12.3 Properties of Logarithms

The Product Rule

Let b, M and N be positive real numbers with $b \neq 1$.

$$\log_b MN = \log_b M + \log_b N$$

The logarithm of a product is the sum of the logarithms of the factors.

Example 1: Use the product rule to expand each logarithmic expression. Assume all variables and variable expressions represent positive numbers.

a. $\log_4(7x)$

b. $\log_4(7x(x-2))$

c. $\log(10x)$

The Quotient Rule

Let b, M and N be positive real numbers with $b \neq 1$.

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

The logarithm of a quotient is the difference of the logarithms.

Example 2: Use the quotient rule to expand each logarithmic expression. Assume all variables and variable expressions represent positive numbers.

a.. $\log_2 \frac{8}{x}$

b. $\log \frac{10^2}{5}$

c. $\ln \frac{8.7}{e^5}$

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The Power Rule

Let b and M be positive real numbers with $b \neq 1$, and let p be any real number.

$$\log_b M^p = p \log_b M$$

The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.

Example 3: Use the power rule to expand each logarithmic expression. Assume all variables and variable expressions represent positive numbers.

a. $\log_3 x^6$

b. $\log_2 (7x)^4$

c. $\log_6 \sqrt{x}$

Expanding Logarithmic Expressions

Summary of Properties for Expanding Logarithmic Expressions

1. $\log_b MN = \log_b M + \log_b N$ Product Rule
2. $\log_b \frac{M}{N} = \log_b M - \log_b N$ Quotient Rule
3. $\log_b M^p = p \log_b M$ Power Rule

Expanding logarithmic expressions may require that you use more than one property.

Example 4: Use logarithmic properties to expand each expression as much as possible. Assume all variables and variable expressions represent positive numbers.

a. $\log_2 (2x^2)$

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b. $\log \frac{10^{1.5}}{\sqrt{x}}$

c. $\log_b x^4 \sqrt[3]{y}$

d. $\log_4 \frac{\sqrt{x}}{25y^3}$

Condensing Logarithmic Expressions

To condense a logarithmic expression, we write a sum or difference of two logarithmic expressions as a single logarithmic expression. Use the properties of logarithms to do so.

Restatement of Properties of Logarithms:

1. $\log_b M + \log_b N = \log_b MN$ Product Rule
2. $\log_b M - \log_b N = \log_b \frac{M}{N}$ Quotient Rule
3. $p \log_b M = \log_b M^p$ Power Rule

Example 5: Write as a single logarithm. Assume all variables and variable expressions represent positive numbers.

a. $\log 25 + \log 4$

b. $\log 2x + \log 4$

c. $\log(x-1) + \log(x+4)$

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d. $2\log x - \log 4$

e. $7\log_4 5x - \log_4 8$

f. $\frac{1}{2}\log_2 x + 2\log_2 5y^2$

The Change-of-Base Property

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}$$

The logarithm of M with base b is equal to the logarithm of M with any new base divided by the logarithm of b with that new base.
Since calculators generally have keys for only common or natural logs, the change of base formula must be used to evaluate logarithms with bases other than 10 or e.

If the new base, a, is chosen to be 10 or e, the change-of-base formula becomes:

$$\log_b M = \frac{\log M}{\log b} \qquad \text{or} \qquad \log_b M = \frac{\ln M}{\ln b}$$

Example 6: Evaluate each logarithm. Round your answer to the nearest hundredth.

