Section 7.4 Adding and Subtracting Rational Expressions with Different Denominators

In order to add or subtract rational expressions that have different denominators, you must first find a least common denominator and then rewrite each rational expression as an equivalent rational expression that has the common denominator.

Finding the Least Common Denominator
1. Factor each denominator completely.
2. List the factors of the first denominator.
3. Add to the list in step 2 any factors of the second denominator that do not appear in the list. (NOTE: If a factor appears once in the first denominator and more than once in the second denominator, then you must add to the list the "extra" factors from the second denominator.)
4. Form the product of each different factor from the list in step 3. This product is the least common denominator.

Example 1: Find the least common denominator for each of the following pairs of rational expressions.

a. $\dfrac{7}{15x^2}, \dfrac{13}{24x}$

Factors of 1st denominator : $3, 5, x, x$

Factors of 2nd denominator : $2, 2, 2, 3, x$

Add to the first list any "missing" factor from the 2nd list : $3, 5, x, x, 2, 2, 2$

$LCD = 3*5*x*x*2*2*2 = 120x^2$

b. $\dfrac{2}{x-5}, \dfrac{3}{x^2-25}$

c. $\dfrac{3}{x^2-25}, \dfrac{x}{x^2-10x+25}$

> **Adding and Subtracting Rational Expressions That Have Different Denominators:** To add or subtract rational expressions that have different denominators:
> 1. Find the LCD of the rational expressions.
> 2. Rewrite each rational expression as an equivalent expression whose denominator is the LCD. To do so, multiply the numerator and denominator of each rational expression by any factor(s) needed to convert the denominator into the LCD.
> 3. Add or subtract the numerators, placing the resulting expression over the LCD.
> 4. If necessary, simplify the resulting expression.

Example 2: Add or subtract, as indicated. Express your result in simplest form.

a. $\dfrac{5}{6x} + \dfrac{7}{8x}$

b. $\dfrac{5}{x} + 3$

c. $\dfrac{4x}{x^2 - 25} + \dfrac{x}{x + 5}$

d. $\dfrac{x}{x^2 - 2x - 24} - \dfrac{x}{x^2 - 7x + 6}$

Adding and Subtracting Rational Expressions When Denominators Contain Opposite Factors: When one denominator contains the opposite factor of the other, first multiply either rational expression by $\frac{-1}{-1}$. Then apply the procedure for adding or subtracting rational expressions that have different denominators to the rewritten problem.

Example 3: Add or subtract, as indicated. Express your result in simplest form.

a. $\dfrac{x+7}{9x-12} + \dfrac{x}{16-9x^2}$

b. $\dfrac{y}{y^2-1} + \dfrac{5y}{y-y^2}$

Answers Section 7.4

Example 1:
   a. LCD is $120x^2$
   b. LCD is (x − 5)(x + 5)
   c. LCD is (x − 5)(x + 5) (x − 5)

Example 2:
   a. $\dfrac{41}{24x}$
   b. $\dfrac{5+3x}{x}$
   c. $\dfrac{x^2-x}{x^2-25}$
   d. $\dfrac{-5x}{(x-6)(x+4)(x-1)}$

Example 3:
   a. $\dfrac{3x^2+22x+28}{3(3x-4)(4+3x)}$
   b. $\boxed{\dfrac{-(4y+5)}{(y+1)(y-1)}}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.