Intermediate Algebra
Chapter 12 Review

Set up a table of coordinates and graph the given functions. Give equations of horizontal or vertical asymptotes.

1. $f(x) = 2^x$
2. $g(x) = \left(\frac{1}{3}\right)^x$
3. $f(x) = \log_2 x$
4. $g(x) = \log_{\frac{1}{2}} x$

Graph f and g on the same coordinate system. Describe, in terms of shifts, how the graph of g is related to the graph of f. Give the equation of the horizontal asymptote for g. Label at least three points on the graph of g.

5. $f(x) = 3^x \quad g(x) = 3^{x+2}$
6. $f(x) = 3^x \quad g(x) = 3^x + 2$
7. $f(x) = 3^x \quad g(x) = 3^{x-1} + 3$

8. Given f and g, find $f \circ g$, f^{-1}, and g^{-1}.

$f(x) = 7x - 2 \quad g(x) = 2x^3 + 1$

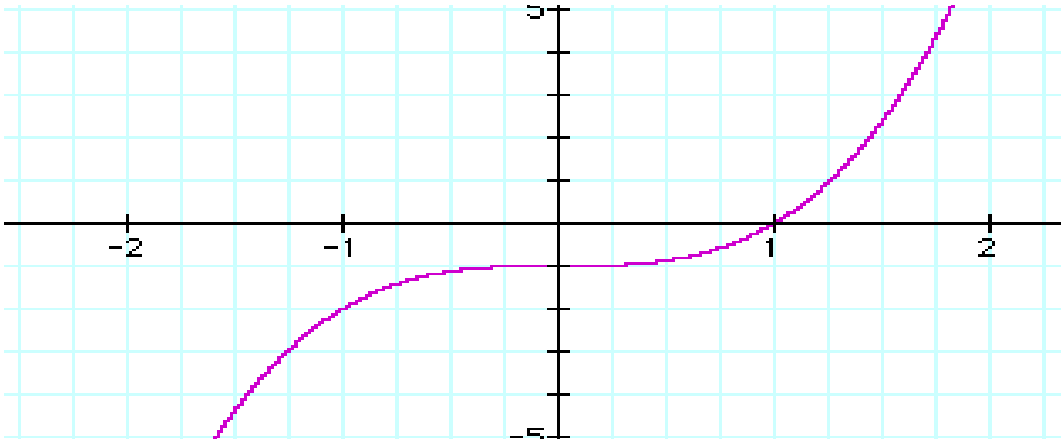
Find $f(g(x))$ and $g(f(x))$, and determine if f and g are inverses of each other.

9. $f(x) = 4x + 9 \quad g(x) = \frac{x-9}{4}$
10. $f(x) = \sqrt[3]{x-4} \quad g(x) = x^3 + 4$

11. What test do you use to determine if a given graph represents a function that has an inverse function? State the test.

12. Determine whether each graph represents a function that has an inverse function. State your conclusion.

a.



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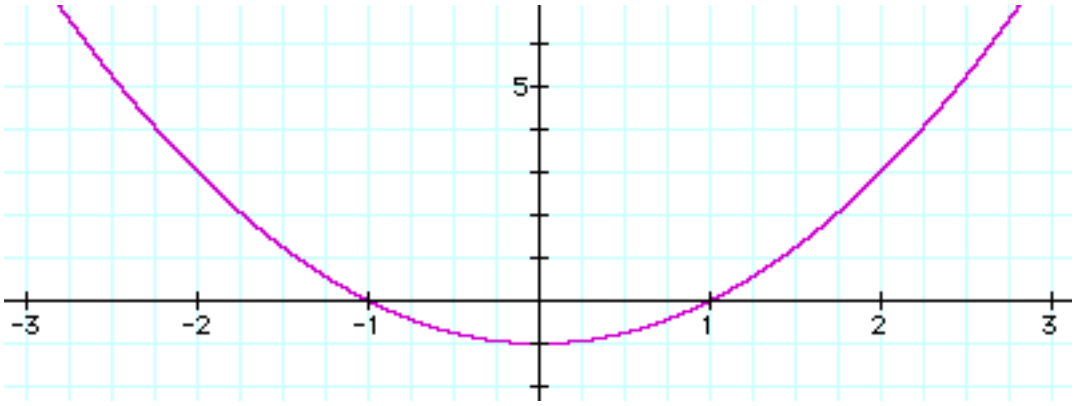