a. $\log_2 133$

b. $\log_{0.5} 23.5$

c. $\log_6 458$

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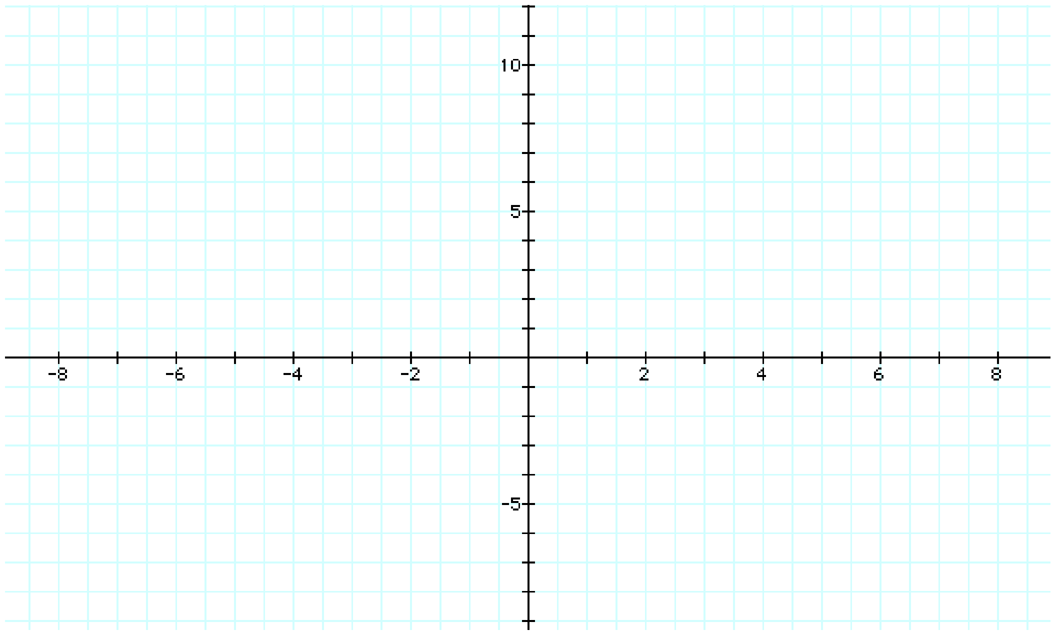
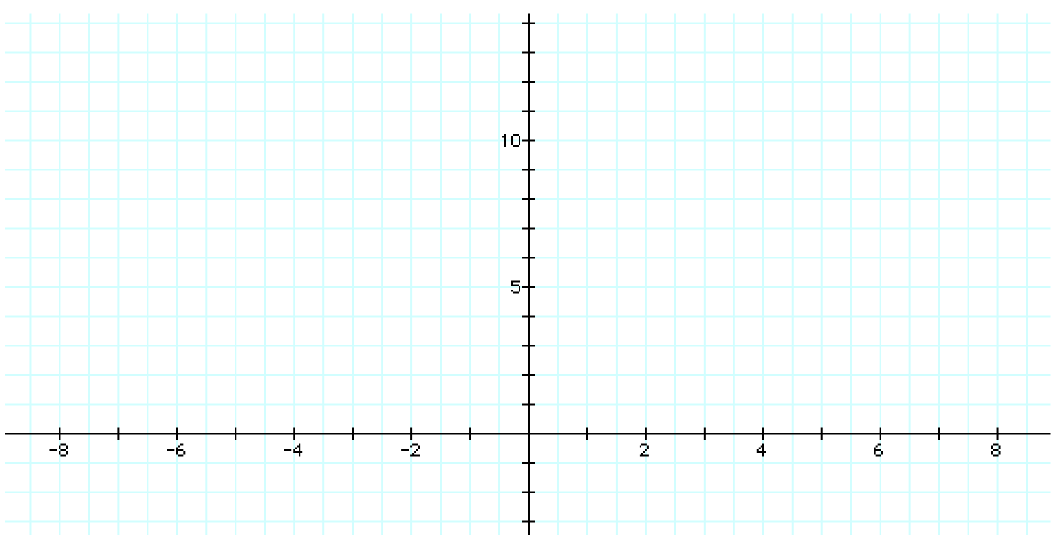
Example 7: Use the change-of-base formula and your graphing calculator to graph $f(x) = \log_2(x-1)$. Indicate any vertical asymptotes with a dotted line.



Example 8: Use your graphing calculator to graph $f(x) = 2\log x$ and $f(x) = \log x^2$. Show the graphs on the grid below Explain why the graphs are different.



