8.4 Composite and Inverse Functions

The Composition of Functions

The composition of the function f with g is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x))$$

The domain of the composite function $f \circ g$ is the set of all x such that

1. x is in the domain of g and
2. g(x) is in the domain of f.

Forming Composite Functions

Example 1: Given f(x) = 5x + 2 and g(x) = 3x – 4, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Inverse Functions

Let f and g be two functions such that

f(g(x)) = x for every x in the domain of g, and

g(f(x)) = x for every x in the domain of f.

The function g is the inverse of the function f, and is denoted by f^{-1} (read "f-inverse"). Thus $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1} and vice versa.

Verifying Inverse Functions

Example 2: Verify that each function is the inverse of the other:

$$f(x) = 6x \text{ and } g(x) = \frac{x}{6}$$

Example 3: Verify that each function is the inverse of the other.

$$f(x) = 4x + 9 \text{ and } g(x) = \frac{x-9}{4}$$

Finding the Inverse of a Function

The equation for the inverse of a function can be found as follows:

1. Replace f(x) with y in the equation for f(x).
2. Interchange x and y.
3. Solve for y. If this equation does not define y as a function of x, the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x, the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 with $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example 4: Find the inverse of f(x) = 6x + 3

Example 5: Find the inverse of f(x) = $(x+1)^3$

Example 6: Find the inverse of f(x) = x^3-4.

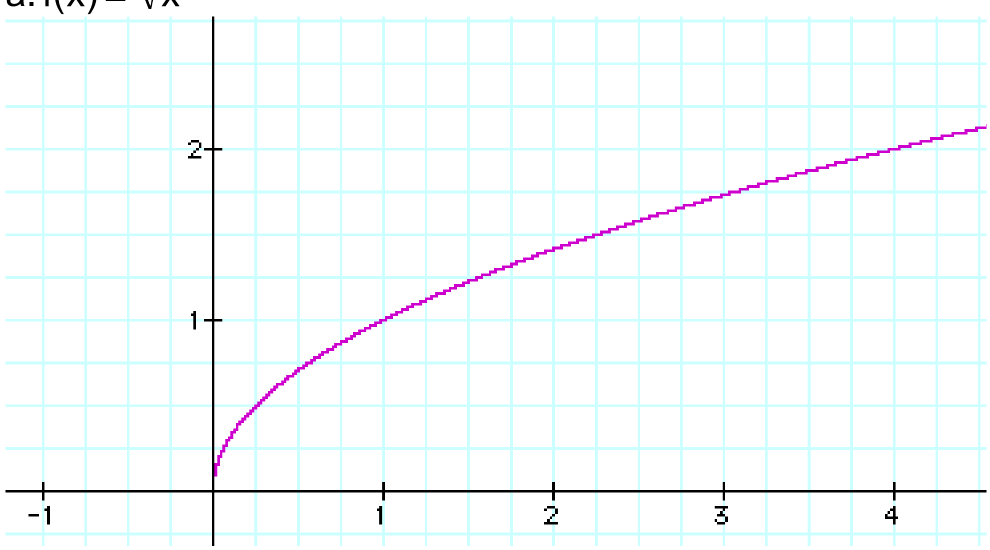
The Horizontal Line Test and One-to-One Functions

The Horizontal Line Test for Functions

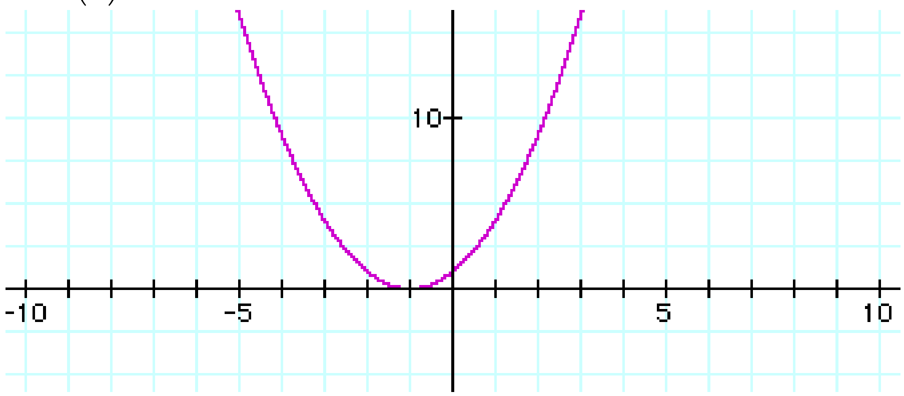
A function f has an inverse that is a function, f^{-1}, if there is no horizontal line that intersects the graph of the function f at more than one point.

