## 8.3 The Algebra of Functions

### The Domain of a Function

Functions that model data often have their domains given, either on the horizontal axis of the graph of the function or along with the function's equation. However, for most functions, only an equation is given and the domain is not specified. In cases like this, the domain is the largest set of numbers for which the value of f(x) is a real number.

**Finding the Domain of a Function:** If a function f is given by an equation and the domain is not given, find the domain by choosing all real numbers except:

- Any x-value that makes a denominator equal to zero,
- Any x-value that results in a negative number under a square root (or any other even root)
- Any x-value that makes the argument of a logarithmic function negative or zero (We will study these functions in chapter 12).

Example 1: Find the domain of each function:

a. $f(x) = x + 7$

b. $f(x) = \sqrt{x-1}$

c. $f(x) = \dfrac{x}{x^2 - 1}$

d. $f(x) = \sqrt{2x-7}$

e. $f(x) = \dfrac{2x-4}{x^2 - 2x - 3}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

## The Algebra of Functions

Two functions can be added, subtracted, multiplied or divided as long as there are numbers common to the domains of both functions. The common domain for the sum, difference, product or quotient of two functions is the set of numbers that are common to the domains of both functions.

### The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The sum f + g, the difference f – g, the product fg, and the quotient $\frac{f}{g}$ are functions whose domains are the set of all real numbers common to the domains of f and g. They are defined as follows:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \bullet g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

Example 2: Let $f(x) = 2x + 1$ and $g(x) = x^2 - x$. Find:

a. $(f + g)(2)$

b. $(f - g)(-1)$

c. $(f - g)(x)$

d. $(fg)(3)$

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## Answers Section 8.3

Example 1:

a. Domain: All real numbers

b. Domain: {x/ x is a real number and $x \geq 1$}

c. Domain: {x/ x is a real number and $x \neq 1$ and $x \neq -1$}

d. Domain: {x/ x is a real number and $x \geq \frac{7}{2}$}

e. Domain: {x/ x is a real number and $x \neq 3$ and $x \neq -1$}

Example 2:

a. (f + g)(2) = 7

b. (f – g)( –1)= –3

c. $(f - g)(x) = -x^2 + 3x + 1$

d. (fg)(3) = 42

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