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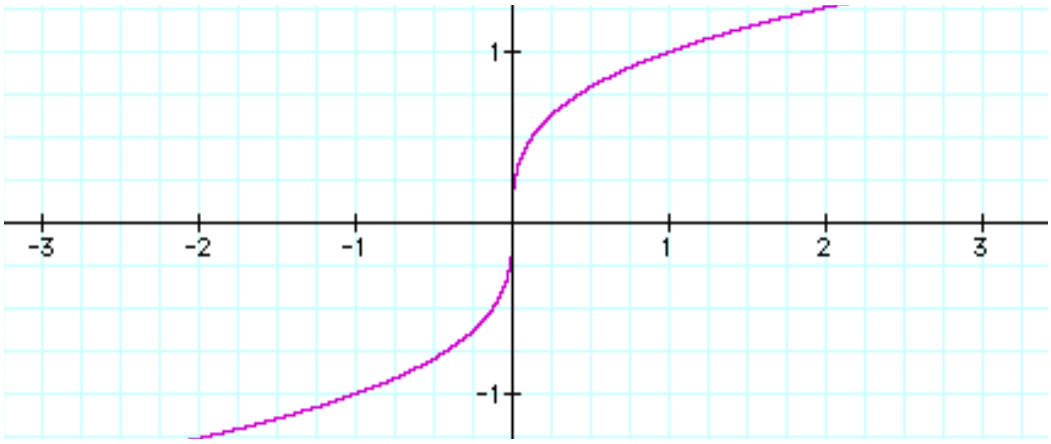
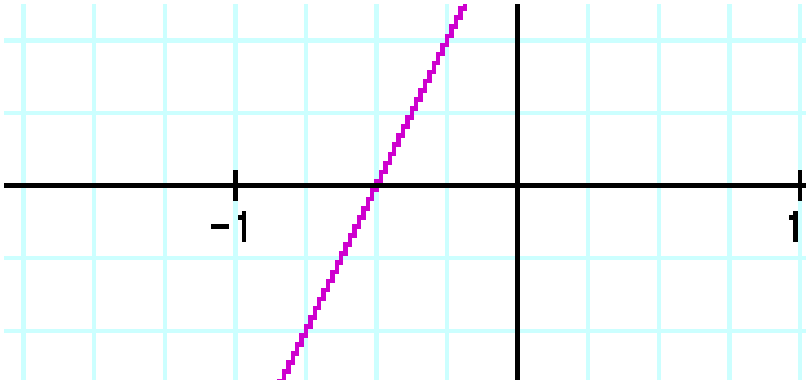


d.



Evaluate each of the following without using a calculator. Leave your answers in exact form without any logarithms.

13. $\log_5 1$
14. $6^{\log_6 5}$
15. $\log_{25} 5$
16. $\log_4 \frac{1}{16}$
17. $\log_7 \sqrt{7}$
18. $\log 1000$
19. $\ln e^6$
20. $\ln e$
21. $\log 10$
22. $\log \sqrt{10}$
23. $e^{\ln 300}$
24. $\ln e^{(11x+1)}$
25. $10^{\log(8x)}$

Find the domain of each of the following functions.

26. $f(x) = \sqrt{2x-3}$
27. $f(x) = 3^x$
28. $f(x) = \left(\frac{1}{3}\right)^x$
29. $f(x) = \log_6(2x+4)$
30. $f(x) = \log(6-2x)$
31. $f(x) = \ln(x+1)^2$

32. State the product rule, quotient rule, and power rule for logarithms. (See section 3, chapter 12.)

Use the appropriate properties of logarithms to expand each logarithm as much as possible. Where possible, evaluate logarithm expressions without using a calculator.

