12.4 Exponential and Logarithmic Equations

Exponential Equations

An exponential equation is an equation containing a variable in an exponent. We solve exponential equations in by one of the following methods:

Method 1: Express both sides of the equation as a power of the same base.

1. Rewrite each side as a power of the same base.
2. Equate the exponents. (If $b^M = b^N$, then M=N. Note: b > 0.)
3. Solve the resulting equation.

Method 2: Take the natural logarithm of both sides of the equation.

1. Isolate the exponential expression.
2. Take the natural logarithm on both sides of the equation.
3. Simplify using one of the following properties:
   $\ln b^x = x \ln b$ or $\ln e^x = x$
4. Solve for the variable.

Example 1: Solve each exponential equation. Give exact answers.

$a.\ \ 2^{4x+1} = 16$

$b.\ \ 3^{2x-1} = 81$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

$c.\ \ 4^{4x-1} = 8$

$d.\ \ 9^{2x+3} = 27$

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Example 2: Solve each exponential equation. Give an exact answer, and then use your calculator to approximate your answer to two decimal places.

a. $10^x = 14$

b. $4e^x = 17$

c. $15^{2+3x} = 122$

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Logarithmic Equations

A logarithmic equation is an equation that contains a variable in a logarithmic expression. To solve a logarithmic equation:

1. Collect all of the terms involving logarithms on one side of the equation. Rewrite that logarithmic expression as a single logarithm using the properties of logarithms.
2. Rewrite the equation in its equivalent exponential form.
3. Solve the resulting equation.
4. Check proposed solutions and exclude any that produce the logarithm of a negative number or the logarithm of 0.

Example 3: Solve the given logarithmic equations. Give exact answers.

$a.\ \ \log_2 x = -4$

$b.\ \ \log_4(x+5) = 3$

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c. $\log_6(x+5)+\log_6 x=2$

Applications

Example 4: Use the formula $R = 6e^{12.77x}$, where x is the blood alcohol concentration and R, given as a percent, is the risk of having a car accident to find the blood alcohol concentration that corresponds to a 50% risk of having a car accident.

Example 5: If A is the accumulated value of an investment P after t years at r, the annual interest rate in decimal form and n, the number of compounding periods per year, then

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Find the accumulated value of an account in which $10,000 was invested for 10 years at 5% interest, compounded daily (360 times per year).

Example 6: If A is the accumulated value of an investment P after t years at r, the annual interest rate in decimal form and with continuous compounding, then

$$A = Pe^{rt}$$

Find the accumulated value of $10,000 invested at 5% interest, compounded continuously for 10 years.

Answers Section 12.4

Example 1:

a. $\left\{\frac{3}{4}\right\}$

b. $\left\{\frac{5}{2}\right\}$

c. $\left\{\frac{5}{8}\right\}$

d. $\left\{-\frac{3}{4}\right\}$

Example 2:

a. {1.15}

b. {1.45}

c. {–0.08}

Example 3:

a. $\{\frac{1}{16}\}$

b. {59}

c. {4}

Example 4: 0.17, A blood alcohol content of .17 corresponds to a 50% risk of having a car accident.

Example 5: The accumulated value of an account in which $10,000 was invested for 10 years at 5% interest, compounded daily is $16,486.64.

Example 6 The accumulated value of $10,000 invested at 5% interest, compounded continuously for 10 years is $16,487.21.