## Section 9.2 Compound Inequalities

A ***compound inequality*** is formed by joining two inequalities with the word _____ , or the word _____ .

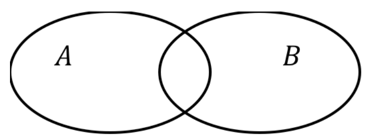
For example: $3 < x$ *AND* $x < 5$

$x + 4 > 3$ OR $2x - 3 \leq 6$

**Definition of the intersection of two sets----*AND***

For any two sets $A$ and $B$, the intersection of $A$ and $B$, symbolized $A \cap B$, is defined as follows:

$$A \cap B = \{ x \mid x \text{ is an element of } A \text{ \_\_\_\_\_ } x \text{ is an element of } B \}$$




Example 1: Let $A = \{ 1, 2, 3, 4, 5 \}$ and $B = \{ 2, 4, 6\}$. Find $A \cap B$.

Example 2: Let $C = \{ 1, 3, 5, 7\}$ and $D = \{ 4, 6, 8\}$. Find $C \cap D$.

A number is a **solution of a compound inequality** formed by the word ***AND*** if it is a solution of both inequalities. Thus, the solution set is the intersection of the solution sets of the two inequalities.

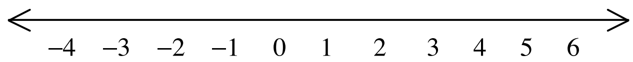
**Steps for solving a compound inequalities involving *AND***

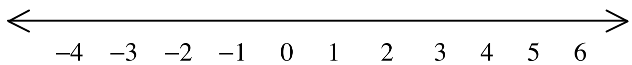
Step 1. Solve each inequality individually.

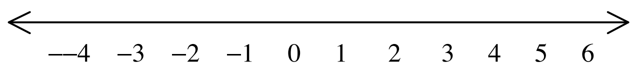
Step 2. Graph the solution set to each inequality on a number line and take the intersection of these solution sets. This intersection appears as the portion of the number line that the two graphs have in common.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

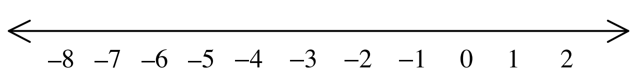
Example 3: Solve: $-3 < x$ AND $x < 5$

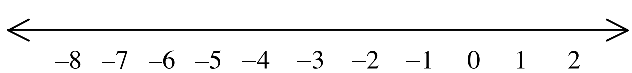


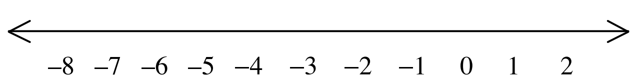




Example 4: Solve: $-3x - 2 > 5$ AND $5x - 1 \leq -21$

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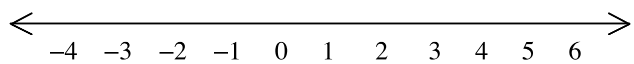
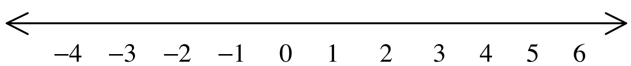
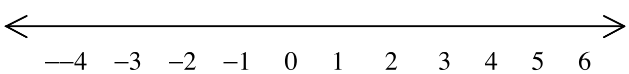
Example 5: Solve: $3x + 2 \le 11$ AND $-2x - 3 < 5$




If a < b, the compound inequality $a < x$ *AND* $x < b$ can be written in the shorter form $a < x < b$. For example, the compound inequality $-5 < 2x + 1$ *AND* $2x + 1 < 3$ can be abbreviated $-5 < 2x + 1 < 3$.

The word *AND* does not appear when the inequality is written in the shorter form, although it is implied. The shorter form enables us to solve both inequalities at once. By performing the same operations on all three parts of the inequality, our goal is to **isolate *x* in the middle**.