Example 7: For each of the following functions, use the given graph of the function and the horizontal line test to determine if the function has an inverse.

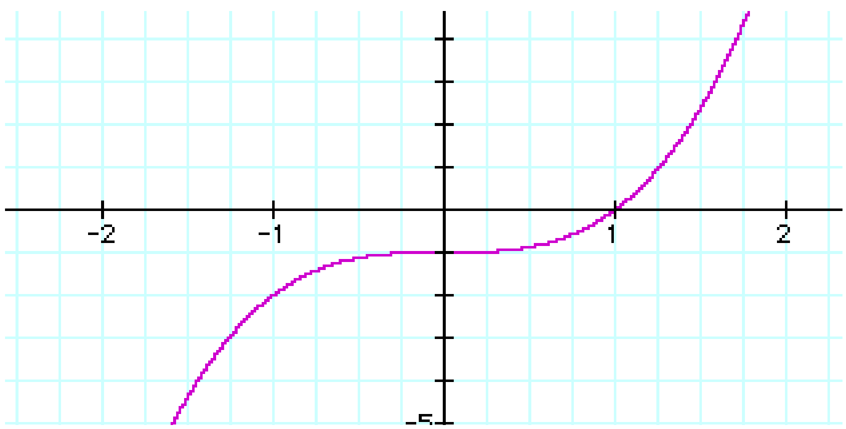
a. $f(x)=\sqrt{x}$



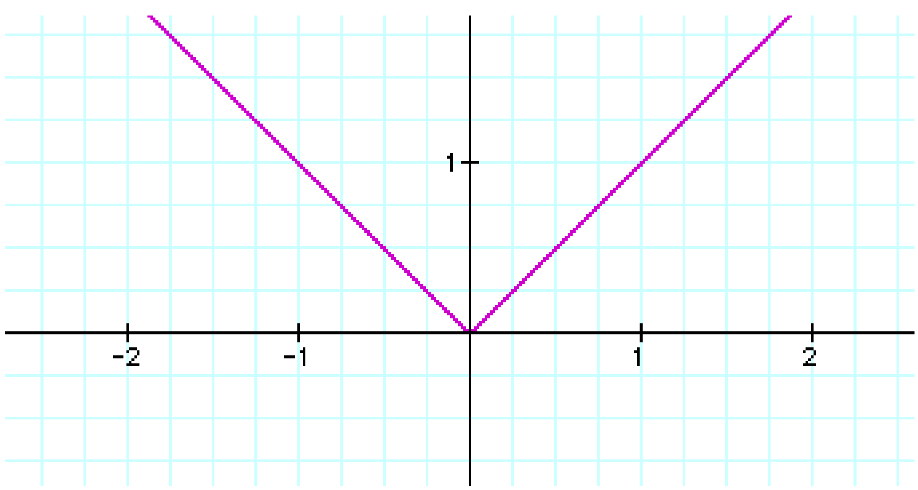
b. $f(x) = x^2 + 2x + 1$



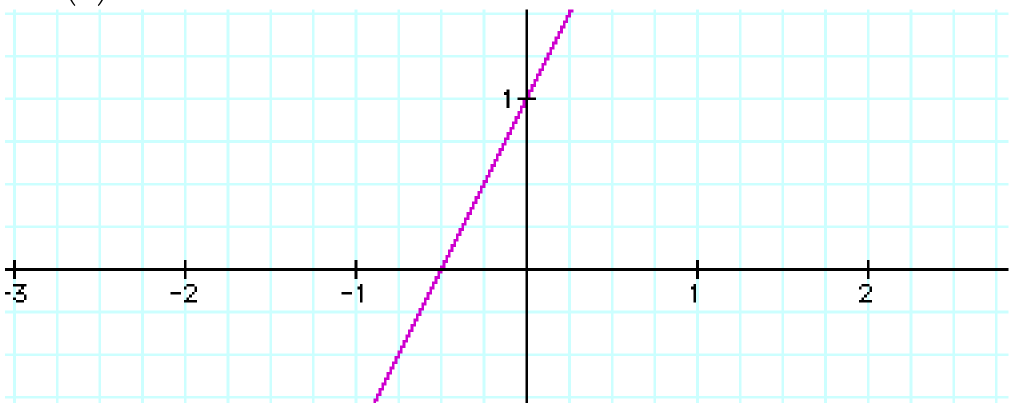
c. $f(x) = x^3 - 1$



d. $f(x) = |x|$



e. $f(x) = 2x + 1$

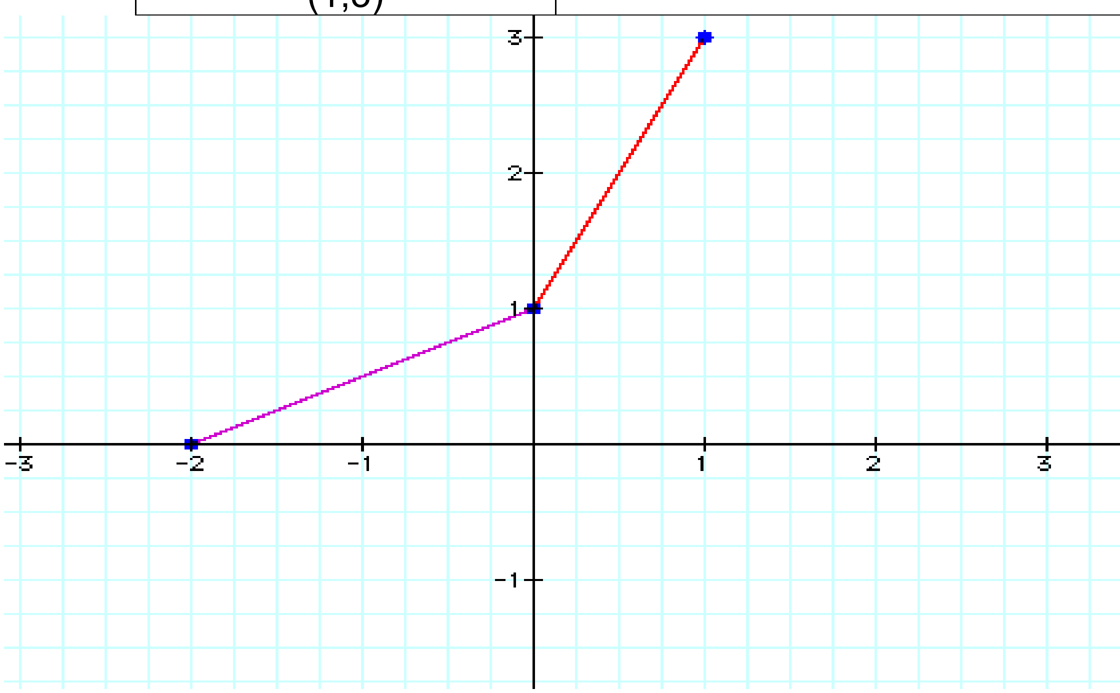


Graphs of f and f^{-1}

The graphs of f and f^{-1} are reflections of one another through the line y = x. Points on the graph of f^{-1} can be found by reversing the coordinates of the points on the graph of f.

Example 8: Consider the graph of the function f traced by joining the points given below with straight-line segments. Sketch the graph of f and the graph of f^{-1}.

Points on y = f(x)	Points on $y = f^{-1}(x)$
(–2,0)	
(0,1)	
(1,3)	



Answers Section 8.4

Example 1: $(f \circ g)(x) = 15x - 18$ and $(g \circ f)(x) = 15x + 2$

Example 2: $(f \circ g)(x) = (g \circ f)(x) = x$ f and g are inverses of one another.

Example 3: $(f \circ g)(x) = (g \circ f)(x) = x$ f and g are inverses of one another.

Example 4: $f^{-1}(x) = \dfrac{x-3}{6}$

Example 5: $f^{-1}(x) = \sqrt[3]{x} - 1$

Example 6: $f^{-1}(x) = \sqrt[3]{x+4}$

Example 7:

a. The graph passes the horizontal line test, and thus the function graphed has an inverse function.
b. The graph fails the horizontal line test, and thus the function graphed does not have an inverse function.
c. The graph passes the horizontal line test, and thus the function graphed has an inverse function.
d. The graph fails the horizontal line test, and thus the function graphed does not have an inverse function.
e. The graph passes the horizontal line test, and thus the function graphed has an inverse function.

Example 8: