Section 9.3 Equations and Inequalities Involving Absolute Value

If an equation involves an unknown quantity that is enclosed in absolute value bars, to solve the equation you must rewrite the equation without the absolute value bars and then solve using an appropriate technique.

Rewriting an Absolute Value Equation without Absolute Value Bars

If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to X = c or X = –c.

Example 1: Solve the given absolute value equations.

a. $|3x - 7| = 1$

$3x - 7 = 1$ or $3x - 7 = -1$

b. $|2x + 4| = 10$

c. $|4x - 5| + 3 = 17$ (Hint: First subtract 3 from both sides to isolate the absolute value term, and then rewrite the equation without absolute value bars using the rule in the box above Example 1.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Solve Inequalities Involving Absolute Value

To solve an inequality involving absolute value, if the variable appears inside the absolute value symbols:

- Isolate the term involving the absolute value symbols.
- Rewrite the inequality without absolute values using the following rules:

  $|u| < a$ is equivalent to $-a < u < a$ (Also true for "$\leq$")

  $|u| > a$ is equivalent to $u < -a$ or $u > a$. (Also true for "$\geq$")

- Solve the resulting inequalities.

Example 2: Solve the given inequalities. Express your solution in interval notation. Graph your solution on the number line.

a. $|x+4|+3<5$

b. $|2-3x|-1>0$

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Extra Practice:
Solve the given equations or inequalities. Express your answer in interval notation.
Example 3:
a. $1 + 3x > 4 + x$

b. $|3x+4| \geq 10$

c. $|4-2x| = 18$

d. $|4\text{x}-3| + 2 \leq 23$

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Answers Section 9.3

Example 1a: $\{\frac{8}{3},\ 2\}$

Example 1b: { 3, –7}

Example 1c: $\left\{\frac{19}{4},\ -\frac{9}{4}\right\}$

Example 2a: $(-6,-2)$ OR $\{x/-6 < x < -2\}$

Example 2b: $\left(-\infty,\frac{1}{3}\right)$ or $(1,\infty)$

OR

$\left\{x/x < \frac{1}{3} \text{ or } x > 1\right\}$

Extra Practice:

Example 3a: $\left(\frac{3}{2},\infty\right)$

Example 3b: $\left(-\infty,-\frac{14}{3}\right]\cup[2,\infty)$

Example 3c: $\{-7,11\}$

Example 3d: $\left[-\frac{9}{2},6\right]$

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