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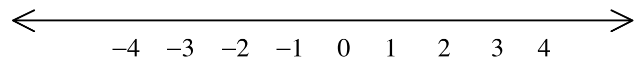
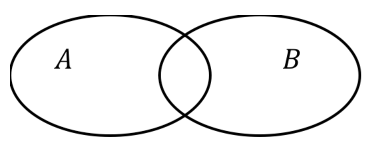
Example 6: Solve: $-5 \le 2x + 1 < 3$




**Definition of the union of two sets----*OR***

For any two sets $A$ and $B$, the union of $A$ and $B$, symbolized $A \cup B$ , is defined as follows:

$$A \cup B = \{ x \mid x \text{ is an element of } A \text{ \_\_\_\_\_ } x \text{ is an element of } B \}$$




Example 7: Let $A = \{ 1, 2, 3, 4, 5 \}$ and $B = \{ 2, 4, 6\}$. Find $A \cup B$.

Example 8: Let $C = \{ 1, 3, 5, 7\}$ and $D = \{ 4, 6, 8\}$. Find $C \cup D$.

A number is a **solution of a compound inequality** formed by the word ***OR*** if it is a solution of either inequality. Thus, the solution set of a compound inequality formed by the word *OR* is the union of the solution sets of the two inequalities.

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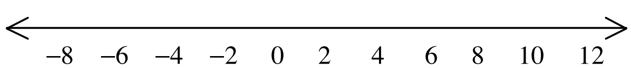
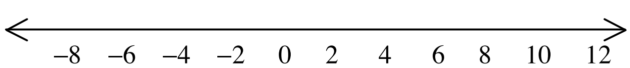
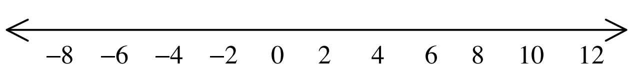
**Steps for solving a compound inequalities involving *OR***

| |
|---|
| Step 1. Solve each inequality individually. |
| Step 2. Graph the solution set to each inequality on a number line and take the union of these solution sets. This union appears as the portion of the number line representing the total collection of numbers in the two graphs. |

Example 9: Solve: $2x - 3 < 7$ OR $35 - 4x \leq 3$

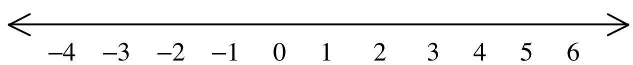


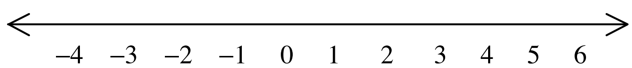



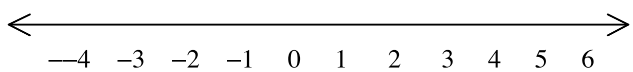




Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

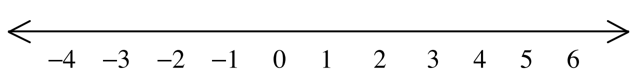
Example 10: Solve: $3x - 5 \leq 13$ OR $5x + 2 > -3$

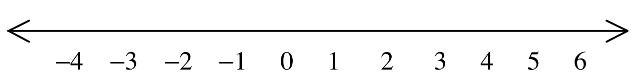


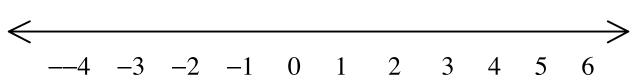




Example 11: Solve: $2x - 7 > 3$ AND $5x - 4 < 6$




Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

## Answers Section 9.2

Example 1: $\{2,4\}$

Example 2: $\varnothing$ or $\{\quad\}$

Example 3: $(-3,5)$

Example 4: $(-\infty,-4]$

Example 5: $(-4,3]$

Example 6: $[-3,1)$

Example 7: $\{1,2,3,4,5,6\}$

Example 8: $\{1,3,4,5,6,7,8\}$

Example 9: $(-\infty,5)\cup[8,\infty)$

Example 10: $(-\infty,\infty)$

Example 11: $\varnothing$ or $\{\quad\}$

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