33. $\log_2 \frac{32}{x}$

34. $\ln \frac{5}{e^2}$

35. $\log_5 \frac{25}{\sqrt{x+1}}$

36. $\log_b \frac{x^2 y}{z+1}$

37. $\log_b x(2x-1)^2$

Write as a single logarithm whose coefficient is 1. Where possible, evaluate logarithm expressions without using a calculator.

38. $3\log_b x + 2\log_b y$

39. $\frac{1}{5}\ln x + 2\ln y$

40. $6\log_b(x+1) - 3\log_b y$

41. $3\log_2 x + \frac{1}{2}\log_2(y+1)$

Solve each equation using an appropriate method. Do not use a calculator, and give all answers in exact form without logarithms, where possible.

42. $4^x = 16$

43. $4^x = 32$

44. $4^x = \frac{1}{4}$

45. $25^x = 5$

46. $6^{2x+1} = 36$

For this portion of the review, you may use your calculator. Follow all directions carefully.

Use your calculator to approximate the following to the nearest thousandth (three decimal places).

47. $4^{2.1}$
48. $3^{\sqrt{2}}$
49. $e^{-1.25}$
50. $\log_2 17$
51. $\log_{0.5} 2.1$
52. $\log 5.1$
53. $\ln 4.8$

Solve each equation by using an appropriate method. After you have gotten an exact answer, use your calculator to approximate the answer to the nearest thousandth (three decimal places).

54. $5^x = 3$
55. $10^x = 8.7$
56. $e^{2x} = 6.1$
57. $20 - 2.1^x = 0$
58. $10^{3x-1} = 3.7$
59. $4^{x+1} = 5^x$
60. $\log_2(3x+1) = 7$
61. $\ln x = -3$
62. $\log_5 x - \log_5(4x-1) = 1$
63. $\log_4(x+2) - \log_4(x-1) = 1$
64. $6\ln 2x = 30$
65. $\ln\sqrt{x+4} = 1$

Solve each of the following application problems. For each problem, you may use your calculator but must show enough work to outline your solution strategy.

66. How much will an investment of $15,000 be worth in 30 years if the annual interest rate is 5.5% and compounding is

a. quarterly?
b. monthly?
c. continuously?

67. (P. 806) The formula $S = C(1+r)^t$ models inflation, where C is the value today, r is the annual inflation rate, and S is the inflated value t years from now. If the inflation rate is 8%, how much will a house now worth $395,000 be worth in 10 years?

68. (P. 851) The function $P(x) = 95 - 30\log_2 x$ models the percentage, P(x), of students who could recall the important features of a lecture as a function of time, where x represents the number of days that have elapsed since the lecture was given.

a. After how many days have the 30% of the students forgotten the important features of the lecture. (Hint: If 30% have forgotten, then 70% could recall.)

b. What percentage of the students recall the important features of the lecture after one week has passed?

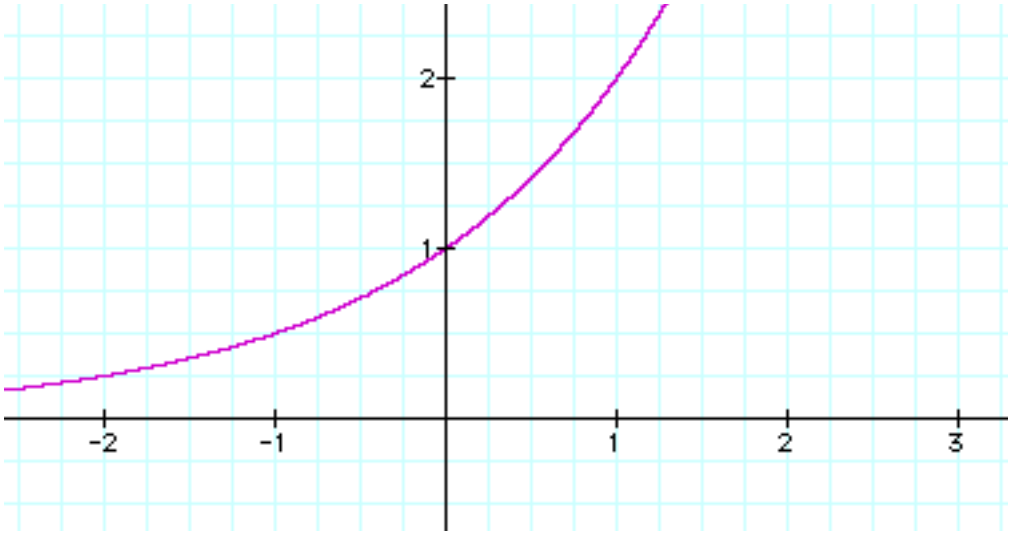
Answers:

1-7. See graphs on following pages.	15. $\frac{1}{2}$
5. shift left 2 units H.A. y=0	16. –2
6. shift up 2 units H.A. y=2	17. $\frac{1}{2}$
7. shift right 1 unit and up 3 units H.A. y=3.	18. 3
8. $(f \circ g)(x) = 14x^3 + 5,$ $f^{-1}(x) = \frac{x+2}{7},\ g^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$	19. 6
9. f(g(x)) = x and g(f(x)) = x so f and g are inverses.	20. 1
10. f(g(x)) = x and g(f(x)) = x so f and g are inverses.	21. 1
11. Use the horizontal line test. HLT: A function f has an inverse function if there is no horizontal line that intersects the graph of the function f at more than one point.	22. $\frac{1}{2}$
12a. Yes, the function has an inverse because no horizontal line intersects its graph in more than one point..	23. 300
12b. No, the function does not have an inverse because at least one (in fact, many) horizontal lines intersect the graph in more than one point.	24. 11x + 1
12c. Yes, the function has an inverse because no horizontal line intersects its graph in more than one point..	25. 8x
12d. Yes, (Same reason as 12c).	26. $\left\{x \middle\| x \geq \frac{3}{2}\right\} = \left[\frac{3}{2}, \infty\right)$
13. 0	27. $\{x \mid \text{all real numbers}\} = (-\infty, \infty)$
14. 5	28. $\{x \mid \text{all real numbers}\} = (-\infty, \infty)$

29. $\{x \mid x > -2\} = (-2, \infty)$	51. –1.070
30. $\{x \mid x < 3\} = (-\infty, 3)$	52. 0.708
31. $\{x \mid x \neq -1\} = (-\infty, -1) \bigcup (-1, \infty)$	53. 1.569
32. See text, ch12, section 3	54. $\left\{\frac{\log(3)}{\log(5)} = \frac{\ln(3)}{\ln(5)} \approx 0.683\right\}$
33. $5 - \log_2(x)$ 34. $\ln(5) - 2$	55. $\{\log(8.7) \approx 0.94\}$
35. $2 - \frac{1}{2}\log_5(x+1)$	56. $\left\{\frac{\ln(6.1)}{2} \approx 0.904\right\}$
36. $2\log_b(x) + \log_b(y) - \log_b(z+1)$ 37. $\log_b(x) + 2\log_b(2x-1)$	57. $\left\{\frac{\log(20)}{\log(2.1)} = \frac{\ln(20)}{\ln(2.1)} \approx 4.038\right\}$
38. $\log_b(x^3 y^2)$ 39. $\ln\left(y^2 \sqrt[5]{x}\right)$	58. $\left\{\frac{1 + \log(3.7)}{3} \approx 0.523\right\}$
40. $\log_b\left[\frac{(x+1)^6}{y^3}\right]$	59. $\left\{\frac{\log(4)}{\log(5) - \log(4)} = \frac{\ln(4)}{\ln(5) - \ln(4)} \approx 6.213\right\}$
41. $\log_2\left(x^3 \sqrt{y+1}\right)$	60. $\left\{\frac{2^7 - 1}{3} \approx 42.333\right\}$
42. $\{2\}$	61. $\{e^{-3} \approx 0.05\}$
43. $\left\{\frac{5}{2}\right\}$	62. $\left\{\frac{5}{19} \approx 0.263\right\}$
44. $\{-1\}$	63. $\{2\}$
45. $\left\{\frac{1}{2}\right\}$	64. $\left\{\frac{e^5}{2} \approx 74.207\right\}$
46. $\left\{\frac{1}{2}\right\}$	65. $\{e^2 - 4 \approx 3.389\}$
47. 18.379	66a. $77,231.65
48. 4.729	66b. $77,810.52 66c. $78, 104.70
49. 0.287	67. $852,775.37
50. 4.087	68a. 1.8 days 68b. 11% of students recall

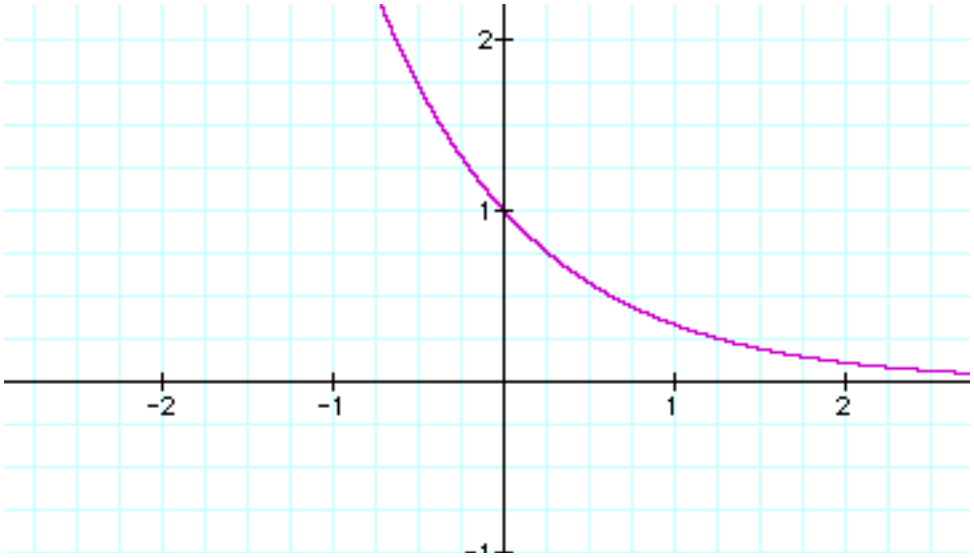
1. $f(x) = 2^x$, H.A. y = 0

x	y
-1	1/2
0	1
1	2



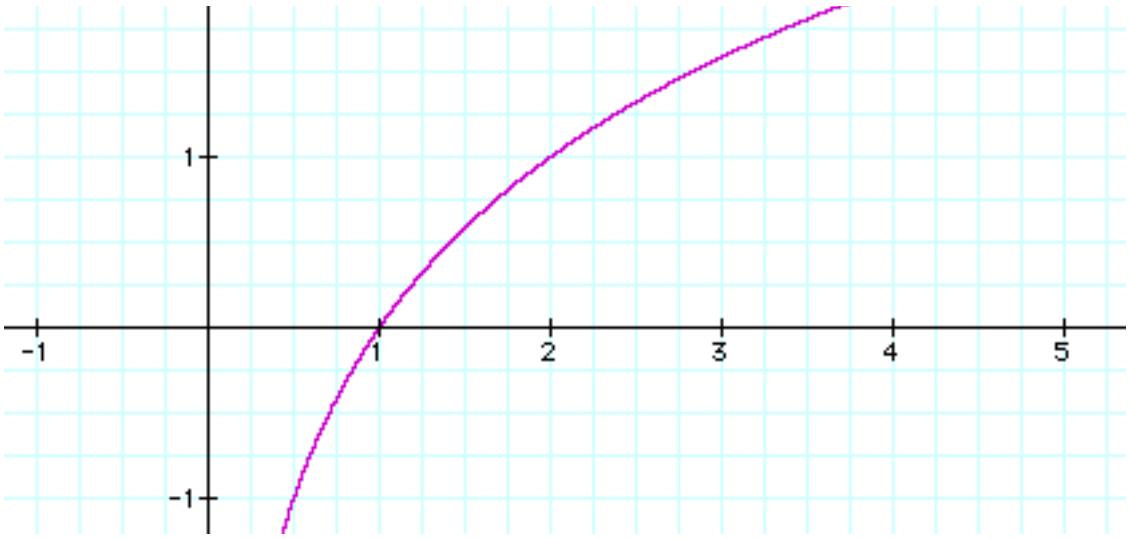
2. $g(x) = \left(\frac{1}{3}\right)^x$, H.A. y = 0

