Section 9.1 Solving Linear Inequalities

We know that a linear equation in x can be expressed as $ax+b=0$. A **linear inequality in *x*** can be written in one of the following forms: $ax+b<0$, $ax+b\le 0$, $ax+b>0$, or $ax+b\ge 0$ In each form, $a\ne 0$.

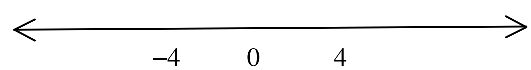
If an inequality does not contain fractions, it can be solved using the following procedure. Notice how similar this procedure is to the procedure for solving a linear equation.

Steps for solving a linear inequality

Step 1. Simplify each side.
Step 2. Collect variable terms on one side and constant terms on the other *(use addition property of inequalities)*
Step 3. Isolate the variable and solve *(use multiplication property of inequalities, change the sense of the inequality when multiplying, or dividing both sides by a negative number)*
Step 4. Express the solution set in interval notation or set-builder notation and graph the solution set on a number line.

Example 1: Solve and graph the solution set on a number line:

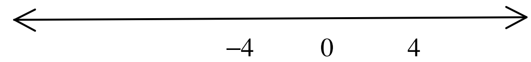
$3x-5>-17$



Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 2: Solve and graph the solution set on a number line:

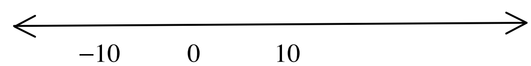
$-2x-4 > x+5$



If an inequality contains fractions, begin by multiplying both sides by the least common denominator. This will clear the inequality of fractions.

Example 3: Solve and graph the solution set on a number line:

$$\frac{x-4}{2} \geq \frac{x-2}{3} + \frac{5}{6}$$



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Example 4: You are choosing between two telephone plans. Plan A has a monthly fee of \$15 with a charge of 8 cents per minute for all calls. Plan B has a monthly fee of \$3 with a charge of 12 cents per minute for all calls. How many minutes of calls in a month make plan A the better deal? *(Define a variable, create an inequality, solving using algebra, and answer in a sentence.)*

APPLICATION: For a business to realize a profit, the revenue (or income), R, must be greater than the __________, C. That is, a profit will be obtained only when $R > C$. The company breaks __________ when R = C.

If you sell x units of a product at a certain price p, then your revenue function is $R(x) =$ ________ .

The cost of your business may include a fixed cost (like rental fees, initial cost of equipment, etc.) and the cost of making each item.

$C(x)$ = fixed cost + __________ .

The profit $P(x)$, generated after producing and selling x units of a product is given by the profit function:

$\boldsymbol{P(x)} =$ __________ – __________

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Example 5: Technology is now promising to bring light, fast, and beautiful wheelchairs to millions of disabled people. A company is planning to manufacture these radically different wheelchairs. Fixed cost will be $500,000 and it will cost $400 to produce each wheelchair. Each wheelchair will be sold for $600.

a) Write the cost function, C, of producing x wheelchairs.

b) Write the revenue function, R, of producing x wheelchairs.

c) Write the profit function, P, from producing and selling x wheelchairs.

d) How many wheelchairs must be produced and sold for the business to make money?

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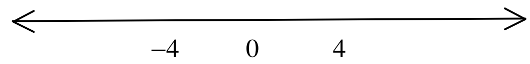
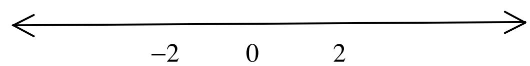
Extra Practice: Solve the given inequalities and graph the solution set. Express your answer in interval notation.

Example 6:

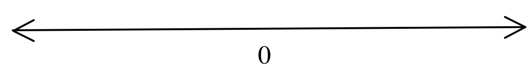
a. $-4(x+2) > 3x+20$



b. $\dfrac{3x}{10}+1 \geq \dfrac{1}{5}-\dfrac{x}{10}$



c. $\dfrac{4x-3}{6} \geq \dfrac{2x-1}{12}-2$



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Answers Section 9.1

Example 1: $(-4,\infty)$

Example 2: $(-\infty,-3)$

Example 3: $[13,\infty)$

Example 4:

let $x =$ minutes used in call plan, $15+0.08<3+0.12x,\ x>300,$

Plan A is a better deal when you use more than 300 minutes of calls.

Example 5a: $C(x)=500{,}000+400x$

Example 5b: $R(x)=600x$

Example 5c: $P(x)=200x-500{,}000$

$200x-500{,}000>0,\quad x>2{,}500,$

Example 5d: More than 2,500 wheelchairs must be produced and sold for the business to make money.

Extra Practice:

Example 6a: $(-\infty,-4)$

Example 6b: $[-2,\infty)$

Example 6c: $\left[-\frac{19}{6},\infty\right)$

Common Student Error: *Students often forget to change the sense, (the ____________________), of the inequality when multiplying or dividing by a _______________ number.*

Given: $-3x<6$ $\quad \frac{-3x}{-3}<\frac{6}{-3} \rightarrow$ *x < –2 is WRONG*

$\frac{-3x}{-3}>\frac{6}{-3} \rightarrow$ *x > –2 is CORRECT*

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