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Answers Section 12.3

Example 1:

a. $\log_4 7 + \log_4 x$

b. $\log_4 7 + \log_4 x + \log_4 (x-2)$

c. $1 + \log x$

Example 2:

a. $3 - \log_2 x$

b. $2 - \log 5$

c. $\ln 8.7 - 5$

Example 3:

a. $6\log_3 x$

b. $4\log_2(7x)$

c. $\frac{1}{2}\log_6 x$

Example 4:

a. $1 + 2\log_2 x$

b. $1.5 - \frac{1}{2}\log x$

c. $4\log_b x + \frac{1}{3}\log_b y$

d. $\frac{1}{2}\log_4 x - \log_4 25 - 3\log_4 y$

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Example 5:

a. 2

b. $\log(8x)$

c. $\log[(x-1)(x+4)]$

d. $\log\left(\frac{x^2}{4}\right)$

e. $\log_4\left(\frac{(5x)^7}{8}\right)$

f. $\log_2\left(25y^4\sqrt{x}\right)$

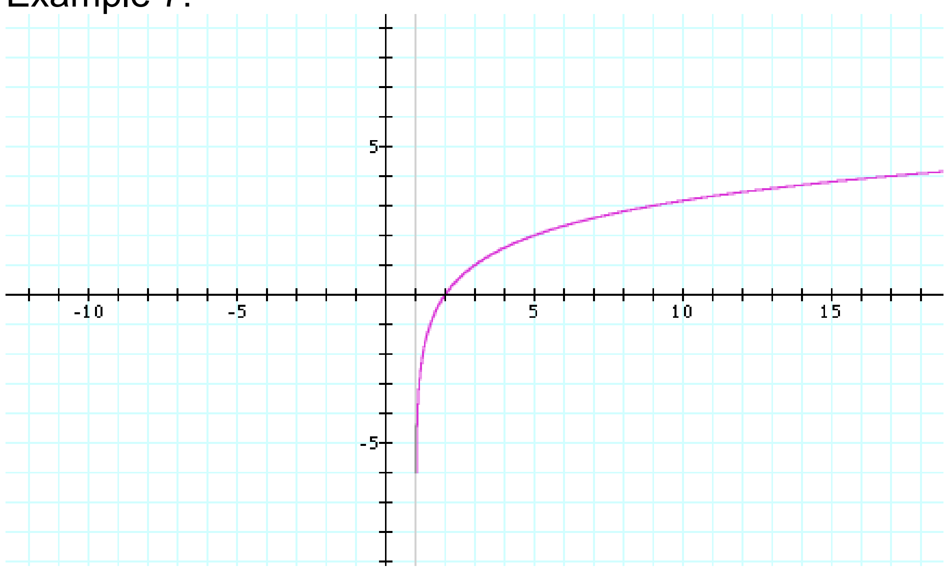
Example 6:

a. 7.06

b. –4.55

c. 3.42

Example 7:



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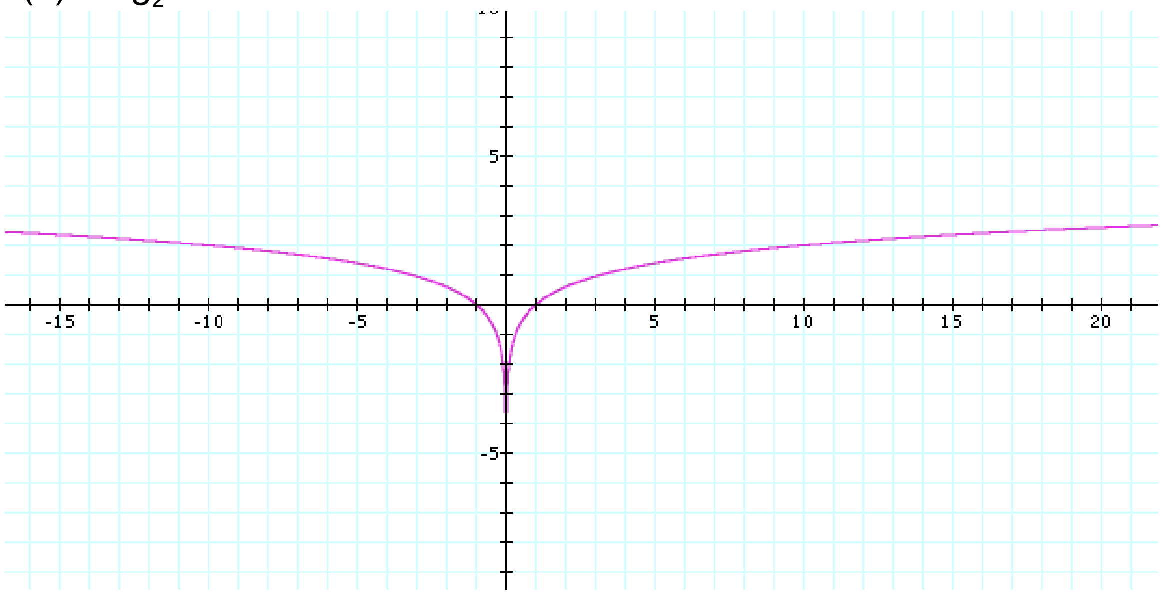
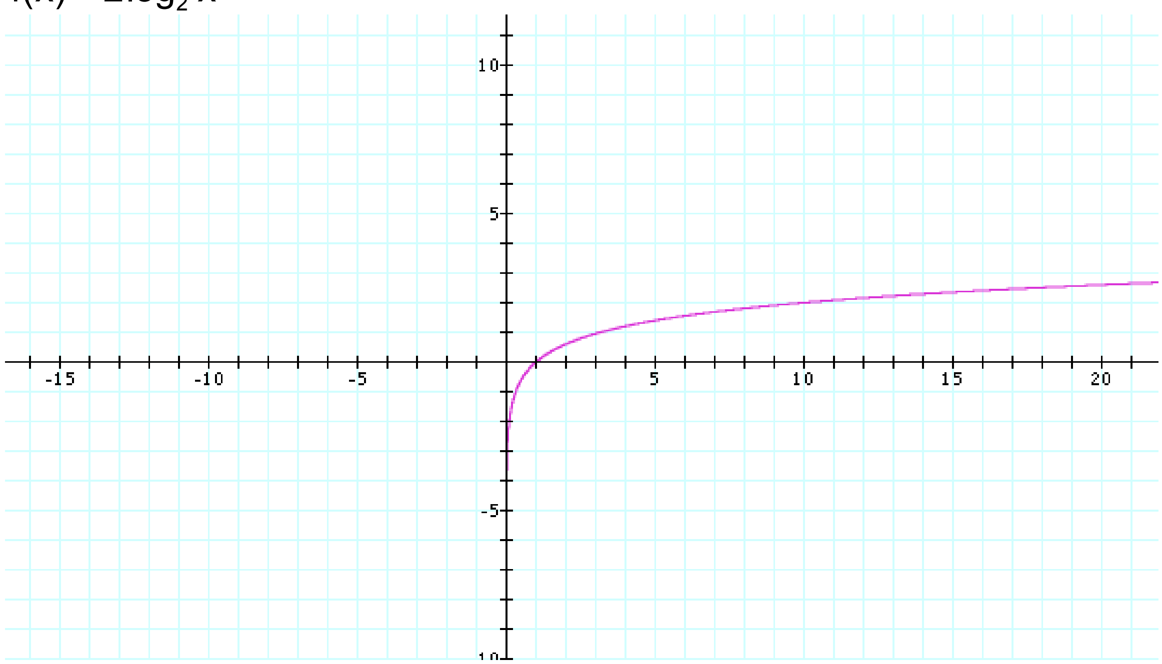
Example 8:

The domain of $f(x)=\log_2 x^2$ is the set of all real numbers except 0 but the domain of $f(x)=2\log_2 x$ is the {x/ x>0}.

$f(x)=\log_2 x^2$



$f(x)=2\log_2 x$



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