.

Example 9: Find the domain of the function.

a. $f(x) = \ln(x+3)$

Example 10: Simplify each expression.

a. $\ln e$

b. $\ln e^4$

c. $e^{\ln 7}$

d. $\ln e^{1.5x}$

e. $e^{\ln 3x}$

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Summary of Properties of Logarithms

General Properties	Common Logarithms	Natural Logarithms
1. $\log_b 1 = 0$	$\log 1 = 0$	$\ln 1 = 0$
2. $\log_b b = 1$	$\log 10 = 1$	$\ln e = 1$
3. $\log_b b^x = x$	$\log 10^x = x$	$\ln e^x = x$
4. $b^{\log_b x} = x$	$10^{\log x} = x$	$e^{\ln x} = x$

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Answers Section 12.2

Example 1:

a. $x = 2^4$

b. $x = 3^{-1}$

c. $2^y = 8$

Example 2:

a. $\log_2 x = 6$

b. $\log_b 81 = 4$

c. $\log_2 128 = y$

Example 3:

a. $\log_{10} 100 = 2$

b. $\log_{25} 5 = \frac{1}{2}$

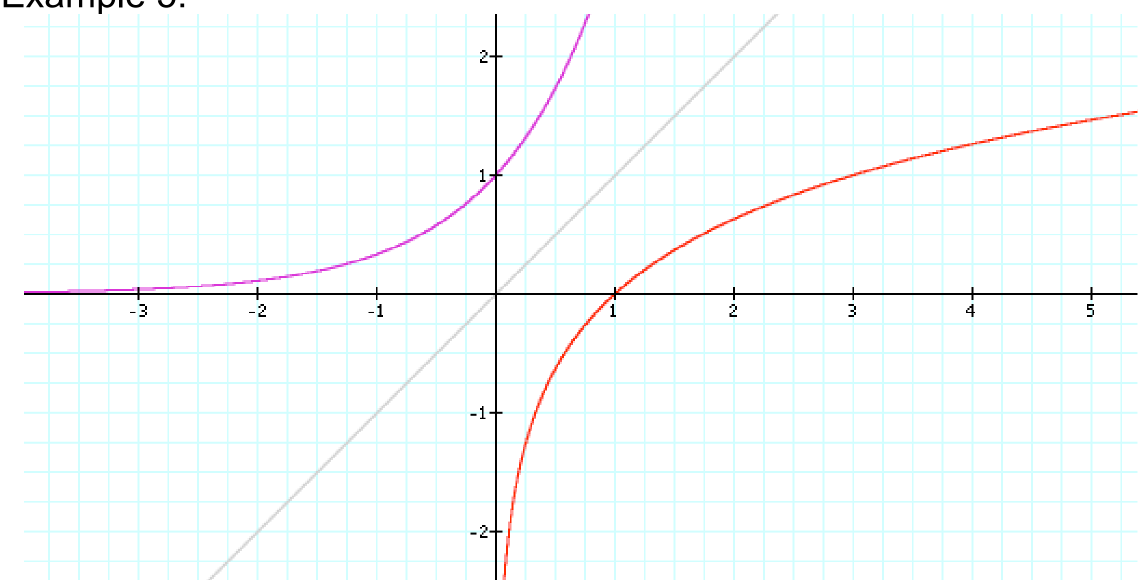
c. $\log_5 \frac{1}{5} = -1$

d. $\log_2 \frac{1}{16} = -4$

Example 4:

a. 1
b. 1
c. 0
d. 0
e. 5
f. 2.3
g. 8.3

Example 5:



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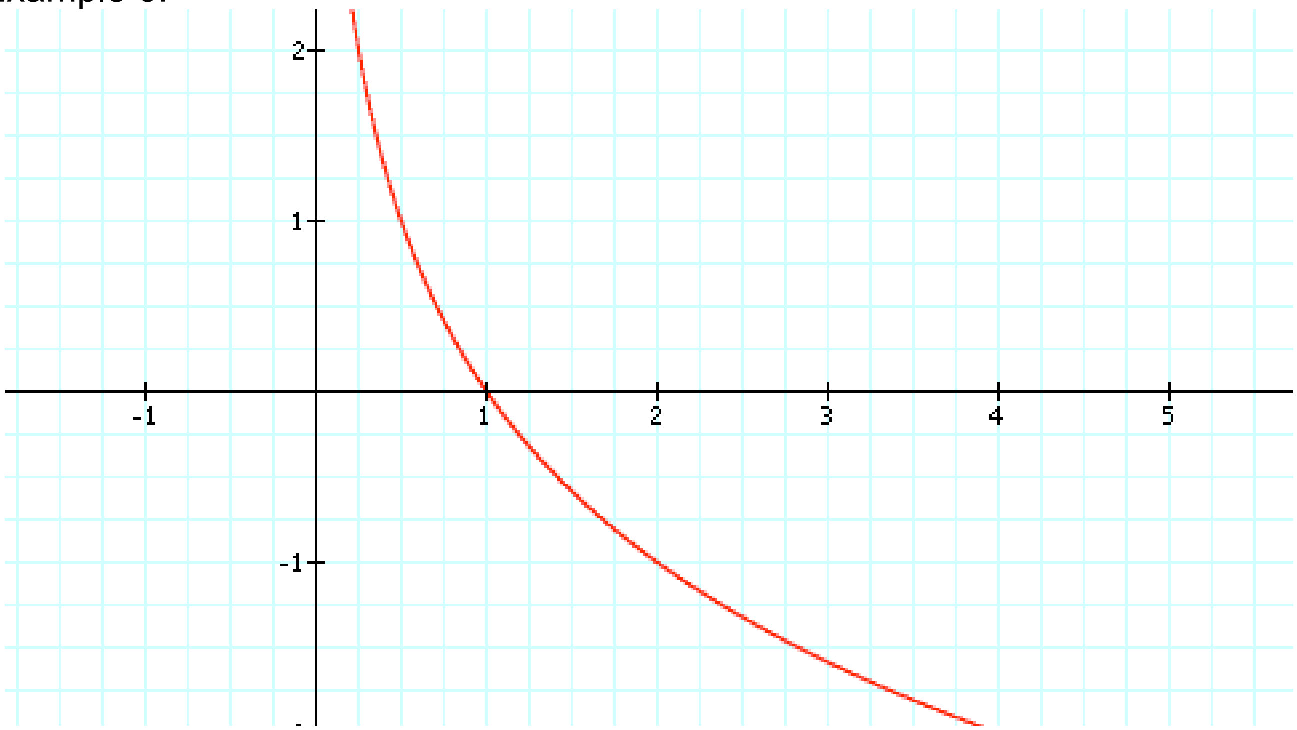
Example 6:



Example 7: {x| x > 7}

Example 8: f(10) ≅ 89.23 A 10-yr old girl has attained about 89% of adult height.

Example 9: {x| x > –3}

Example 10:

a. 1
b. 4
c. 7
d. 1.5x
e. 3x

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