x	y
-1	3
0	1
1	1/3



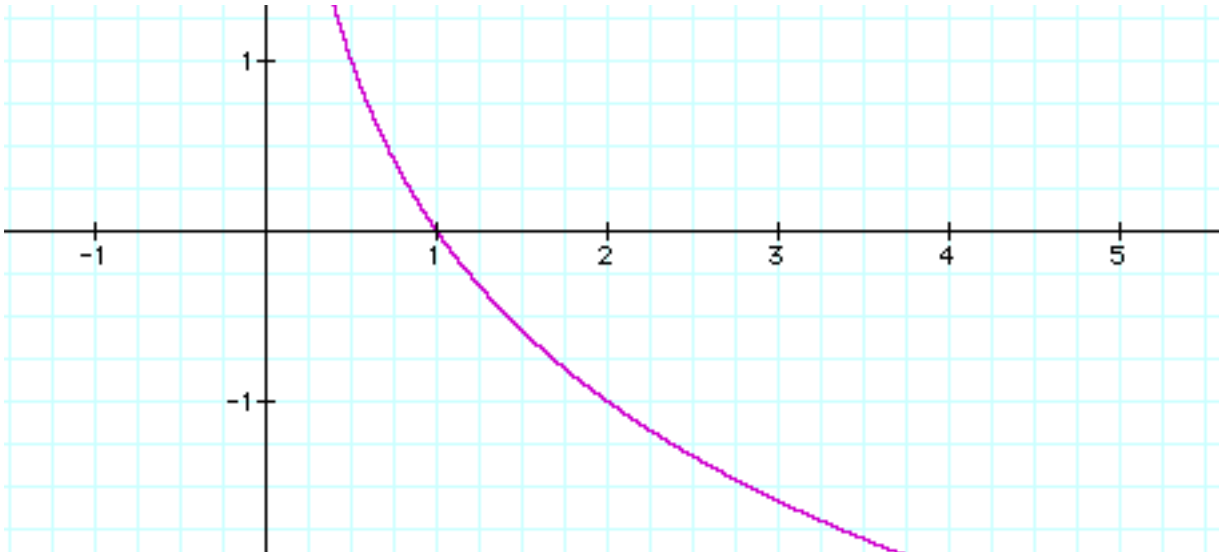
3. $f(x) = \log_2 x$, V.A. x = 0

x	y
1/2	-1
1	0
2	1



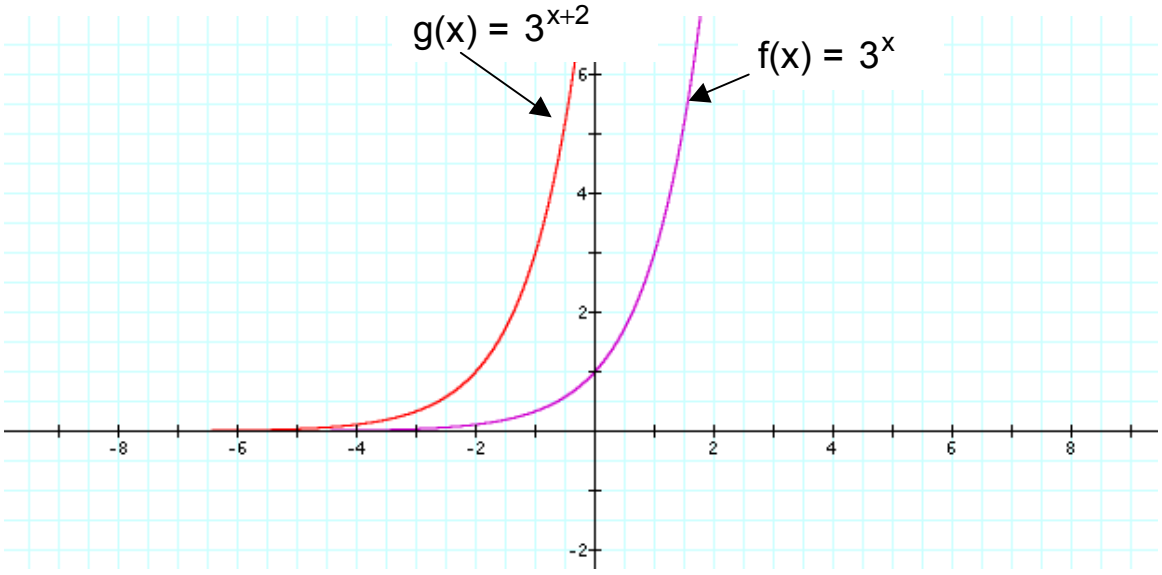
4. $g(x) = \log_{\frac{1}{2}} x$, V.A. x = 0

x	y
2	-1
1	0
1/2	1



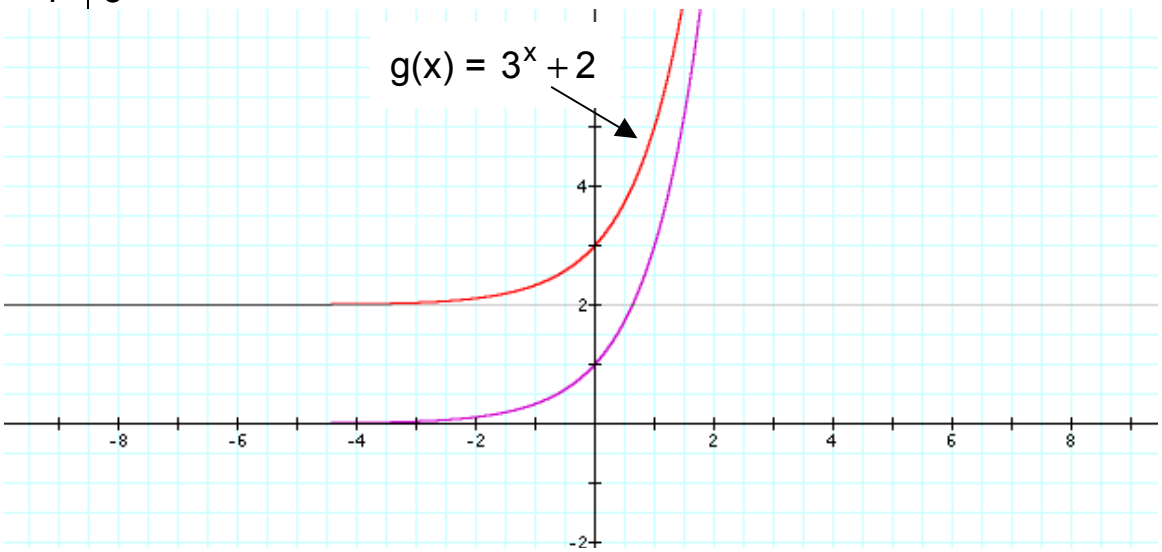
5. $f(x) = 3^x$, $g(x) = 3^{x+2}$. Shift left two units. Horizontal asymptote does not move—it remains y = 0.

x	g(x)
-3	1/3
-2	1
-1	3



6. $f(x) = 3^x$, $g(x) = 3^x + 2$. Shift up two units. Horizontal asymptote moves up two units to y = 2.

x	g(x)
-1	5/3
0	3
1	5



7. $f(x) = 3^x$, $g(x) = 3^{x-1} + 3$. Shift right one unit and up three units. Horizontal asymptote moves up two units to y = 3.

x	g(x)
0	10/3
1	4
2	6