MATH 155 - Chapter 8.7 - Indeterminate Forms and L'Hopital's Rule

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1. **Indeterminate Forms:** Indeterminate forms are of the form

$$\frac{0}{0}, \quad \frac{\pm\infty}{\pm\infty}, \quad 0\cdot\infty, \quad 1^{\infty}, \quad 0^{0}, \quad \infty-\infty$$

Note: The followings are determinate.

$\infty+\infty=\infty.$

$-\infty-\infty=-\infty.$

$0^{\infty}=0.$

$0^{-\infty}=\frac{1}{0^{\infty}}=\frac{1}{0}=\infty.$

2. **Theorem: L'Hopital's Rule:**

Let f and g be functions that are differentiable on an open interval (a,b) containing c, except possibly at c itself. Assume that $g'(x)\neq 0$ for all x in (a,b), except possible at c itself. If the limit of $\frac{f(x)}{g(x)}$ as x approaches c produces the indeterminate forms $\frac{0}{0}$, or $\frac{\pm\infty}{\pm\infty}$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$

L'Hopital's Rule can also be applied to one-sided limits. ie.

$$\lim_{x\to c^{\pm}}\frac{f(x)}{g(x)}=\lim_{x\to c^{\pm}}\frac{f'(x)}{g'(x)}$$