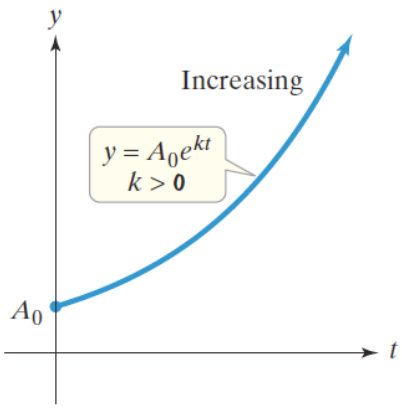
12.5 Exponential Growth and Decay; Modeling Data

Exponential Growth and Decay Models

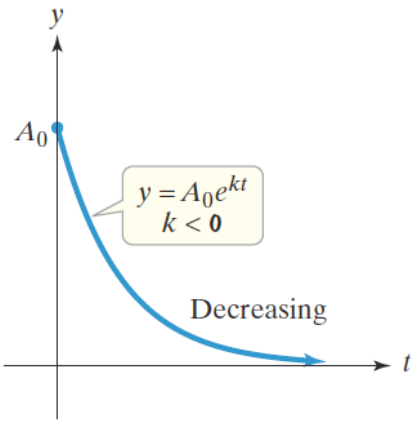
The mathematical model for **exponential growth** or **decay** is given by $f(t) = A_0 e^{kt}$, or $A = A_0 e^{kt}$.

• **If $k > 0$ the function models the amount, or size, of a *growing* entity.** A_0 is the original amount, or size, of the growing entity at time $t = 0$, A is the amount at time t and k is a constant representing the growth rate.

• **If $k < 0$ the function models the amount, or size, of a *decaying* entity.** A_0 is the original amount, or size, of the decaying entity at time $t = 0$, A is the amount at time t and k is a constant representing the decay rate.



(a) Exponential growth

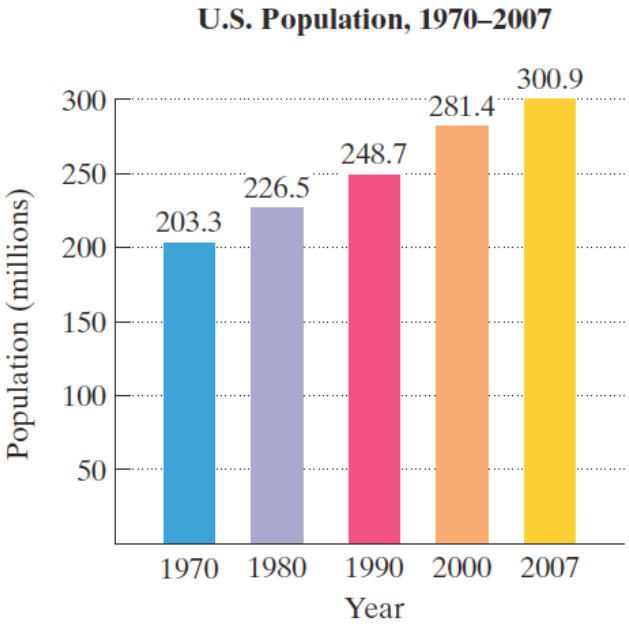


(b) Exponential decay

Sometimes we need to use given data to determine k, the rate of growth or decay. After we compute the value of k, we can use the formula $A = A_0 e^{kt}$, to make predictions.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer

Example 1: The graph below shows the U.S.population, in millions, for five selected years from 1970 through 2007. In 1970, the U.S. population was 203.3 million. By 2007, it had grown to 300.9 million.



a. Find an exponential growth function that models the data for 1970 through 2007.

b. By which year will the U.S. population reach 315 million?

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Example 2: In 1990, the population of Africa was 643 million and by 2006 it had grown to 906 million.

a. Use the exponential growth model $A = A_0 e^{kt}$, in which is the number of years after 1990, to find the exponential growth function that models the data.

b. By which year will Africa's population reach 2000 million, or two billion?

Our next example involves exponential decay and its use in determining the age of fossils and artifacts. The method is based on considering the percentage of carbon-14remaining in the fossil or artifact. Carbon-14 decays exponentially with a half-life of approximately 5715 years. The **half-life** of a substance is the time required for half of a given sample to disintegrate. Thus, after 5715 years a given amount of carbon-14 will have decayed to half the original amount. Carbon dating is useful for artifacts or fossils up to 80,000 years old. Older objects do not have enough carbon-14 left to determine age accurately.

Example 3: **a.** Use the fact that after 5715 years a given amount of carbon-14 will have decayed to half the original amount to find the exponential decay model for carbon-14.

b. In 1947, earthenware jars containing what are known as the Dead Sea Scrolls were found by an Arab Bedouin herdsman. Analysis indicated that the scroll wrappings contained 76% of their original carbon-14. Estimate the age of the Dead Sea Scrolls.

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Example 4: Strontium-90 is a waste product from nuclear reactors. As a consequence of fallout from atmospheric nuclear tests, we all have a measurable amount of strontium-90 in our bones.

a. Use the fact that after 28 years a given amount of strontium-90 will have decayed to half the original amount to find the exponential decay model for strontium-90.

b. Suppose that a nuclear accident occurs and releases 60 grams of strontium-90 into the atmosphere. How long will it take for strontium-90 to decay to a level of 10 grams.